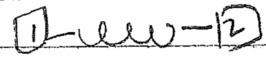


$F_1 = k(x_2 - x_1 - \Delta)$ $\qquad \Delta$ = rest sep.

$\quad = m_1\ddot{x}_1$

$F_2 = k(x_1 - x_2 + \Delta)$ $\quad (= -F_1)$

$\quad = m_2\ddot{x}_2$

Define $x = x_2 - x_1 - \Delta$, $\quad x_c = \dfrac{m_1x_1 + m_2x_2}{\underbrace{m_1+m_2}_{M}}$

Then

$F_2 = -kx = m_2\ddot{x}_2$

$F_1 = kx = m_1\ddot{x}_1$

Then $\quad m_1\ddot{x}_1 + m_2\ddot{x}_2 = M\ddot{x}_c = 0 \qquad \dot{x}_c$ = constant

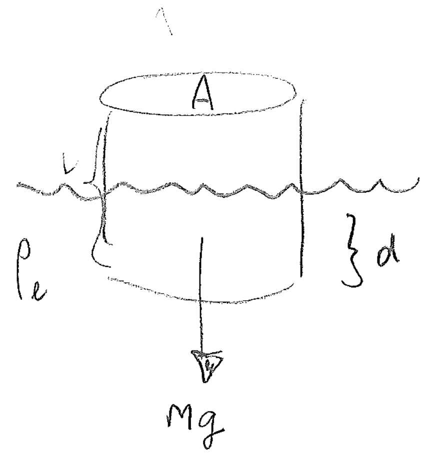
Also, $\quad \ddot{x}_2 - \ddot{x}_1 = \ddot{x} = -\dfrac{k}{m_2}x - \dfrac{k}{m_1}x$

OR $\quad \mu\ddot{x} = -kx \quad$ where $\mu = \dfrac{m_1m_2}{m_1+m_2}$

reduced mass

So freq $= \sqrt{\dfrac{k}{\mu}}$

3-7.



Buoyant force = mass displaced × g

$= V \rho_L g$

$= A \cdot d \rho_L g$

Net force $= A d \rho_L g - mg$

At eqm, $mg = A d_0 \rho_L g$, so

$F_{net} = -A x \rho_L g$ where $x = d_0 - d$.

$M = A d_0 \rho_L$, so $\omega = \sqrt{\dfrac{gA}{V}}$

$= V \rho_L$

(V = volume displaced) so $T = 2\pi \sqrt{\dfrac{V}{gA}}$

with $V = 0.8\ cm^3$, $A = 1\ cm^2$, $g = 10\ \frac{m}{s^2} = 1000\ \frac{cm}{s^2}$

$T = 0.18\ s$.

Cycloid Problem

$$V = \frac{ds}{dt} \qquad ds = (dx^2 + dy^2)^{1/2} \qquad = d\phi \left(\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dy}{d\phi}\right)^2 \right)^{1/2}$$

$$x = a(\phi - \sin\phi) \qquad dx = \frac{dx}{d\phi}\, d\phi \quad \text{etc}$$

$$y = a(\cos\phi - 1)$$

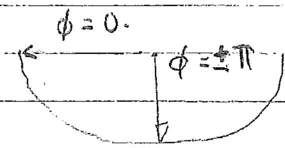
$$\frac{dx}{d\phi} = a(1 - \cos\phi)$$

$$\frac{dy}{d\phi} = a(-\sin\phi)$$

so

$$ds = a\, d\phi \left[1 - 2\cos\phi + \cos^2\phi + \sin^2\phi \right]^{1/2}$$

$$ds = \sqrt{2}\, a\, d\phi\, \left[1 - \cos\phi\right]^{1/2}$$



$$V = \sqrt{2}\, a\, \dot\phi\, \left[1 - \cos\phi\right]^{1/2}$$

Now $\frac{1}{2}\cancel{m}V^2 = \cancel{m}g(y_0 - y)$

$$a^{\cancel{2}}\dot\phi^2 (1 - \cos\phi) = g a\left[\cos\phi_0 - 1 - (\cos\phi - 1)\right]$$

$$= g\cancel{a}\left[\cos\phi_0 - \cos\phi\right]$$

$$\dot\phi = \left[\frac{g}{a}\,\frac{\cos\phi_0 - \cos\phi}{1 - \cos\phi}\right]^{1/2} = \frac{d\phi}{dt}$$

⟵ Square root introduces $\pm$ ambiguity

$$\int dt = \int \left[\frac{a}{g}\,\frac{1 - \cos\phi}{\cos\phi_0 - \cos\phi}\right]^{1/2} d\phi$$

What are limits? ~~ϕ_0~~ to ϕ_0 is 1/4 cycle

Note that $\frac{d\phi}{dt} < 0$ here

(b)

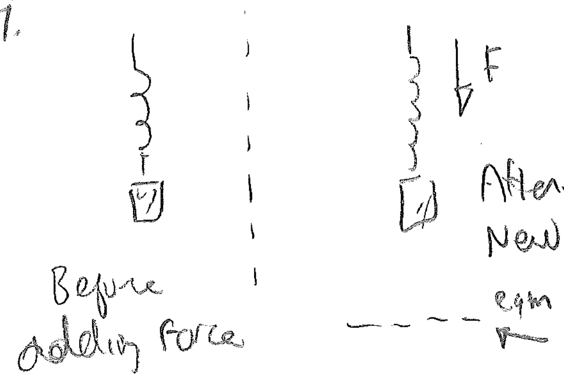
$$t = \sqrt{\frac{a}{g}}\,(-2)\arcsin\left[\frac{\sqrt{2}\cos x/2}{\sqrt{\cos\phi_0 + 1}}\right]\Bigg|_{x=-\pi}^{x=\phi_0}$$

$$= \sqrt{\frac{a}{g}}\,(-2)\arcsin\left[\sqrt{\frac{1+\cos x}{1+\cos\phi_0}}\right]\Bigg|_{x=-\pi}^{x=\phi_0}$$

$$= (-2)\left[\underbrace{\arcsin(1)}_{\frac{\pi}{2}} - \underbrace{\arcsin(0)}_{0}\right]$$

$$t = \sqrt{\frac{4a}{g}}\cdot\frac{\pi}{2} = \sqrt{\frac{l}{g}}\,\frac{\pi}{2}$$

3-9.



at $x = x_0 + \frac{F}{k}$.

Motion:

$$x - \left(x_0 + \frac{F}{k}\right) = -\frac{F}{k}\cos wt$$

or

$$x = x_0 + \frac{F}{k} - \frac{F}{k}\cos wt$$

$$\dot{x} = \frac{wF}{k}\sin wt.$$

After force is removed,
we have initial conditions $x(t_0)$, $\dot{x}(t_0)$

Motion is $x - x_0 = A\cos w(t-t_0) + B\sin w(t-t_0)$ at $t = t_0$, $x(t_0) = A$

$\dot{x}(t) = -Aw\sin w(t-t_0) + wB\cos w(t-t_0)$ at $t = t_0$, $\dot{x}(t_0) = wB$.

so

$$A = \frac{F}{k}(1 - \cos wt_0)$$

$$B = \frac{F}{k}\sin wt_0.$$

thus,

$$x - x_0 = \frac{F}{k}\left[(1-\cos wt_0)\cos w(t-t_0) + \sin wt_0 \sin w(t-t_0)\right]$$

$$= \frac{F}{k}\left[\cos w(t-t_0) + \underbrace{\sin\alpha\sin\beta - \cos\alpha\cos\beta}_{= -\cos(\alpha+\beta)}\right]$$

$\alpha = (t - t_0)w$
$\beta = wt_0$

So

$$x - x_0 = \frac{F}{k}\left[\cos w(t-t_0) - \cos wt\right]$$

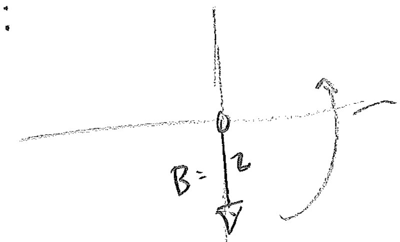
5.8 a) $\sqrt{\frac{k}{m}} = \omega = \sqrt{\frac{80}{0.2}} = 20\ s^{-1}$ $f = \frac{20}{2\pi}\ s^{-1} = 3.2$ cycles/s.

$T = \frac{1}{f} = 0.314$ s.

b) $x_0 = 0$ so $A = 0$ in $x = A\cos\omega t + B\sin\omega t$.

$v_0 = 40$ m/s so $\omega B = v_0$ $B = 2$ m.

Phasor diagram:



Thus, $x(t) = 2\cos(\omega t - \pi/2)$ (m).

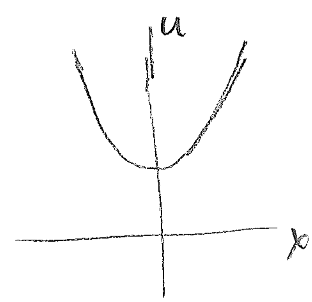
5.9. If $x = A\cos\omega t$, $\dot{x} = -\omega A\sin\omega t$.

A = max displacement = 0.2 m

ωA = max speed = 1.2 m/s so $\omega = 6\ s^{-1}$. $T = \frac{2\pi}{\omega} \approx 1$ s.

5.10 $F = -F_0 \sinh \alpha x$

$u = -\int F \cdot dx = F_0 \int \sinh \alpha x\, dx = \frac{F_0}{\alpha}\cosh \alpha x$



$u' = +F_0 \sinh \alpha x$

$u'' = F_0 \alpha \cosh \alpha x$ $u''(0) = F_0\alpha\left(\frac{e^0 + e^{-0}}{2}\right) = F_0\alpha = k$.

$\omega = \sqrt{\frac{F_0\alpha}{m}}$.

5.11 The energy of an oscillator is $\frac{1}{2}kx^2 + \frac{1}{2}mv^2$.

taking $x = A\cos\omega t$, $E = \frac{1}{2}kA^2\cos^2\omega t + \frac{1}{2}m\omega^2A^2\sin^2\omega t$

$\dot{x} = -\omega A\sin\omega t$

$$k x_1^2 + m v_1^2 = k A^2 =$$

① $$\omega^2 x_1^2 + v_1^2 = \omega^2 A^2 = \omega^2 x_2^2 + v_2^2 . \qquad \left(\omega = \sqrt{\frac{k}{m}}\right)$$

$$\omega^2 (x_1^2 - x_2^2) = v_2^2 - v_1^2$$

$$\omega = \sqrt{\frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}}$$

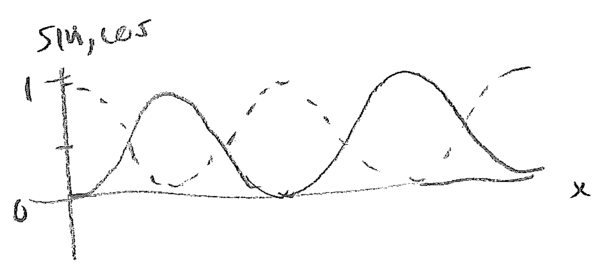
Substitute in ① to find $$\frac{v_2^2 - v_1^2}{x_1^2 - x_2^2} \cdot x_1^2 + v_1^2 = \frac{v_2^2 - v_1^2}{x_1^2 - x_2^2} A^2$$

$$(v_2^2 - v_1^2) \cancel{x_1^2} + v_1^2 (\cancel{x_1^2} - x_2^2) = (v_2^2 - v_1^2) A^2$$

$$\sqrt{\frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}} = A$$

5.12 $\langle \sin^2 \rangle = \langle \cos^2 \rangle = 1/2$

Can be seen by plotting.



$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x .$$

$$x = A\cos(\omega t - \delta)$$

$$v = -\omega A \sin(\omega t - \delta)$$

$$\langle T \rangle = \frac{1}{T}\int_0^T \frac{1}{2} m v^2 \, dt$$

$$= \frac{1}{T} \int_0^T \frac{1}{2} \cdot m . \, \omega^2 A^2 \sin^2(\omega t - \delta) dt = \frac{1}{T}\left[\frac{1}{2} m \omega^2 A^2 \cdot \frac{T}{2} \right]$$

$$= \frac{1}{2}\left\{ \frac{1}{2} m \omega^2 A^2 \right\} = \frac{1}{2} E .$$

$$\langle U \rangle = \frac{1}{T}\int_0^T \frac{1}{2} k x^2 \, dt = \frac{1}{T}\int_0^T \frac{1}{2} k * A^2 \cos^2(\omega t - \delta)\, dt = \frac{1}{2}\left\{ \frac{1}{2} k A^2 \right\} = \frac{1}{2} E .$$

5.13. Eqm. $\frac{dU}{dr} = 0 = U_0\left(\frac{1}{R} + -\lambda^2 \frac{R}{r^2}\right) \rightarrow \frac{\lambda^2}{r^2} = \frac{1}{R^2} \quad r^2 = R^2\lambda^2$

$r = \lambda R.$

Taylor expand about $r = \lambda R$.

$$\frac{d^2U}{dr^2} = 2\lambda^2 \frac{R}{r^3} U_0 \bigg|_{\lambda R} \rightarrow 2\lambda^2 \frac{R U_0}{\lambda^3 R^3} = \frac{2 U_0}{\lambda R^2} = k$$

$$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{2U_0}{\lambda R^2 m}}$$