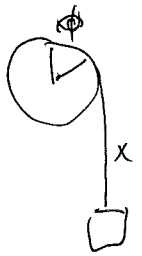
Quiz 14

Name Solution

Lagrange equations with undetermined multiplier
$L = T - U$

$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} + \lambda \frac{\partial f}{\partial q_i} = 0$ where the constraint is $f(q_i) = 0$

1. A mass m hangs by a string, the other end of which is wrapped around a frictionless wheel with radius R and moment of inertia I. Write down the Lagrangian in terms of x, the distance the mass has fallen, and ϕ, the angle the wheel has rotated.



$$\mathcal{L} = T - U$$
$$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \dot{\phi}^2 + mgx$$

2. Write down the equation that represents the constraint that the string not stretch.

$x = R\phi$ or $f(x, \phi) = x - R\phi = 0$

3. Write down the Lagrange equations with an undetermined multiplier, so as to be able to find the (generalized) constraint forces.

$$\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} + \lambda = 0 \qquad mg - m\ddot{x} + \lambda = 0 \quad (1)$$

OVER

$$\frac{\partial \mathcal{L}}{\partial \phi} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \underbrace{+ \lambda R}_{F_\phi} = 0 \qquad 0 - I\ddot{\phi} - \lambda R = 0 \quad (2)$$

4. Find the generalized constraint force F_ϕ. What is this force, physically?

From ② $\quad \lambda = \dfrac{-I\ddot{\phi}}{R}$

Insert in ① to get $\quad mg - m\ddot{x} - \dfrac{I\ddot{\phi}}{R} = 0$

use constraint $\quad \ddot{x} = R\ddot{\phi}$

$$mg - m\ddot{x} - \frac{I\ddot{x}}{R^2} = 0$$

$$\ddot{x} = \frac{mg}{m + I/R^2}$$

so

$$F_\phi = -\lambda R = R\frac{I\ddot{\phi}}{R} = \frac{I\ddot{x}}{R^2} = \frac{mgI}{mR + I/R}$$

This is a torque.