Solution to HW#11

May 7, 2007

23-15

a) The critical angle is $\theta_c = \sin^{-1}(n_2/n_1) = 43.8^\circ$.
The polarizing angle for external reflection is $\theta_p = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.458/1) = 55.6^\circ$.
The polarizing angle for internal reflection is $\theta'_p = \tan^{-1}(1/1.458) = 34.4^\circ$.

b) At normal incidence:

\[ R_{TE} = r_{TE}^2 = \left(\frac{1 - n}{1 + n}\right)^2 = \left(\frac{1 - 1.458}{1 + 1.458}\right)^2 = 0.0347 \]

\[ T_{TE} = 1 - R_{TE} = 0.9653 \]

At $\theta = 45^\circ$,

\[ R_{TM} = \left(\frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}}\right)^2 = 0.0821 \]

\[ T_{TE} = 1 - R_{TE} = 0.9179 \]

c) At normal incidence: $R_{TM} = R_{TE} = 0.0347$, $T_{TE} = 0.9653$

At $\theta = 45^\circ$,

\[ R_{TM} = \left(\frac{-n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}}\right)^2 = 0.0067, \quad T_{TM} = 0.9933 \]
d) Use Eqns. (23-28) and (23-29):

For incident angles less than $\theta_p'$, $\phi_{TM} = \phi_{TE} = 0$. So for $\theta = 0^\circ, 20^\circ$, the phase difference between TE and TM modes is zero.

For $\theta = 40^\circ$ which lies between $\theta_p'$ and $\theta_c'$, $\phi_{TM} = \phi_{TE} = \pi$.

For $\theta > \theta_c$, ($n = 1/1.458$)

$$\phi_{TM} = \phi_{TE} = -2\tan^{-1}\left(\frac{\sin^2 \theta - n^2}{n^2 \cos \theta}\right) + \pi + 2\tan^{-1}\left(\frac{\sin^2 \theta - n^2}{\cos \theta}\right)$$

- $\theta = 50^\circ$: $\phi_{TM} = \phi_{TE} = 139^\circ$
- $\theta = 70^\circ$: $\phi_{TM} = \phi_{TE} = 152^\circ$
- $\theta = 90^\circ$: $\phi_{TM} = \phi_{TE} = 180^\circ$

23-18 Using Eq. (23-65), for the TE case:

$$R_{TE} = \left|\frac{\cos \theta - \sqrt{(n_R^2 - n_I^2 - \sin^2 \theta) + i(2n_Rn_I)}}{\cos \theta + \sqrt{(n_R^2 - n_I^2 - \sin^2 \theta) + i(2n_Rn_I)}}\right|^2$$

and for the TM case,

$$R_{TM} = \left|\frac{-[n_R^2 - n_I^2 + i(2n_Rn_I)]\cos \theta + \sqrt{(n_R^2 - n_I^2 - \sin^2 \theta) + i(2n_Rn_I)}}{[n_R^2 - n_I^2 + i(2n_Rn_I)]\cos \theta + \sqrt{(n_R^2 - n_I^2 - \sin^2 \theta) + i(2n_Rn_I)}}\right|^2$$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$R_{TE}$</th>
<th>$R_{TM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi/6$</td>
<td>0.847</td>
<td>0.801</td>
</tr>
<tr>
<td>$\pi/3$</td>
<td>0.909</td>
<td>0.695</td>
</tr>
</tbody>
</table>

23-20 Consider Eqn. (23-47):

$$1 = r^2 + n \left(\frac{\cos \theta_l}{\cos \theta}\right) t^2, \quad n = n_2/n_1$$

a) External Reflection: $n > 1$; $\theta_l < \theta_i$; $\cos \theta_l > \cos \theta$. Thus,

$$1 = r^2 + n \left(\frac{\cos \theta_l}{\cos \theta}\right) t^2 > t^2 \Rightarrow t^2 < 1 - r^2,$$  since $r^2 < 1$, $t^2 < 1$

Internal Reflection: $n < 1$; $\theta_l > \theta_i$; $\cos \theta_l < \cos \theta$. Thus,

$$1 = r^2 + n \left(\frac{\cos \theta_l}{\cos \theta}\right) t^2 < t^2 \Rightarrow t^2 > 1 - r^2, \quad \text{No upper limit}$$
b) For the angle of incidence equal to the critical angle, \( \sin \theta = \sin \theta_c = n, \)

\[
t_{TE} = \frac{2 \cos \theta}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}} = \frac{2 \cos \theta}{\cos \theta} = 2
\]

\[
t_{TM} = \frac{2n \cos \theta}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}} = \frac{2}{n}
\]

Figure 1: 23-20 part c

23-21

a) The penetration depth is

\[
|z|_{1/e} = \frac{1}{\alpha} = \frac{\lambda}{2\pi} \frac{1}{\sqrt{\sin^2 \theta / n^2 - 1}} = 0.164 \ \mu m
\]

b) Since irradiance is proportional to the square of the field amplitude and with \( \alpha = 1/|z|_{1/e} = 6.089 \ \mu m^{-1}, \)

\[
\frac{I}{I_0} = e^{-2\alpha|z|} = 5.1 \times 10^{-6}
\]