Solution of chap. 24.

24.3.

\[ E_{inc} = \frac{1}{2} E_{01} \left( e^{i \omega_1 t} + c.c. \right) + \frac{1}{2} E_{02} \left( e^{i \omega_2 t} + c.c. \right) + \frac{1}{2} E_{03} \left( e^{i \omega_3 t} + c.c. \right) \]

\[ P_2 \propto \chi_2 E_{inc}^2 \]

so the second-order polarization is proportional to \( E_{inc}^2 \)

which is

\[ E_{inc}^2 = \frac{1}{4} E_{01} \left( e^{i(2 \omega_1 t)} + e^{i(-2 \omega_1 t)} + 2 \right) + \frac{1}{4} E_{02} \left( e^{i(2 \omega_2 t)} + e^{i(-2 \omega_2 t)} + 2 \right) + \frac{1}{4} E_{03} \left( e^{i(2 \omega_3 t)} + e^{i(-2 \omega_3 t)} + 2 \right) \]

\[ + \frac{1}{2} E_{01} E_{02} \left( e^{i(\omega_1 + \omega_2) t} + e^{i(\omega_1 - \omega_2) t} + c.c. \right) \]

\[ + \frac{1}{2} E_{01} E_{03} \left( e^{i(\omega_1 + \omega_3) t} + e^{i(\omega_1 - \omega_3) t} + c.c. \right) \]

\[ + \frac{1}{2} E_{02} E_{03} \left( e^{i(\omega_2 + \omega_3) t} + e^{i(\omega_2 - \omega_3) t} + c.c. \right) \]

2. possible radiating frequencies are:

0 (DC), 2\( \omega_1 \), 2\( \omega_2 \), 2\( \omega_3 \) (SHG) \( \omega_1 + \omega_2 \), \( \omega_1 + \omega_3 \), \( \omega_2 + \omega_3 \) (SFG)

\( |\omega_1 - \omega_2|, |\omega_1 - \omega_3|, |\omega_2 - \omega_3| \) (DFG)

24.4. (24.7): \[ \frac{1}{\eta^2} = \frac{1}{\eta_0^2} + r \mathcal{E} \]

(linear EO effect)

for crystals who are \textit{inversely} inversion-symmetric

\[ \frac{1}{\eta^2}(+) = \frac{1}{\eta^2}(-) \]

\[ \frac{1}{\eta_0^2} + r \mathcal{E} = \frac{1}{\eta^2} - r \mathcal{E} \]

\[ r = 0 \]
This means that there is no linear EO effect in inversion-symmetric crystals.

24.5.

a) Coherent length defined as

\[ L_c = \frac{\lambda}{4 \Delta n} = \frac{6.94 \text{ nm}}{4 \times 1.834 - 1.805} = 3.98 \text{ cm} \]

b) \[ \Delta n = \frac{4L_c}{\lambda_0} = \frac{1.06 \text{ cm}}{4 \times 0.3 \text{ cm}} = 0.045 \]

Without phase matching, the crystal must be kept within \( L_c \).

24.6. From Table 24-2, for ADP @ 546 nm,

\[ r = 8.56 \text{ pm/V}, \quad n_0 = 1.48 \]

\[ V_{\text{HN}} = \frac{\lambda_0}{2n_0^3} = \frac{546 \times 10^{-9} \text{ m}}{2 \times 8.56 \times 10^{-12} \text{ m/V} \cdot (1.48)^3} = 9.34 \text{ KV} \]

Can't determine its length.

24.8. Pockel's effect:

\[ J = I_{\text{max}} \sin^2 \left( \frac{\pi V}{2V_{\text{HN}}} \right) \]

\[ V_{\text{HN}} = \frac{\lambda_0}{2n_0^3} \quad J = I_{\text{max}} \sin^2 \left( \frac{\pi V}{2} \cdot \frac{2n_0^3}{\lambda_0} \right) \]

\[ = I_{\text{max}} \sin^2 \left( \frac{\phi}{2} \right) \]
a) \( I_0 \) if \( \phi/2 = m\pi \) \((m=1,2,3,...)\)

\[
\phi = 2m\pi,
\]

Corresponding to \( V = 2mV_{\text{th}} \).

b) A THWP introduces another \( \pi \) phase difference between the two polarization components.

\[
\Phi = \frac{\pi V}{V_{\text{th}}}, \pi
\]

\( V=0 \), \( \Phi = \pi \), \( I = I_{\text{max}} \sin^2(\phi/2) = I_{\text{max}} \)

\( V=V_{\text{th}} \), \( \Phi = 2\pi \), \( I = I_{\text{max}} \sin^2(\Phi/2) = 0 \)

24-10, Kerr effect:

\[
V_{\text{th}} = \frac{d}{\sqrt{2KL}} \quad K: \text{Kerr constant}
\]

From Table 24-3, \( K = \frac{0.036 \times 10^{-12} \text{ m/V}^2}{\text{CS}_2} \)

\[
L = \frac{(d/V_{\text{th}})^2}{2K} = \frac{(1.5 \times 10^2 \text{ m}/36 \times 10^3 \text{ V})^2}{2 \times 0.036 \times 10^{-12} \text{ m/V}^2} = 3.47 \text{ m}
\]

It's not a practical cell.
Eq. (24-17)

\[ \Delta \omega = \omega_s = \frac{2\pi V_s}{\lambda_s} \]

\( V_s \): Speed of sound in medium

Bragg condition relates \( \lambda_s \) to \( \lambda \) (light in medium)

as \( \lambda = 2\lambda_s \sin \theta \)

\[ \Delta \omega = \omega_s = \frac{2\pi V_s \cdot 2\sin \theta}{\lambda} = \frac{4\pi V_s \sin \theta}{\lambda} \]

From a Doppler shift point of view,

\[ \Delta \nu = \frac{2\nu_u}{\nu} = \left( \frac{c}{\lambda_o} \cdot \nu_u \right) \]

\( \nu_u = V_s \cdot \sin \theta \)

\[ \Delta \omega = 2\pi \Delta \nu = \frac{2\pi \cdot 2 \cdot V_s \sin \theta}{\lambda_o / \nu} = \frac{4\pi V_s \sin \theta}{\lambda} \]

Thus the equivalence is shown.

24-14.

\[ V_s = \frac{V_s}{\lambda_s} \quad \lambda = \lambda_s \cdot 2 \sin \frac{\phi_p}{2} \quad \phi_D = 1 \times 7/180 \]

\[ V_s = \frac{V_s \cdot 2 \sin \frac{\phi_p}{2}}{\lambda_o / \nu} = 110 \text{ m Hz} \]
24.15:

\[ \theta_0 \]

\[ \lambda = 2 \lambda_0 \sin \theta \]

\[ \lambda_0 / n = 2 \lambda_0 \sin \theta_0 \]

\[ \lambda_0 = 2 \lambda_0 \sin \theta_0 \cdot n \]

Snell's law: \( n \sin \theta = \sin \theta_0 \).

\[ \therefore \lambda_0 = 2 \lambda_0 \sin \theta_0 \]

24.18. \( \beta = VBd \) : \( V = \beta / Bd \)

\( \beta = 3098 \) G, \( d = 2.73 \) cm

a) \( \lambda = 632.8 \) nm.

\[ V = \frac{900 \text{ m/s}}{3098 \text{ G} \cdot 2.73 \text{ cm}} = 0.0647 \text{ m/s} \]

b) \( \lambda = 543.5 \) nm.

\[ V = \frac{1330 \text{ m/s}}{3098 \text{ G} \cdot 2.73 \text{ cm}} = 0.956 \text{ m/s} \]

24.20.

Pulse broadening is due to the fact that the blue part of the pulse travels at a different velocity than the red part of the pulse. In normal dispersion, red parts leads the blue part.
Ordinary mirror reflects red first before the blue part. However, a PCM will allow the blue part to travel less optical path length than the red part, thus compensating for its slower speed.

Pulse coming out.