Useful Equations

$$\boxed{\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta}$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

PHYC303 Quiz 2 Name SOLUTIONS

1. (3pt) A particle moves in a plane along a path $r = k\theta$ at constant speed v, starting from the origin.

What is $\dot{r}$ when $r=k$?

$v^2 = \dot{r}^2 + r^2\dot{\theta}^2$. BUT $\dot{r} = k\dot{\theta}$ so $v^2 = \dot{r}^2 + \frac{r^2}{k^2}\dot{r}^2$

When $r=k$, $v^2 = 2\dot{r}^2$

$\dot{r} = \frac{v}{\sqrt{2}}$

2. (3pt) Evaluate the integral $\int \vec{A}\cdot\ddot{\vec{A}}\,dt$, ***given that the vector $\dot{\vec{A}}$ has constant length*** *(call it v.)*

$$\frac{d}{dt}(\vec{A}\cdot\dot{\vec{A}}) = \vec{A}\cdot\ddot{\vec{A}} + \dot{\vec{A}}\cdot\dot{\vec{A}} = \vec{A}\cdot\ddot{\vec{A}} + v^2.$$

Take the integral of every term:

$$\int \frac{d}{dt}(\vec{A}\cdot\dot{\vec{A}})\,dt = \int \vec{A}\cdot\ddot{\vec{A}}\,dt + \int v^2\,dt$$

$$\vec{A}\cdot\dot{\vec{A}} = \int \vec{A}\cdot\ddot{\vec{A}}\,dt + v^2 t + C$$

$$\hookrightarrow = \vec{A}\cdot\dot{\vec{A}} - v^2 t + C.$$

-Turn the quiz over-

3. (3pt) A particle moves through space, described by a vector from the origin $\vec{r}(t)$. The length of $\vec{r} = r = \sqrt{\vec{r}\cdot\vec{r}}$.

Evaluate $\frac{d}{dt}(r^2\vec{v})$ in terms of $r, \vec{r}, \vec{v}, \vec{a}$.

$$\frac{d}{dt}(r^2\vec{v}) = \frac{d}{dt}\left[(\vec{r}\cdot\vec{r})\vec{v}\right]$$
$$= (\vec{r}\cdot\vec{v} + \vec{v}\cdot\vec{r})\vec{v} + (\vec{r}\cdot\vec{r})\vec{a}$$
$$= 2(\vec{r}\cdot\vec{v})\vec{v} + r^2\vec{a}.$$

4. (1 pt) An object's position in a plane is described by polar coordinates r, θ. The object is initially stationary. A force is applied with F_θ = constant (not zero), F_r = 0. Does r remain the same during the object's motion? Explain your answer by referring to the "useful equations" above.

No. r will change.

$$\ddot{r} - r\dot{\theta}^2 = F_r = 0.$$

Since $\dot{\theta} \neq 0$, it must be that $\ddot{r} \neq 0$.