Quiz 11

Name: Solutions

$\int_a^b f\{y,y';x\}\,dx$ is stationary when $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$. Note that y is a function of x, and $y' \equiv \frac{dy}{dx}$.

1. Given $f\{y,y';x\}=\frac{1}{2}y'^2-gy$, where g is a positive constant. Find the differential equation that makes $\int_a^b f\{y,y';x\}\,dx$ stationary along a path *y(x)*. (It will actually be a minimum.)

$$\frac{\partial f}{\partial y} = -g = \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = \frac{d}{dx}\left(\cancel{2}\cancel{\tfrac{1}{2}}y'\right) = y''$$

$$\boxed{-g = y''}$$

2. Solve the differential equation by integration. You will need two undetermined constants.

$$y'' = \frac{d}{dx}y' = -g$$

$$\int dy' = \int -g\,dx$$

$$y' = -gx + C$$

$$\frac{dy}{dx} =$$

$$\int dy = \int -gx\,dx + C\,dx$$

$$y = -\frac{1}{2}gx^2 + Cx + D$$

3. Evaluate the constants to find the minimizing path from x=0, y=0 to x=2, y=0.

$(0,0) \rightarrow D = 0$

$(2,0) \rightarrow \quad 0 = -\frac{1}{2} g \cdot 2^2 + c \cdot 2$

$= -2g + 2c \qquad c = g$

$$\boxed{y = -\frac{1}{2} g x^2 + g x}$$

4. What is the largest value of y along this path?

$\frac{dy}{dx} = -gx + g = 0$ for max. $x = 1$

$y_m = -\frac{1}{2} \cdot g + g = \frac{1}{2} g.$