surface will reflect high-frequency, short-wavelength sound diffusely. These facts have not been lost on designers of acoustic spaces, especially concert halls. The effect of the scale of the roughness compared to the wavelength is illustrated in the *Ripple* simulation shown in figure 2.8. A point source some distance from the surface sends long- and short-wavelength sound toward a surface; much more "damage" is done by the rough surface to the short-wavelength sound, which reveals clumps of waves traveling in nonspecular directions.

2.6 Refraction

Wave energy often progresses in a well-defined direction, but that direction can change more or less slowly. Such *refraction* often goes unnoticed for sound waves, but refraction is actually quite ubiquitous outdoors over distances of about 100 m and beyond. Refraction results from the variation of the speed of the wave within the medium in which it is traveling; the bending or curving of the wave is always toward regions of slower wave speed.

Several factors cause the wave speed to vary in air, all understandable in terms of the concepts introduced in chapter 1: the drunken messenger model, the air cell impedance picture, or both.

Temperature. Lower temperature means the molecular messengers are moving more sluggishly, reducing the speed with which pressure fluctuations are propagated. Every gas atom or molecule has on average the same energy as its neighbors, independent of its mass. Energy is defined as $E = 1/2\,mv^2$, where m is the mass of the molecule, and v is its velocity. Energy per molecule in a gas is proportional to temperature, expressed in kelvin, or K (room temperature is 295 K). Thus at 273 K, the freezing point of water, the speed of sound in any gas should be $\sqrt{273/295} = 0.965$ times as fast as it is at room temperature, 295 K. At 295 K, the speed of sound in air is 343 m/s; thus we predict it to be $343 \times 0.965 = 331$ m/s at 273 K or 0°C. This is indeed the measured value.

Composition. Differences in chemical composition change the messengers themselves. At the same temperature, lighter messengers are speedier. Again, the energy of any molecule due to its speed is the same as that of any other molecule (this is called *equipartition of energy*), and since $E = 1/2mv^2$, the average speed must be higher if m is smaller. The speed of sound in air, a mixture of nitrogen (about 80%) and oxygen (about 20%) with an average mass of 29 grams/mol is 343 m/s at 20°C (room temperature). As discussed earlier, we would expect the sound speed in helium gas, mass 4, to be about $\sqrt{29/4} = 2.7$ times faster than air, or $343 \times 2.7 = 929$ m/s. The measured value is 972 m/s. Sulfur hexaflouride, SF_6, should have a speed of $343 \times \sqrt{29./146} = 153$ m/s; the measured value is 150. In the atmosphere, water vapor content is the most common cause of composition changes from one place to another.

Motion of medium. Last, if in some region the messengers are moving en masse in the same direction, the wave propagates slower or faster (over the ground) according to whether it is moving with or against the mass movement. This will speed up or slow down the wave arrival merely by a fraction, except if the speed variation differs from place to place, in which case the variations *also cause refraction of the waves.* Temperature, composition, and speed gradients are common factors affecting sound outdoors; they will come up again in chapter 28.

A way to quickly (if qualitatively) follow wavefronts to see how sound (or light) propagates was invented by Christian Huygens more than 300 years ago. This is a third way of understanding sound propagation, in addition to the drunken messengers model and the cellular method (chapter 1).

Huygens's method works as follows: we start with a wavefront representing some wave incident on a "scene" that may include different materials. We want to construct the wave farther along the direction of propagation. Along the initial wavefront, we locate the centers of arcs of constant radius; the "envelope" of the new arcs is the new wavefront, also of constant phase. If the arcs are half a wavelength in radius, the new wavefront will be a crest if the old one was a trough. To understand refraction, we slightly modify Huygens's original illustration. In figure 2.9, a wave is traveling in the direction of the line segment D-A, with perpendicular wavefronts—for example, A-C and the lines K-L. Suppose τ is the time it takes for the wavefront to travel the distance from C to the leftmost L. This distance is the radius of the Huygens arcs used to propagate wavefronts in the upper medium. (We have drawn a few of these in red.) Along the interface between the two media, the wavefronts will advance between the adjacent points labeled K in a given time τ. Inside the medium indicated by the rectangle, the speed is lower, and we draw correspondingly smaller radius arcs. The arc whose center is the rightmost point labeled K has such a smaller radius and represents the progress of part of the wavefront from that point in one unit of time τ. Huygens has drawn an arc of twice this radius from the adjacent point K to its left, representing a time 2τ since the wavefront entered there, and three times the radius from the point to the left of that. This is the correct procedure, as can be seen by the intermediate wavefronts inside the medium (colored blue) given by the lines labeled K-O. The new wavefronts are not parallel to the old wavefronts outside and above the medium. We have thus constructed the new wavefront inside the medium. Employing the rule that the energy flow is perpendicular to the wavefronts, we see that there is a new direction A-N inside the medium compared to the old direction D-A of propagation above the medium. We have shown that the wave refracts as it enters the medium of slower wave speed. Notice that the ray bends *toward* the medium with slower wave speed—this is a useful rule to remember about refraction.

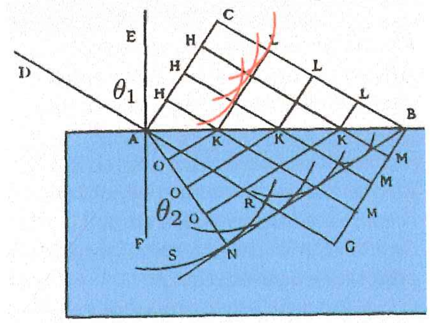


Figure 2.9

Christian Huygens's depiction of the geometry of refraction (from Huygens's *Traité de la Lumiere*, 1678), as determined by his construction using arcs to advance a wavefront, with the addition of the red arcs.

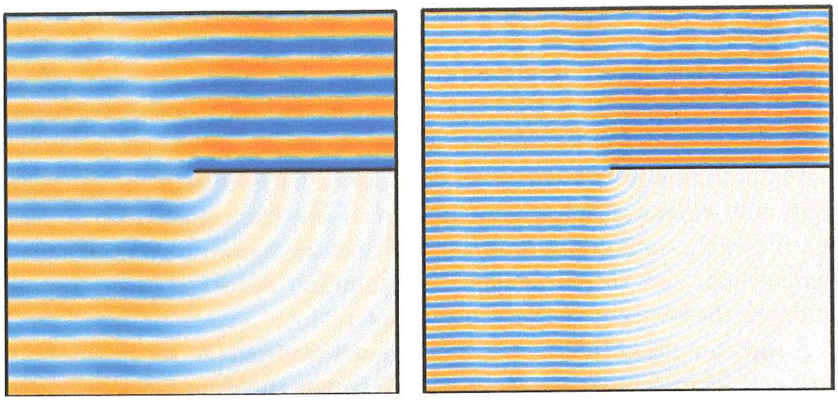


Figure 2.10
A plane wave having long wavelength arrives from the top of the image and is interrupted by a reflecting wall (left). The geometric shadow is a vertical line heading straight down from the end of the wall. Waves penetrating beyond this line into the shadow region are by definition "diffracting." More diffraction occurs for longer-wavelength (lower-frequency) sound; the diffracted power is proportional to the wavelength. This figure is taken from a *Ripple* simulation; claims about the amount of diffracted intensity can be checked by setting up probes at appropriate positions.

If you are sitting inside the lower medium, the wave crests arrive one after the other with the same period as in the medium above. (If a person above the surface of a pool is waving her arm once per second, the period will be one second whether you are looking from above or below the surface.) Because the wave is moving more slowly in the lower (blue) medium, the wavelength must be shorter to keep the frequency f the same. The rule is $f = c_1/\lambda_1 = c_2/\lambda_2$, where the c's are the wave speeds and the λ's are the wavelengths in the two media. It is then quite simple to show geometrically, using the fact that the wavefronts must agree at the interface, that

$$\lambda_2 \sin\theta_1 = \lambda_1 \sin\theta_2, \tag{2.3}$$

where the angles θ_1 and θ_2 are the angles of incidence and refraction indicated in figure 2.9. This is Snell's law, derived in many elementary physics textbooks, usually in the context of light waves. There are many practical implications for the refraction of sound that we will encounter along the way in this book.

2.7 Diffraction

Diffraction is another way that waves, including sound, can take a non-straight-line path. For projectiles launched from a distant point, a solid object forms a "hard shadow," a refuge from the projectiles. Waves do not respect such hard shadows; they can bend around edges of obstructions, making it possible to "hear around corners." Diffraction is less pronounced for short wavelengths, so if a marching band is approaching out of sight behind a building, the low thumping of drums will be heard well before the piccolos. This fact is reinforced by the simulations in figures 2.10 and 2.11.

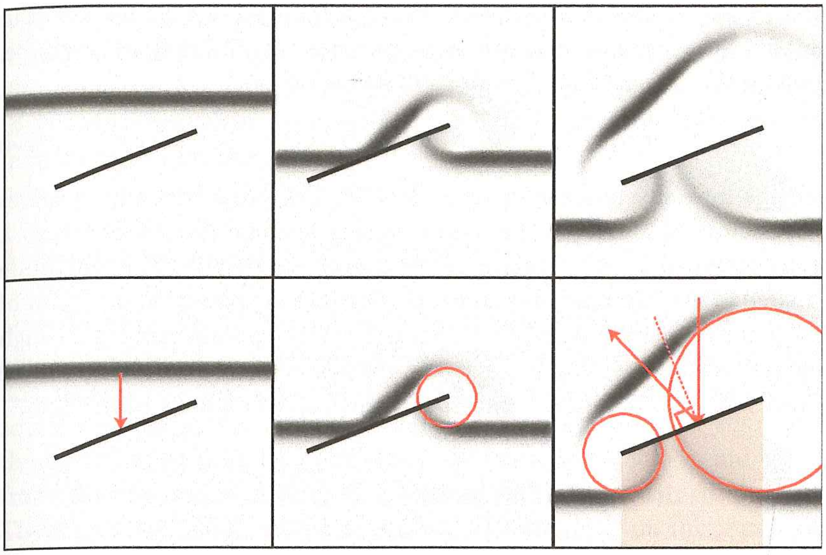


Figure 2.11

(Top) Successive snapshots of a sound pulse arriving from above a wall segment. (Bottom) Diffraction occurs from the end points and is similar to a pulsed wave from those points leaving as the initial pulse arrives, as indicated by the red circles. The analysis shows that the main portion of the reflected wave obeys the rule angle of incidence = angle of reflection. The "hard shadow" region, shaded pink, would be free of sound if sound traveled in straight lines, without diffraction. Taken from screenshots in a simulation run in *Ripple*.

Figure 2.10 shows diffraction around a wall segment for two different wavelengths of sound. We see that long wavelengths diffract more easily, or put another way, there is a closer approximation to a hard shadow for very short wavelengths.

Diffraction at an Edge

We may visualize diffraction due to an edge in terms of the cell picture of sound propagation. If we examine figure 2.10 closely, it is apparent that the end of the wall is very much like a point source of sound. A circular wave emanates from that point, as is especially evident in the lower-right quadrant, where nothing else is present. Faint evidence of this circular wave can also be seen in the other three quadrants. See also figure 2.11.

The diffraction from the edge is understandable from the cell model as follows: a cell just above the wall, but more than about half a wavelength from the end point of the wall, won't "feel" the end point, since it is too far away to send and receive information (within one period) from the end point at the speed of sound. The impedance of such a cell is therefore not reduced because it does not sense the lower resistance to pressure changes near the wall. More than half a wavelength beyond the end of the wall also acts like any other cell in free space, again because it is too far away to send and receive the information that the wall is present. There is one special cell acting strangely, straddling the end of the wall. This cell is busy scattering sound because its impedance is different than any adjacent cell. It scatters whatever impinges on it. The scattered (diffracted) wave may be understood as coming from this small region, which therefore acts like a small, or *point* source of sound. Like any source, there will be a falloff in amplitude as distance increases from it; in this two-dimensional example, we have the diffracted amplitude $a_d(r)$ falling off as

$$a_d(r) \propto \sqrt{\lambda/r}, \tag{2.4}$$

and the intensity thus declining as $\propto \lambda/r$. The proportionality involves only factors of 2 and π. The power passing through the cell of length λ is proportional λ—it's a bigger cell at longer wavelength. The amplitude squared of the diffracted wave is proportional to the power, so the factor of $\sqrt{\lambda}$ in the numerator correctly accounts for the *amplitude* passing through one cell. As the wavelength gets larger, the odd cell at the end, which is one wavelength across, diffracts more power in proportion to its wavelength.

Brush with the Law of Similarity

For a thin wall, the scale of the picture is set by the wavelength. The left panel in figure 2.10 is about 10 wavelengths across in both directions. If someone declares that the frame is physically 100 m across, then the wavelength is about 10 m; from $f\lambda = c$, the frequency is about 34 Hz. If the frame is only 10 m across, the wavelength is about 1 m and the frequency is 344 Hz. *The picture is correct either way.* By this principle, a mockup only 1 meter high could be used to study the diffraction of sound around a highway barrier that in reality is going to be 10 m high, *provided all the sound frequencies are increased correspondingly by a factor of 10.* This is our first encounter with the law of similarity, which allows us to scale up and scale down studies of wave propagation, diffraction, and so on by scaling the physical dimensions such as wavelength λ by some factor such as $1/10$, and scaling the frequency by a factor of 10 *so that* $\mathrm{f}\lambda = \mathrm{c}$ *both before and after the scaling.*

Similarity implies that sharp edge diffraction is always the same: you need only one picture! It's just a matter of scaling the picture up or down. But then what is the justification for claiming that long wavelengths diffract more than short ones?

We have already shown why the diffracted power increases in proportion to the wavelength—that is, more is diffracted for lower frequencies—the *drum and piccolo effect.* The similarity argument we are now making reinforces this: measuring distances in a picture such as figure 2.10 in terms of wavelengths, not meters, the amplitude $a_d(r)$ in equation 2.4 falls off at the same rate. For example, suppose r is 10 wavelengths away—that is, $r = 10\lambda$. If the wavelength is 10 m, r is 100 m from the wall; if the wavelength is 1 m, r is only 10 m from the wall. This confirms that there is more diffraction in the case of the longer wavelength, *since it has the same sound intensity in the shadow region 100 m from the edge as does the shorter wavelength 10 times closer to the edge, at 10 m.* This also confirms with the notion that a region about one wavelength wide is responsible for the diffraction. The longer the wavelength, the more energy is thrown into diffraction, since power passing through an opening 10 m wide is 10 times that passing through an opening 1 m wide. The law of similarity is taken up in more detail in section 7.6.

Active Noise Reduction of Diffracted Sound

Long highway sound barriers are now routinely placed between traffic and residential areas, although they are quite expensive—about two million dollars per mile. The walls are usually very solid, so most of the sound arriving at the houses (if the wall blocks the line of sight with the traffic) must have been diffracted. We now know this sound comes from the top edge—that is, it is diffracted near the top of the wall. The amplitude falloff will be a function of distance from the top of the fence.

This makes possible an "active" sound attenuation strategy, namely, to *put an out-of-phase source right where the culprit "edge diffraction source" lies*. This can be accomplished by mounting microphones and loudspeakers along the top of the sound barrier. Sound impinging on the edge is detected by the microphone, processed by a computer chip and re-emitted *out of phase with the incident sound* by loudspeakers aimed toward the quiet side of the barrier. The speakers could be efficient horn loudspeakers (see section 7.3) and solar powered. The speakers and microphones need to be placed densely along the wall, but compared to two million dollars per mile, it might prove cheap if the attenuation worked well enough.

A *Ripple* simulation of a similar situation is shown in figure 2.12. A space between two vertical walls, with no roof, is filled with loud sound due to

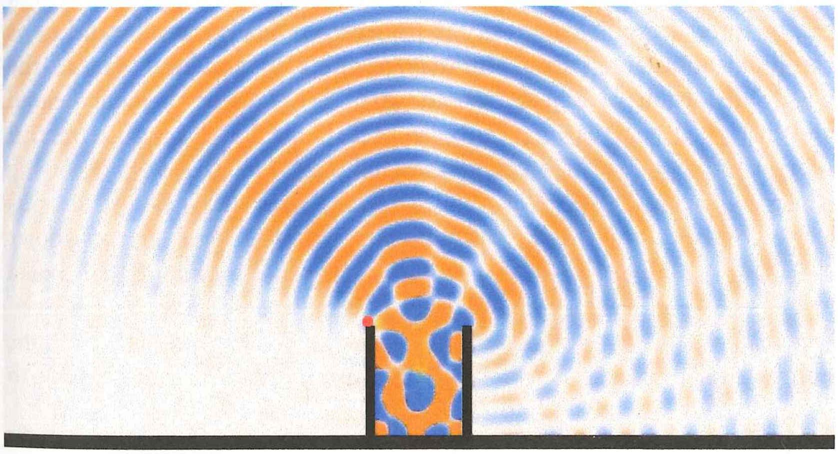


Figure 2.12
Ripple simulation of active noise cancellation from a noisy area (between the walls), using destructive interference. The red dot is the location of a point source out of phase with the diffracted wave, which is for the most part successfully canceled on the left.

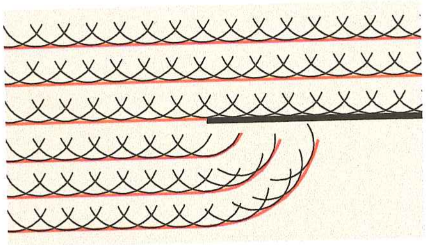


Figure 2.13

Huygens's wavefront construction of the diffraction that results when a plane wave collides with a wall. The wavefront constructed from the wavelets (black curves) becomes curved near the end of the wall. Subsequent applications of Huygens's rule leads to the propagation shown and the development of the curved diffraction wavefront. Using Huygens's wavefront construction, it is difficult to quantify the amplitude in the diffractive region.

seven sources. An eighth source is placed at the top of one wall; its phase and amplitude are such as to destructively interfere with and therefore attenuate the diffraction reaching the ground, as seen on the left; compare this with the diffraction reaching the ground on the right, for which no cancellation was used. This scenario takes full advantage of the fact that the diffraction from an abrupt edge is itself like a point source.

Diffraction may be understood qualitatively within the Huygens construction. Suppose a wave is incident on a segment of a wall. This leaves a shadow region that however is partly filled with diffracted waves—that is, waves that have deviated from the linear path they were on before they hit the wall (figures 2.11 and 2.13).

To develop an intuition for reflection and diffraction of sound from various objects of different sizes and shapes, it is recommended that you set up various *Ripple* scenarios, drawing obstacles, baffles, objects, and so on and observe the reflection and diffraction of waves of various wavelengths sent at them.

2.8 Schlieren Photography

Schlieren is the German word for optical inhomogeneities in transparent material. Schlieren photography, which makes even slight inhomogeneities visible, was invented by the German physicist August Toepler in 1864. He succeeded in photographing shock waves in air created by *supersonic* (faster than the speed of sound) objects. A sharp *pressure pulse* (shock wave) wave can be created by an electrical spark; we hear this wave as a sudden "snap."

If light propagates through regions of rapidly changing air density, there is a slight deflection toward the denser regions, normally too small to be noticed. In the correct circumstances, especially when the light has to travel large distances to the camera or the eye, small deflections can build up a large effect. Almost everyone has seen "heat waves," the wiggly distortion of objects caused when light passes through heated and disturbed air. Warmer air is less dense and has a lower refractive index than cool air. Cinematographers have a favorite trick for showing something a long way off on a hot day, capturing the wavy distortions caused by refractive index differences of pockets of warmer and cooler air. Light traveling through the turbulent, variable-index medium has a characteristic scintillating and mottled appearance, owing to the schlieren effect. The effect is quite noticeable near very hot objects—for example, near a stove or a candle, where the air density is dramatically reduced due to heating.

Toepler had the idea that a point source of light might be refracted enough by such disturbances to cast lighter and darker bands on a screen

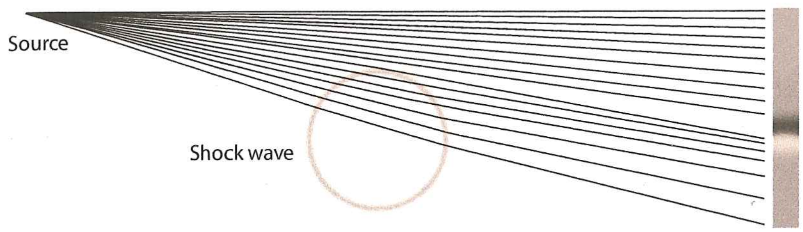


Figure 2.14

Principle of schlieren photography. Slight variations in the index of refraction of air, caused by a propagating circular shock wave (brown), refract light rays. The speed of light is slightly slower in denser air, causing light to bend toward denser regions. Those rays that graze the higher-density disturbance are refracted toward them, deflecting them slightly. This slight deflection is enough to have an effect on the rays cast on a distant screen, causing light and dark bands: light where the extra rays arrived at the screen; dark where they are missing. The rays that graze the shock wave tangential to it are deflected most because they spend the most time near the gradients in the density of air.

some distance away. This works very well indeed, and when the disturbance is localized it can give a very accurate image of it. The principle is illustrated in figure 2.14. Figure 2.15 demonstrates schlieren imaging of a shock wave traveling through tubes of different shapes.

2.9 Ray Tracing

Ray tracing has been around a long time as a substitute for having to solve for the exact wave motion. For example, in describing his speaking trumpet, Sir Samuel Morland used as evidence for the efficacy of ray tracing a pewter parabolic mirror that had not only set a board on fire upon focusing the sun's rays on it, but had also focused a distant man's voice to the same spot (presumably as the shadow of the speaker's head fell across the mirror). Light or sound, ray tracing the waves comes to the same conclusion: they focus at the same spot. A wonderful bit of science for its time.

Optical devices are designed by ray tracing because following the wave motion is far too expensive. A simple rule is used: Rays travel in straight lines unless interrupted by walls or lenses, following the course of the

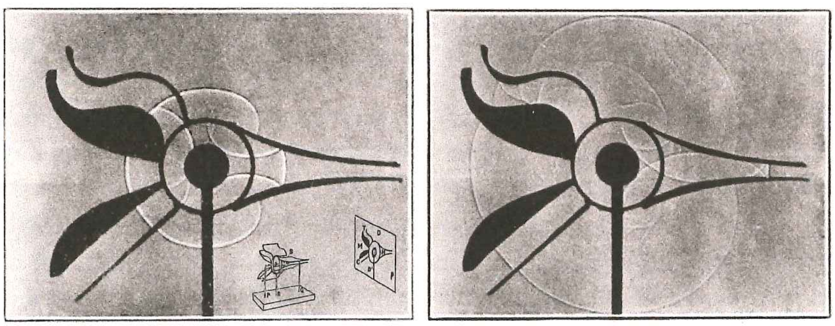


Figure 2.15

In an image created by Arthur Foley and published in *Physical Review* in 1922, sonic (traveling at the speed of sound) shock waves caused by a spark are reflected and guided by three different-shaped tubes. One tube is straight, one is curved, and one decreases in diameter away from the spark. The inner disk hides the spark; the ring surrounding the disk supports the three tube-shaped enclosures and does not lie in the same plane as the spark and does not disrupt the shock waves. The black part of the image is a projection of a three-dimensional object onto a plane (see inset). From Arthur Foley, "A Photographic Study of Sound Pulses between Curved Walls and Sound Amplification by Horns," *Physical Review* 20 (1922), 505–512. © 1922 The American Physical Society.

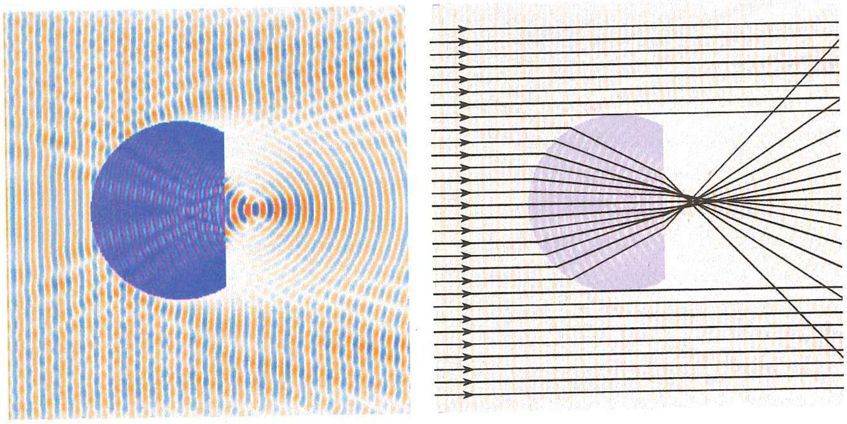


Figure 2.16
Simulation in *Ripple* of a plane wave incident from the left onto a truncated circular lens. On the right, a ray tracing analysis is shown, using only the rays that have not reflected from interfaces.

energy emitted from the source. More precisely, the rays travel perpendicular to the wavefronts, for which they are the surrogates.

At interfaces like that between air and glass, the impedance mismatch tells us that part of the wave is reflected and part is transmitted. This is not a problem for ray tracing: the wavefronts (and with them the rays) split into a transmitted part and a reflected part at interfaces such as air and glass. The part of the wave that penetrates the glass changes direction unless it is incident perpendicular to the interface (refraction). The reflections obey the rule that angle of incidence equals angle of reflection, and the refraction of rays follows Snell's law (equation 2.3). The ray tracing can get quite complicated after several successive encounters with curved surfaces, but it is still much simpler than following the waves themselves.

Ray tracing is only an approximation. It misses diffraction altogether and is accurate and practicable only when the changes in the impedance (and thus the wave speed) in the medium are either very abrupt, in which case there is ray splitting involving reflection and refraction at the interface, or quite slow on the scale of a wavelength, in which case the wavefronts and associated rays curve gracefully.

Figure 2.16 displays a simulation in *Ripple* of a plane wave incident from the left onto a truncated circular lens. On the right, a ray tracing is analysis is shown, using only the rays that do not reflect from interfaces. The full wave simulation shows the effects of interference of the various reflected portions of the waves and the direct waves, as well as diffraction effects.

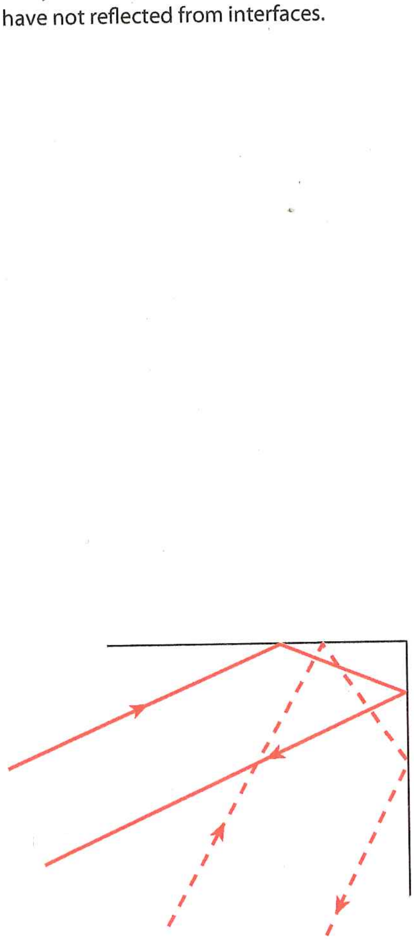


Figure 2.17
A right-angle corner showing two ray paths that bounce once from each wall, returning in a direction exactly opposite to their incident direction. Any ray that bounces from both walls will return exactly parallel to its source.

Corner (Retro-) Reflector

A 90-degree interior corner—three perpendicular walls that meet at right angles—has the interesting and useful property that sound incident on it gets reflected back along the direction it came from, over a wide range of incident angles. This is exactly true in ray tracing analysis, as shown in figure 2.17 for the two-dimensional case. For the three-dimensional

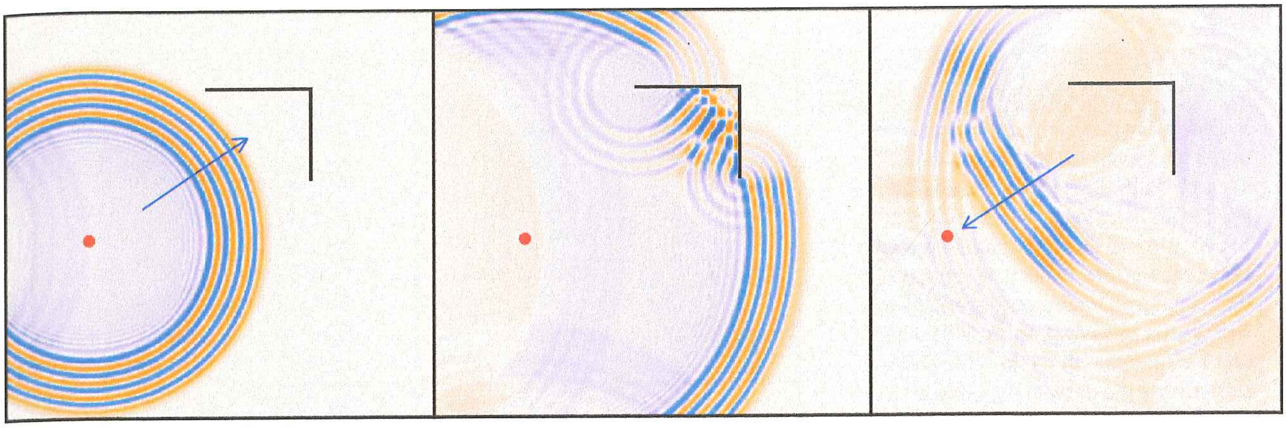


Figure 2.18
Part of a sound pulse from an omnidirectional source (red dot) encounters a corner reflector, which sends some of the pulse back toward its source. In addition, the two weak circular waves seen in the middle frame are the result of diffraction from the tips of the walls (see the discussion for figure 2.19), while the two rather strong side pulses are generated when part of the wave bounces off one wall but misses the opposite wall. The part that hits both walls is reflected back parallel to the direction in which it arrived.

case—that is, the interior corner of a cube—rays can bounce three times, depending on their initial direction, and in every case they return parallel to their incoming path. The sender of a laser pulse will receive a pulse in return. One of the principles of stealth aircraft design is to absolutely avoid right-angle metallic corners that could make the aircraft light up enemy radar screens. On the other hand, small corner radar reflectors are used for boats to make them easily visible on other ship radars. For wavelengths smaller than the dimensions of the reflector, reliable echoes (retroreflections) will be obtained from a corner reflector. A simulation using wave propagation is shown in figure 2.18. The principle has seen wide use, primarily in optics. There are working retroreflectors on the moon, placed there in 1969 by the *Apollo 11* astronauts. Illuminated with laser pulses from the earth, the reflected signal is so strong that the distance to the surface of the moon can be determined very precisely by timing the return pulses.

Sometimes the exterior of buildings will have balconies or other structures that (accidentally) form excellent retroreflectors. The late Professor Frank Crawford of the Physics Department at Berkeley noticed this on the exterior balconies of Latimer Hall on the Berkeley campus. The balconies returned an excellent echo to the sender from a range of different directions.

We conclude our discussion of waves propagating and interacting with different objects with two complex yet informative examples. In figure 2.19, we use a scenario run in *Ripple* to illustrate reflection, refraction, diffraction, and interference, all plainly visible after the wavefronts collide with a block of material with a slower sound speed (which could represent a colder mass of air, or perhaps a heavier gas such as sulfur hexaflouride).

Figure 2.20 shows three different versions (ray tracing, numerical simulation, and hand drawing of an experiment involving liquid mercury) of the result of an off-center source of waves confined to a circular pool.

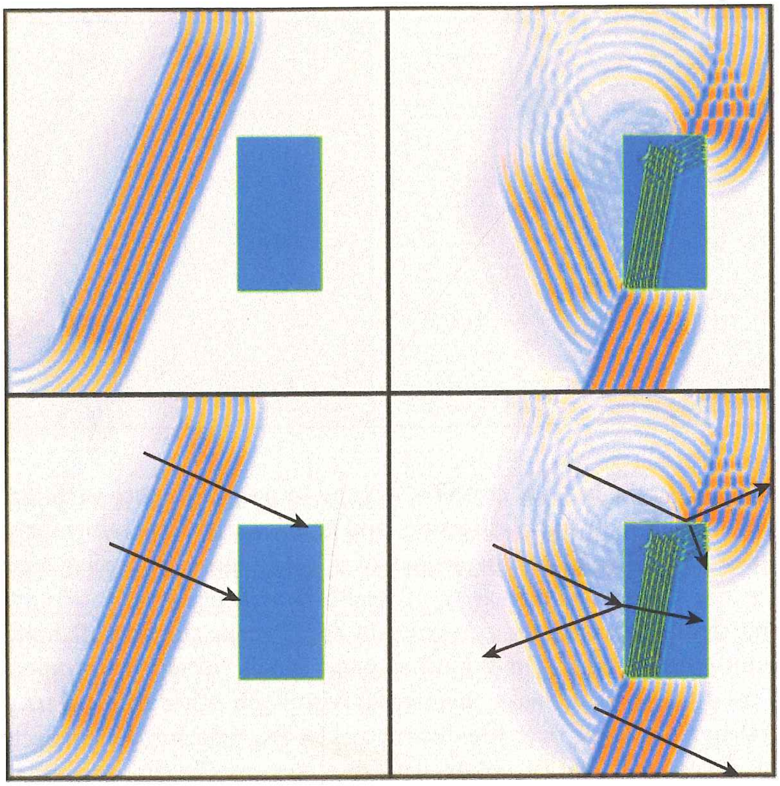
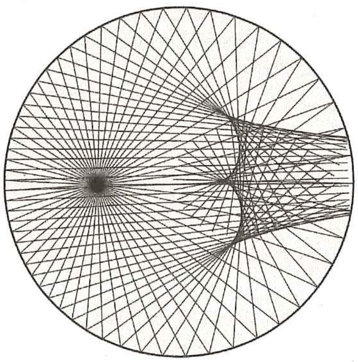
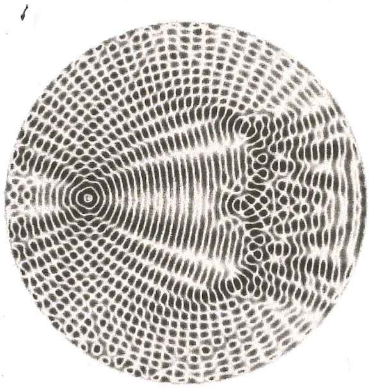


Figure 2.19

A plane wave pulse incident from the upper left travels toward the lower right, in accord with the rule that the energy progresses in a direction perpendicular to the wavefronts. Some ray paths are traced out at the bottom. Reflection, refraction, diffraction, and interference are all plainly visible after the wavefronts collide with a block of material with a slower sound speed. Note the delay in the progress of the wave through the block of material. Also, the wavelength is shorter inside the block. The period must be the same inside the block as it is outside. (Whether you are inside or outside the block, a pulse sent at 1 Hz would have to be received at 1 Hz.) Since the period is the same but the speed is slower, $\lambda = c/f$ implies a shorter wavelength, as seen here.



Ray tracing in *Mathematica*



Ripple simulation

Mercury drops in dish of mercury
Weber brothers, *Wellenlehre*, 1825

Figure 2.20

Three completely different approaches to the same phenomenon: an off-center point source of waves in a circular enclosure. The left image is a ray tracing, obtained by following rays from the off-center source point outward until they hit the circular walls, and then taking specular bounces. The middle image shows the *Ripple* simulation with a sinusoidal source. The right-hand image is the most remarkable, obtained in painstaking detail by watching the wave pattern from liquid mercury dropped periodically at the off-center source point in a circular dish of liquid mercury. The original drawing, published in 1826 in *Wellenlehre*, by Ernst Heinrich Weber and Wilhelm Eduard Weber, consists of about 200,000 individual handmade dots stylistically representing the antinodes as shaded-relief diamonds. Here, we clearly see the relation between two types of modeling (solving numerical equations simulating the waves as in *Ripple*, and following rays from the source) and the "real thing," as drawn in 1825 from an experiment.

Box 2.2

The SOFAR Channel

In the ocean, the speed of sound increases about 4 m/s for every 1°C increase in temperature. Variable temperature in the ocean therefore results in variable sound speed, which has the dual effect of refracting waves and making them arrive sooner or later according to the temperatures through which they have passed. In the 1970s, Walter Munk and Carl Wunsch suggested the idea of ocean acoustic tomography: measuring the temperature of the ocean remotely over large areas and at great depths if desired by sending sound long distances underwater to receiving hydrophones. Sounds are created hundreds of kilometers away from receiving stations, and ray tracing is used to help deduce the temperature profile of the water between the source and the receiver. Depending on the temperature profile, there may be many ways rays can travel from the sound source to the receiver, and generally each path will arrive at a different time and will have passed through different parts of the ocean. This technique is an important part of global earth monitoring of climate change.

Going deeper into the ocean, temperature steadily declines, but the pressure is rising. High pressure increases water density and causes an increase in sound speed. At first, the temperature decline wins, and sound speed decreases with depth. Eventually, at a depth of about 800 meters, the rising pressure overcomes the decline in temperature and causes the sound speed to go up again (see figure 2.21). Thus, as discovered in the United States toward the end of World War II and independently in 1946 in Russia (and kept top secret during the Cold War by both the American and Russian navies), there is a band of minimum sound speed, called the Sound Fixing and Ranging (SOFAR) channel, just under 1 km deep. Since waves always refract toward regions of lower speed, the SOFAR channel is a *waveguide* for sound. Communication over long distances is possible, since the sound energy is held to the channel rather than spreading in three dimensions. Figure 2.22 shows the results of two simple experiments in *Ripple*, with

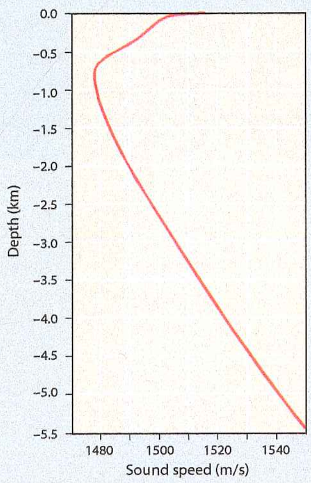


Figure 2.21

Speed of sound versus depth in the ocean. Courtesy Bdushaw.

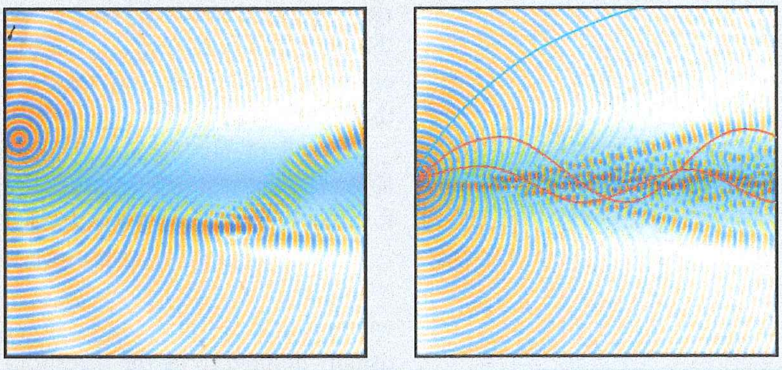


Figure 2.22

Simulations of sound-wave guiding in and near the SOFAR channel, conducted in the *Ripple* Java applet. (Left) Waves are launched from the left, above the middle of the channel (the channel is indicated as a blue hue; the sound speed reaches a minimum in the middle of this region), and pass through the channel. Some of the waves are refracted back and are trapped, oscillating from side to side as they progress down the channel. (Right) Some fraction of the waves launched inside the channel with low-enough angle are captured by the slow-speed channel. Once captured, the waves lose much less amplitude with distance traveled than they would outside the channel. Two representative trapped ray paths are shown in red; an escaping ray is shown in blue. Whales are thought to dive down nearly a kilometer in order to communicate using this sound conduit of the deep, which allows communication perhaps for thousands of kilometers.

The SOFAR Channel *(continued)*

the setting Temperature Gradient 4, which has a band of minimum sound speed, just like the SOFAR channel.

When ships lowered speakers and *hydrophones* (microphones designed to work well underwater) to use the channel, they found they were not alone: strange sounds are heard, believed to be coming from humpback whales diving down to take advantage of the waveguide effects. These whales may communicate with other humpbacks hundreds or perhaps thousands of kilometers away.

2.10 Measures of Sound Power

We have already had a few occasions to discuss sound power. We mentioned the enormous difference between the softest audible sound and sound at the threshold of pain. We have examined the falloff in sound intensity with distance coming from a small source. Whenever a quantity can vary by factors of millions or billions and yet be of significance over its whole range, we need to bring logarithms to the rescue. The *logarithm* measures the *exponent* that gives the number. The *base* of the logarithm is the number we are raising to a power, and the *power* is the logarithm of the number. Thus, by definition $7.3485 = 10^{\log_{10}(7.3485)}$. (We write the base as a subscript on the log.) By trial and error if need be (but of course we now have computers and before that, log tables), you can show that $7.3485 = 10^{0.866199}$, or $\log_{10}(7.3485) = 0.866199$.

Instead of talking about sound intensity, we use the log, which we also call intensity—that is, sound intensity measured in decibels (dB). The formula is simple:

$$I(\text{dB}) = 10 \log_{10}\left(\frac{I}{I_0}\right), \tag{2.5}$$

where $\log_{10}(\ldots)$ is the base 10 logarithm, and I_0 is a reference intensity, usually defined to be the threshold of human hearing. Thus, a 0 dB sound (the logarithm of 1 is 0) is barely audible to those with excellent hearing in a perfectly quiet environment. Since $10\log_{10}(2) \approx 3$, a doubling of sound intensity (for example, two identical instruments instead of one) corresponds to a 3 dB increase. This definition makes clear that a 10 dB increase in sound intensity (measured in dB) corresponds to a ten-fold increase in power. The buzz of a nearby mosquito is about 40 dB, and a normal conversation is about 60 dB. There is 100 times more power in a normal conversational voice than in a mosquito buzz. Still, the mosquito is surprisingly loud, considering a human weighs 10 million times as much as a mosquito.

Other quantities are routinely given as their logarithms. The Richter scale for earthquakes comes to mind; it measures the base 10 log of the *amplitude* of motion of the earth.

Another reason for the decibel measure is that our hearing is essentially logarithmically sensitive. Our impression of loudness is not proportional to sound power, but rather approximately proportional to the *logarithm* of sound power. Table 2.1 gives some typical sounds and their corresponding power in decibels. Sustained exposure to 85 dB sound is considered harmful to hearing; needless to say, a rock concert at 110 dB is almost unquestionably going to do permanent damage.

In section 2.2, we discussed the falloff of sound power with distance from the source. We showed that the power passing through a window drops as $1/r^2$, where r is the distance from the source. There we also introduced the falloff in the *subjective* loudness, which is less rapid. If the distance doubles, from $r \to 2r$, the *objective* sound power I drops from $I = A/r^2$ to $A/4r^2$, where A is characteristic of the source. In terms of decibels, we have

$$I_{dB}(r) - I_{dB}(2r) = 10 \log_{10}\left[\frac{I(r)}{I(2r)}\right] = 10 \log_{10}\left(\frac{4}{1}\right) = 6.02,$$

that is, there is a 6 dB drop in sound intensity measured in decibels for every doubling of distance. This does not account for the effect of the ground, wind gradients, and the like.

Unless they are somehow maintaining a lockstep phase relation, the total intensity of two sources is simply the sum of the individual intensities. For two equally loud trumpets, intensity is $2I$ as compared to I for one trumpet, thus a 3 dB increase is seen when doubling the number of instruments.

Table 2.1

Some Sounds and Their Decibel Equivalents

Decibels	Pressure in pascals	Sound
0	0.00002	Threshold of human hearing
10	0.0000632	Human breathing at 3 m
20	0.0002	Rustling of leaves
40	0.002	Residential area at night
50	0.00632	Quiet home with some appliances on
70	0.0632	Busy traffic
80	0.2	Vacuum cleaner
90	0.632	Loud factory
100	2	Pneumatic hammer at 2 m
110	6.32	Accelerating motorcycle at 5 m
120	20	Rock concert
130	63.2	Threshold of pain
150	632	Jet engine at 30 m (hearing severely damaged)
180	20,000	Rocket engine at 30 m (near-instant death)

Ten trumpets are 10 dB louder than one trumpet, a tenfold increase in power. Because of our logarithmic hearing, ten trumpets subjectively sound only about twice as loud as one.

We are not equally sensitive to sound power at all frequencies—we will delay that discussion until chapter 22.

We can now return to the example at the start of this chapter. Is it reasonable that the sound of a bell can travel 100 miles and be heard aboard a ship? One hundred miles is about 160,000 meters, so the drop in decibels from, say 100 m distance from the town square to 160,000 meters is $10 \log[(100/160,000)^2] \approx 64$ dB. Suppose we say the bell 100 m away was a loud 95 dB. This becomes only about 30 dB at the ship, certainly masked by the probable 50 to 70 dB ambient noise aboard a ship. If we assume the sails can reflect 30% of the sound energy incident on them, and that they concentrate the sound by a factor of 1000 by focusing (see figure 2.7), we have amplification by a factor of 300 at the focal spot on deck.[2] This $10 \log[300] = 25$ dB, so that the sound at the focal spot would be $30 + 25 = 55$ dB; still very soft and probably inaudible except on a very quiet ship, quieter than most houses today.

However, we have been assuming uniform spreading of the sound according to the $1/r^2$ law. On most days, this is not at all the case for outdoor sound propagation over long distances. One reason is gradients in wind velocity, as we shall spell out in more detail in section 28.2. The wind is slowest at ground or sea level, being diminished by friction with the ground, and faster aloft. Sound traveling downwind, therefore, is slowest in its progress at ground level, and faster aloft, since it travels at 344 m/s through air. Sound or indeed any of the usual kinds of waves (light, water waves, and so on) refract toward regions of slower propagation. Therefore, if the wind was somewhat offshore, the downwind portion of the sound, traveling toward the ship, would have been spreading out not in three dimensions, but rather in two dimensions, being prevented from going aloft by refraction. There is only a 34 dB drop for a $1/r$ falloff of sound intensity, as opposed to the 64 dB for $1/r^2$.

In fact, the pattern of sound propagation and intensity downwind is not uniform for another reason, as has been chronicled many times in war and after accidental explosions. This subject is taken up in section 28.2. On a scale of 150 km or more from the source, sound propagation can be controlled by ever-present temperature gradients in the atmosphere, causing a refocusing of the sound that escaped aloft down to the ground about 150 km to 200 km from the source, making this sound many decibels louder than it would have been without the long-range refraction.

[2]Here, we are on the shakiest ground. Clearly, it is impossible for us to know precisely what the sail was doing that day. It is unlikely that it was a perfect shape for concentrating the sound. But decibels are a logarithmic measure, and significant errors of estimation end up as modest changes to the outcome.

Even without these atmospheric focusing effects, the bells might have been just barely audible at the focal point of the sail aboard a very quiet ship. Given more favorable atmospheric conditions, and favorable position and shape of the sail, there seems to be absolutely no doubt that the ringing of the church bells could have been heard 100 miles away.

Box 2.3

How Big?

It can be useful to calculate how big things are. It is easy to be wrong by many orders of magnitude when guessing how far a surface has to move in making a sound wave, or how much energy is in the sound wave, and so on. Grammy Award–winner Eberhard Sengpiel, sound engineer par excellence and lecturer at the Berlin University of the Arts, has compiled a very useful table (table 2.2) that makes it simple to calculate relevant acoustical quantities. Some of the quantities in this table we have not discussed specifically in this book, but we give them here nonetheless for reference.

Please note: The intensity I in the tables is *not* in dB, but bears the relation

$$I\ (\text{dB}) = 10\ \log_{10}\left(\frac{I}{I_0}\right), \tag{2.6}$$

where

$$I_0 = 10^{-12}\ W/\text{m}^2.$$

As an example of the use of the table, suppose we want to discover how far a wall has to move (its displacement amplitude ξ) to create a very loud, 100 dB sound wave of frequency 1000 Hz. First, we find I from equation 2.6 by inserting 100 dB: $100 = 10\ \log_{10}\left[\frac{I}{10^{-12}}\right]$; or $I = 10^{-2}$. Then, from the table, we have

$$\xi = \frac{1}{\omega}\sqrt{\frac{I}{Z}} = \frac{1}{2000\pi}\sqrt{\frac{10^{-2}}{420}} \sim 8 \times 10^{-7}\ \text{m},$$

just under 1 micron. The acceleration a is

$$a = \omega^2\xi = (2000\pi)^2 \times 8 \times 10^{-7} \sim 31\ \text{m/s}^2,$$

or more than three times the acceleration due to gravity.

For a 100 Hz, 100 dB tone, the displacement is 10 times larger and the acceleration is one-tenth as large as for a 1000 Hz, 100 dB tone. If I is 40 dB at 1000 Hz, a soft sound but still well above the threshold for hearing in a home environment, $I = 10^{-8}\ \text{W/m}^2$, a million times smaller than for 100 dB. The displacement ξ is proportional to square root of the intensity, so ξ is 1000 times less at 40 dB than at 100 dB or at 1000 Hz $\xi \sim 10^{-9}$m—just one nanometer, or the width of a few atoms!