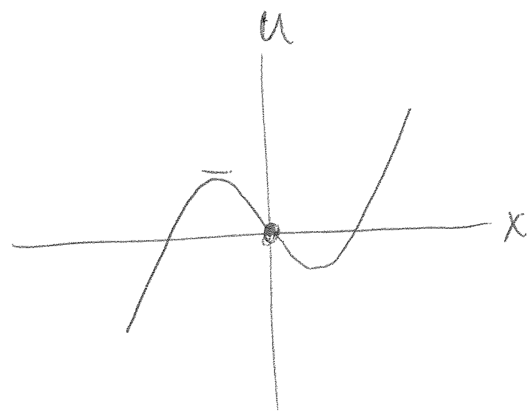
Quiz 6 **Don't forget units.** $\omega = \sqrt{\frac{k}{m}}$ Name SOLUTIONS

1. Consider a 1 dimensional potential U(x) = x³ – 3x (in J). A mass = 1 kg moves in this potential.
a) Where are the equilibrium point(s)?

$\frac{dU}{dx} = 0 = 3x^2 - 3 \quad x^2 = 1 \quad x = \pm 1.$



b) Is each stable or unstable?

$\frac{d^2U}{dx^2} = 6x$ at $x = +1$ stable

$x = -1$ unstable

c) What is the frequency of small oscillations about a stable equilibrium point?

Recall $k = U''$ at min.

$\omega = \sqrt{\frac{6}{1}}\ s^{-1}. \quad f = \frac{1}{2\pi}\sqrt{6}$ cycles/s

d) If the particle is launched from the origin, in the +x direction, what speed would it need to escape to *either* + or – infinity? (Or can it never escape?)

Height of potential hill $U(-1) = 1 - -3 = 2J.$

$\frac{1}{2}mv^2 = 2$

$v^2 = 4$

$v = 2 m/s.$

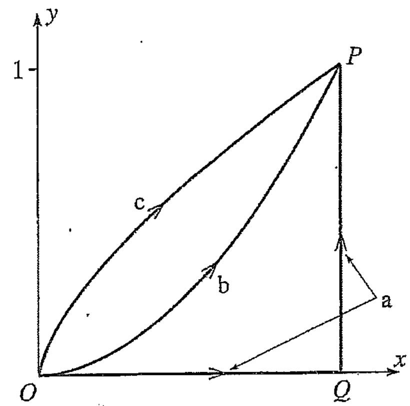
-OVER-

2.

Evaluate the work done

$$W = \int_O^P \mathbf{F} \cdot d\mathbf{r} = \int_O^P (F_x\,dx + F_y\,dy)$$

by the 2-dimensional force F = (x^3, 2xy) along the path c, given parametrically as $x=t^3$, $y=t^2$.



$= 1$

Since
$x = t^3$
$y = t^2$,
when $y = 1$,
$x = 1$
$t = 1$.

$F_x dx = F_x \frac{dx}{dt} dt$ etc.

$$W = \int_0^1 (t^3)^3 \cdot 3t^2\,dt + 2t^3 \cdot t^2 \cdot 2t\,dt$$

$$= \int_0^1 (3t^{11} + 4t^6)\,dt$$

$$= \frac{1}{4}t^{12} + \frac{4}{7}t^7 \Big|_0^1$$

$$= \frac{1}{4} + \frac{4}{7}$$

$$= \frac{23}{28}$$