Solution to HW#1

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14-6 Generally, \( \vec{E} = \vec{E}_\parallel e^{i(\vec{k} \cdot \vec{r} - \omega t)} \), \( \vec{r} = (x, y, z) \)

a) \( \vec{E}_\parallel = E_{0y} \hat{y} + E_{0z} \hat{z} \), \( \vec{k} = k \hat{x} \),
\( \vec{E} = (E_{0y} \hat{y} + E_{0z} \hat{z}) e^{i(\omega t - kx)} = E_0 (\sqrt{3}/2 \hat{y} + 1/2 \hat{z}) e^{i(\omega t - kx)} \), Jones Vector:
\[
\begin{bmatrix}
\sqrt{3}/2 \\
1/2
\end{bmatrix}
\]

b) \( \vec{E}_\parallel = E_{0x} \hat{x} + E_{0z} \hat{z} \), \( \vec{k} = k \hat{y} \), \( |E_{0x}|/|E_{0z}| = 1/2 \), right-elliptically polarized, Jones vector has the form of \( \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -i \end{bmatrix} \),
\( \vec{E} = E_0 (2/\sqrt{5} \hat{z} - i/\sqrt{5} \hat{x}) e^{i(ky - \omega t)} \)

c) \( \vec{E}_\parallel = E_0 \hat{z} \), \( \vec{k} = \sqrt{2}/k (\hat{x} + \hat{y}) \),
\( \vec{E} = E_0 \hat{z} e^{i(\sqrt{2}/k (\hat{x} + \hat{y}) - \omega t)} \), Jones Vector: \( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \)

14-9

a) \( \begin{bmatrix} 3i \\ i \end{bmatrix} = \sqrt{10} i \begin{bmatrix} 3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix} \) Linear (no phase difference). Amplitude: \( \sqrt{10} \),
polarization angle w.r.t. x axis: \( \alpha = \arctan(1/3) = 18.4^\circ \)

b) \( \begin{bmatrix} i \\ 1 \end{bmatrix} = i \begin{bmatrix} 1 \\ -i \end{bmatrix} \) Amplitude: 1 (for CP light, the amplitude refers to the field strength in either one of the two orthogonal components), RCP (the minus sign before i)
c) $\begin{bmatrix} 4i \\ 5 \end{bmatrix} = i \begin{bmatrix} 4 \\ -5i \end{bmatrix}$ right elliptical (the minus sign before $i$), major axis=5, along y-axis, minor axis=4, along x-axis.

d) $\begin{bmatrix} 5 \\ 0 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ Linear (no phase difference), along x-axis. Amplitude:5

e) $\begin{bmatrix} 2 \\ 2i \end{bmatrix} = 2 \begin{bmatrix} 1 \\ i \end{bmatrix}$ LCP (the plus sign before $i$), Amplitude:2

f) $\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \sqrt{13} \begin{bmatrix} 2/\sqrt{13} \\ 3/\sqrt{13} \end{bmatrix}$ Linear (no phase difference). Amplitude:$\sqrt{13}$. Polarization angle w.r.t. x-axis: $\alpha = \arctan(3/2) = 56.31^\circ$

g) $\begin{bmatrix} 2 \\ 6 + 8i \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 3 + 4i \end{bmatrix}$ Left-elliptically polarized (the plus sign before $i$). the relative phase between the x, y components of the electric field is $\Phi_y - \Phi_x = \arctan(4/3) = 53.13^\circ$

$E_{0x} = 2$, $E_{0y} = 2\sqrt{3^2 + 4^2} = 10$

inclination angle of the axis: $\alpha = 1/2\arctan(\frac{2\times1\times5\cos53.13^\circ}{1-25}) = -7^\circ$

14-12 At the reflection surface (normal incidence), there is NO additional phase difference between the TE and TM fields (the polarization parallel and perpendicular to the principle plane). (Fresnel’s equations, Pedrotti page 501 figure 23-8, the external reflection case, angle of incidence = 0).

The incident field on the QWP has equal x and y components. (45 degree between TA of the first polarizer and the main axis of the QWP). Suppose the y-component aligns with the fast axis of the QWP, it will acquire a $\pi/2$ phase advance, after reflection, the y-component still aligns with the fast axis of the QWP, the phase difference becomes $\pi$. A $\pi$ phase difference between the two polarizing components will result in a polarization perpendicular to the initial state, thus, no light will get through the linear polarizer.

In the language of Jones matrices, it can be explained mathematically, \[
\begin{bmatrix}
1 & 1 \\
1 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & -i
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & -i
\end{bmatrix}
\begin{bmatrix}
1 \\
i
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

Here, I started with the polarization right after the first polarizer.

14-15
a) The starting polarization can be expressed as \( \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} \) Jones matrix for the QWP with horizontal SA is \( \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \), the Jones vector after the QWP can be calculated as \( \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 \\ -1/2i \end{bmatrix} \), which is right elliptically polarized with major axis along the x-axis.

b) Jones matrix for the linear polarizer with vertical TA is \( \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \) so the output polarization can be calculated as \( \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 \\ -1/2i \end{bmatrix} = \begin{bmatrix} 0 \\ -1/2i \end{bmatrix} \), which is linearly polarized along y-axis.