The transmission function of a Ronchi ruling can be Fourier expanded as

\[
\tau(x) = \frac{1}{2} + \frac{1}{\pi} \left[ e^{i2\pi(x/d)} + e^{-i2\pi(x/d)} \right] \\
- \frac{1}{3\pi} \left[ e^{i2\pi(3x/d)} + e^{-i2\pi(3x/d)} \right] \\
+ \frac{1}{5\pi} \left[ e^{i2\pi(5x/d)} + e^{-i2\pi(5x/d)} \right] + \ldots
\]

where the exponentials represents plane waves of different propagation directions. (e.g. \( e^{i2\pi(3x/d)} = e^{i\frac{2\pi}{\lambda}(3x\lambda/d)} = e^{i\frac{2\pi}{\lambda} \cdot 3\alpha}, \alpha = \lambda/d \))

\( a) \) first three orders:

\[
Y_1 = \alpha \cdot f = \frac{\lambda}{d} f = \frac{6.328 \times 10^{-4} \ mm \times 500 \ mm}{0.5 \ mm} = 0.6328 \ mm
\]

\[
Y_3 = 3\alpha \cdot f = 3\frac{\lambda}{d} f = 3 \cdot \frac{6.328 \times 10^{-4} \ mm \times 500 \ mm}{0.5 \ mm} = 1.8984 \ mm
\]

\[
Y_5 = 5Y_1 = 3.1640 \ mm
\]

\( b) \) Fundamental spatial frequency \( f_X = 1/d = 2 \ mm^{-1} \) (look at the first exponential term in the Fourier expansion). The wavelength \( \lambda_X = d = 0.5 \ mm \).

\( c) \) Angular spatial frequencies,

\[
2\pi f_{X1} = \frac{2\pi}{d} = 12.56 \ mm^{-1}
\]
\[ 2\pi f_X^3 = 3 \frac{2\pi}{d} = 37.68 \text{ mm}^{-1} \]
\[ 2\pi f_X^5 = 5 \frac{2\pi}{d} = 62.83 \text{ mm}^{-1} \]

d) According to the Fourier expansion of the transmission function, the field strength associated with the first three non-vanishing orders are 1 : 1/3 : 1/5. The ratio of intensity is thus 1 : 1/9 : 1/25.

21-5 By definition of a Fourier transform,
\[ \mathcal{F} [h(x)] = \int h(x)e^{ikx} \, dx \]
Substitute for \( h(x) \) the convolution of \( f \) and \( g \):
\[ \mathcal{F} [h(x)] = \int e^{ikx} \left[ \int f(x-x')g(x') \, dx' \right] \, dx \]
Changing the order of integration,
\[ \mathcal{F} [h(x)] = \int g(x') \left[ \int f(x-x')e^{ikx} \, dx \right] \, dx' \]
Let \( v = x - x' \) so that \( dx = dv \):
\[ \mathcal{F} [h(x)] = \int g(x') \left[ \int f(v)e^{ik(v+x')} \, dv \right] \, dx' \]
so,
\[ \mathcal{F} [h(x)] = \int f(v)e^{ikv} \, dv \cdot \int g(x')e^{ikx'} \, dx' = \mathcal{F} [f(x)] \cdot \mathcal{F} [g(x)] \]

21-6 The two functions look like: Here, \( \rho_{11}(u) = \int f(x-u)f(x) \, dx \). As \( u \) varies, \( \rho_{11}(u) \) measures the area of overlap of the two square waves. For example,
\[ \rho_{11}(0) = \int_{-3}^{3} f(x)^2 \, dx = [3 - (-3)] \cdot 1 = 6 \]
\[ \rho_{11}(3) = \int_{-3}^{3} f(x)f(x-3) \, dx = (3 - 0) \cdot 1 = 3 \]
For \( u \geq 6 \), the integral will yield zero since there will be no overlap between the two functions. Note that \( \rho_{11}(u) \) is symmetric, the triangle function can be derived. (This problem can also be solved by the convolution theorem and using the contour integral to do the inverse Fourier transform.)
21-10

a) During one pass, the total mirror movement is

\[ x = (71.5 \, \text{nm/s})(\text{time}) = 71.5 \, \text{nm/s} \cdot \frac{256}{(1.28 \, \text{readings/s})} = 14300 \, \text{nm} \]

\[ x_w = 2x = 2.86 \times 10^3 \, \text{cm} \]

b) \n
\[ \lambda_{\text{min}} = \frac{\lambda^2}{x_w} = \frac{(400 \times 10^{-7} \, \text{cm})^2}{2.86 \times 10^{-3} \, \text{cm}} = 5.59 \times 10^{-7} \, \text{cm} = 5.59 \, \text{nm} \]

c) To avoid aliasing,

\[ \lambda_{\text{min}} = \frac{2(2.86 \times 10^{-3} \, \text{cm})}{255} = 224 \, \text{nm} \]

d) To resolve \( \lambda = 360 \, \text{nm} \) (set by the filter),

\[ \lambda_s = \frac{2V}{\text{sampling rate}} \]

peak frequency = \[ \frac{2V}{\lambda_s} = 0.397 \, \text{reading/s} \]

According to Nyquist’s sampling theory, sample rate must be at least twice as that, thus 0.8 \( \text{reading/s} \)