5.23. $\frac{dE}{dt} = \frac{1}{2}m \cdot 2\dot{x}\ddot{x} + \frac{1}{2}k \cdot 2x\dot{x}$

Since $m\ddot{x} + b\dot{x} + kx = 0$, $\frac{dE}{dt} = \dot{x}(-b\dot{x}) = F_{damp} \cdot V$.

5.26. General case $T_1 = T_0 + \delta$. Since $T = \frac{2\pi}{\omega}$, $dT = \delta = -\frac{2\pi}{\omega^2} d\omega$

also $\omega_1 = (\omega_0^2 - \beta^2)^{1/2} = \omega_0\left(1 - \frac{\beta^2}{\omega_0^2}\right)^{1/2} \simeq \omega_0\left(1 - \frac{\beta^2}{2\omega_0^2}\right)$ so $d\omega = -\frac{\beta^2}{2\omega_0}$

Combine to get $+\frac{\beta^2}{2\omega_0} = +\frac{\delta\omega_0^2}{2\pi} = \frac{\delta}{T}\omega_0$ or $\beta = \sqrt{\frac{2\delta}{T}} \cdot \omega_0 = \sqrt{\frac{2\delta}{T}} \cdot \frac{2\pi}{T}$

Here, $T = 1$, $\frac{\delta}{T} = 0.001$, so $\beta = 0.28\ s^{-1}$

Log decrement $= \beta T = 0.28$ 10 cycles, $A = A_0 e^{-10\beta T} = 6\%$.

Much easier to see effect of damping on amplitude than period!

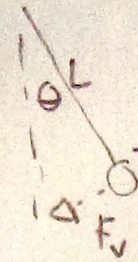
5.28 Critical: $x = Ae^{-\beta t} + Bte^{-\beta t}$ $x_0 = 0.5 = A$

$\dot{x} = -\beta A e^{-\beta t} + B\left[-\beta t e^{-\beta t} + e^{-\beta t}\right]$ $\dot{x}_0 = 0$ $B = \beta A$

So $x = 0.5\,e^{-\beta t}\left[1 + \beta t\right]$. Need more info! From the final position, we find $kx_0 = mg$ so $\sqrt{\frac{k}{m}} = \omega_0 = \sqrt{\frac{g}{x_0}}$. Critical damp $\rightarrow \beta = \omega_0$

So $x = x_0 e^{-\beta t}\left[1 + \beta t\right]$ $\beta t = \sqrt{\frac{g}{x_0}} \cdot t = \sqrt{20} \cdot 1 = 4.47$ $\boxed{x = 0.031\ m}$

3.12



$\tau = I\alpha$

$-mgL\sin\theta = mL^2 \cdot \ddot{\theta} \rightarrow \ddot{\theta} + \frac{g}{L}\theta = 0 \quad \frac{g}{L} = \omega_0^2.$

If viscous medium provides force $2m\sqrt{gL}\,\dot{\theta}$, this is a torque $\vec{\tau}_v = -2mL\sqrt{gL}\,\dot{\theta}$. So

$-mgL\sin\theta - 2mL\sqrt{gL}\,\dot{\theta} = mL^2\ddot{\theta}$ Divide by L^2, m.

$\ddot{\theta} + 2\sqrt{\frac{g}{L}}\,\dot{\theta} + \frac{g}{L}\theta = 0$ Critical damping $\beta = \omega_0$

3.29 $L \leftrightarrow m = 0.01\,H$ $\quad \beta = \frac{b}{2m} = \frac{100\,\Omega}{0.02\,H} = 50\,s^{-1}$

$R \leftrightarrow b = 100\,\Omega$

$C \leftrightarrow 1/k = ?$

$\omega_1^2 = \omega_0^2 - \beta^2 \quad \omega_0^2 = \omega_1^2 + \beta^2$

$\omega_1 = 2\pi \cdot 1\,kHz = 6283\,s^{-1}$

$\omega_0 = 6283.2 = \sqrt{\frac{k}{m}} = \frac{1}{\sqrt{LC}} \quad C = 2.53\,\mu F$

$q = e^{-\beta t}[A\cos\omega_1 t + B\sin\omega_1 t)$

$\dot{q} = i = e^{-\beta t}[-\omega_1 A\sin\omega_1 t + \omega_1 B\cos\omega_1 t] + -\beta e^{-\beta t}[\,]$

at $t=0$ $V_{cap} = 10V$ $\quad q = CV = 25.3\,\mu C.$

$25.3 = A\ (\mu C)$

$\dot{q}(0) = 0 = \omega_1 B + -A \quad B = 25.3/6283 = 0.004$

B is so small, I will ignore it!

$\dot{q}(0.2ms)$: $\beta t = 0.01, \quad \omega_1 t = 1.2566$

$\dot{q} = e^{-0.01}[-6283 \cdot 25.3\times10^{-6} \cdot \sin 1.2566] - 50e^{-0.01} \cdot [25.3\times10^{-6} \cos 1.2566]$

$= -0.15\,A.$

5.44 a) $E = \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2$

at $x=0$, all energy is kinetic $E = \frac{1}{2}m\dot{x}^2$

$x = A_0 e^{-\beta t}\cos\omega_1 t$ with β small, $\dot{x} \approx \omega_1 A_0 e^{-\beta t}\sin\omega_1 t$

so $E = \frac{1}{2}m\omega_1^2 \underbrace{A_0^2 e^{-2\beta t}}_{\equiv A^2}$ at max

b) $\Delta E_{dis} = \int_{cycle} F\cdot v\, dt$ only F_{damp} matters, others avg to 0.

$= \int -b\dot{x}\cdot\dot{x}\, dt = -\int_{cycle} \beta\cdot 2m\omega_1^2 A_0^2 \underbrace{\cancel{e^{-2\beta t}}}_{\approx 1} \sin^2\omega_1 t\, dt$

$\beta = \frac{b}{2m}$

$= -2\pi\cdot m\beta\omega^3 A^2.$

c) $Q = \frac{\pi}{\beta} = 2\pi\frac{E}{|\Delta E|}$

5.41 $D = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2 + 4\beta^2\omega^2}}$

Marion used D for A.

$D^2 = A^2 \propto \frac{1}{(\omega_0^2-\omega^2)^2 + 4\beta^2\omega^2}$

Half max when $4\beta^2\omega^2 = (\omega_0^2-\omega^2)^2$

$2\beta\omega = (\omega_0-\omega)\underbrace{(\omega_0+\omega)}_{\approx 2\omega}$

$\beta = \omega_0 - \omega$ at half max.

$2\beta = FWHM.$

3-45. Energy of a simple pendulum is $\frac{mgl}{2}\theta^2$ where θ is the amplitude.

For a slightly damped oscillation $\theta(t) \approx \theta \exp(-\beta t)$.

Initial energy of pendulum is $\frac{mgl}{2}\theta^2$.

Energy of pendulum after one period, $T = 2\pi\sqrt{\frac{l}{g}}$, is

$$\frac{mgl}{2}\theta(T)^2 = \frac{mgl}{2}\theta^2 \exp(-2\beta T)$$

So energy lost in one period is

$$\frac{mgl}{2}\theta^2\left(1-\exp(-2\beta T)\right) \approx \frac{mgl}{2}\theta^2 2\beta T = mgl\theta^2\beta T$$

So energy lost after 7 days is

$$mgl\theta^2\beta T\frac{(7\text{ days})}{T} = mgl\theta^2\beta\,(7\text{ days})$$

This energy must be compensated by potential energy of the mass M as it falls h meters:

$$Mgh = mgl\theta^2\beta(7\text{ days}) \quad\Rightarrow\quad \beta = \frac{Mh}{ml\theta^2(7\text{ days})} = 0.01\text{ s}^{-1}$$

Knowing β we can easily find the coefficient Q (see Equation (3.64))

$$Q = \frac{\omega_R}{2\beta} = \frac{\sqrt{\omega_0{}^2 - 2\beta^2}}{2\beta} = \frac{\sqrt{\frac{g}{l} - 2\beta^2}}{2\beta} = 178$$