Quiz 11
Euler-Lagrange

Name Solution

$S = \int_{t1}^{t2} \mathcal{L}[q,\dot{q},t]dt$ stationary when $\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = 0$

$\mathcal{L} = \text{T-U}$

1. Consider a mass m moving on a frictionless plane inclined at an angle α with the horizontal. Write down the Lagrangian in terms of the coordinates x, measured horizontally across the plane, and y, measured down the slope. (Treat the system as two-dimensional, but include gravitational potential energy.)

$\mathcal{L} = T - U$

$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + mgy\sin\alpha$

2. Solve your equations for a particle with initial velocity v_{x0} in the x-direction, with $v_{y0} = 0$.

$\frac{\partial \mathcal{L}}{\partial x} = 0 \quad m\dot{x} = \text{constant}$ $\boxed{x = v_{x_0}t + x_0}$

$\frac{\partial \mathcal{L}}{\partial y} = mg\sin\alpha = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) = m\ddot{y}$

$\dot{y} = g\sin\alpha \cdot t + \cancel{C}^{\,0}$ $\quad \sin\alpha$ $\quad v_{y_0} = 0$

$y = \frac{g\sin\alpha}{2}t^2 + \cancel{C}t + D$

$\boxed{y = \frac{g\sin\alpha}{2}t^2 + y_0}$