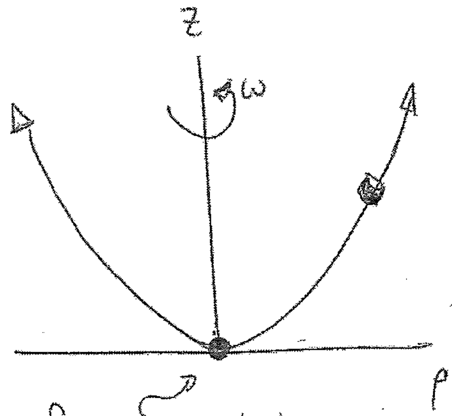
Quiz 13 Name SOLUTIONS

Euler-Lagrange

$S=\int_{t1}^{t2}\mathcal{L}[q,\dot{q},t]dt$ stationary when $\frac{\partial\mathcal{L}}{\partial q}-\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}}=0$

$\mathcal{L}=T-U$



for part 3, axis is here, out of the page

1. A bead slides on a frictionless wire with a parabolic shape. The equation of the wire is $z=\frac{1}{2}k\rho^2$. The wire is rotating about the z axis at a constant angular speed, ω. Write down the Lagrangian for the bead, using ρ as the generalized coordinate.

$\mathcal{L}=T-U$

$T=\frac{1}{2}m(\dot{\rho}^2+\dot{z}^2+(\rho\omega)^2)$ $\quad U=mgz$

$\dot{z}=k\rho\dot{\rho}$ $\Rightarrow$ $\mathcal{L}=\frac{1}{2}m\{\dot{\rho}^2+k^2\rho^2\dot{\rho}^2+\rho^2\omega^2-gk\rho^2\}$

2. Above a critical rotational speed, there is no stable equilibrium for the bead. What is this speed, ω_c?

$\frac{\partial\mathcal{L}}{\partial\rho}=\frac{1}{2}m\{2k^2\rho\dot{\rho}^2+2\rho\omega^2-2\rho gk\}$

$=$

$\frac{\partial\mathcal{L}}{\partial\dot{\rho}}=\frac{1}{2}m\{2\dot{\rho}+2k^2\rho^2\dot{\rho}\}$

$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\rho}}=\frac{1}{2}m\{2\ddot{\rho}+2k^2\rho^2\ddot{\rho}+4k^2\rho\dot{\rho}^2\}$

Eqm. $\ddot{\rho}=\dot{\rho}=0$

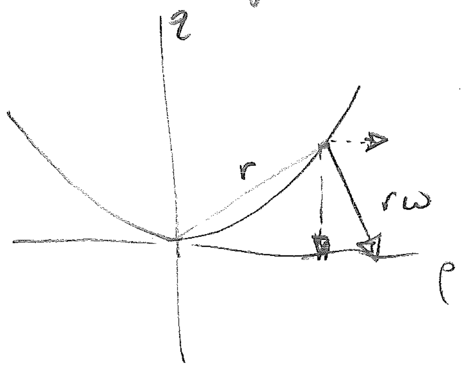
$\rho\omega^2-\rho gk=0$

$\rho=0$ or $\omega^2=gk$

$\omega=\sqrt{gk}=\omega_c$

3. Suppose the wire is rotated in the plane of the figure, about an axis through the origin, perpendicular to the page (at speed ω.) Write down the kinetic energy of the bead in terms of the generalized coordinate ρ and its derivative $\dot{\rho}$ (and k and ω.)

I'll assume my coordinates ρ, z rotate with the wire.



The velocity along ρ is $\dot{\rho} + rw\cdot\left(\frac{z}{r}\right)$

" along z is $\dot{z} - rw\left(\frac{\rho}{r}\right)$

so $T = \frac{1}{2}m\left[(\dot{\rho} + zw)^2 + (\dot{z} - \rho w)^2\right]$

with $z = \frac{1}{2}k\rho^2$ $\dot{z} = k\rho\dot{\rho}$ and $T = \frac{m}{2}\left[(\dot{\rho} + \frac{1}{2}k\rho^2 w)^2 + (k\rho\dot{\rho} - \rho w)^2\right]$

If the coordinates don't rotate, I need to find $\rho, \dot{\rho}$ in this frame from a rotation.

$\rho' = \rho\cos wt + z\sin wt$

$z' = -\rho\sin wt + z\cos wt$

this isn't especially hard, but it is especially ugly.