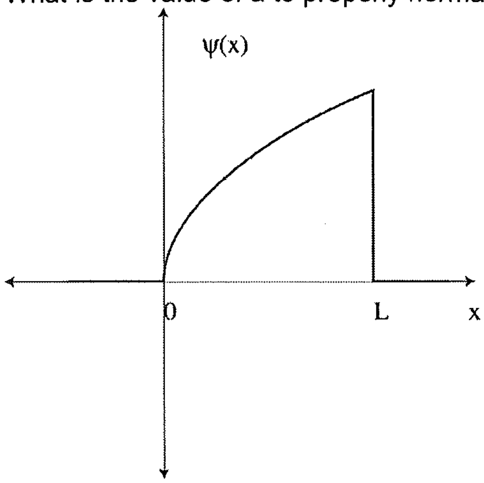
Solutions

Physics 262 Fall 2010 Practice Exam #6

1&2. A wavefunction for a particle is shown:

$\psi(x) = a\sqrt{x}$, for 0<x<L. L=0.01 m.

What is the value of a to properly normalize this wavefunction? (in m^{-1})



$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

$$\int_0^L a^2 x\, dx = a^2 \frac{L^2}{2} = 1$$

$$a = \frac{\sqrt{2}}{L} = \frac{1.414}{0.01} = 141.4\ m^{-1}$$

In %

3&4. What is the probability the particle will be found between 0 and L/2 ?

$$P = \int_0^{L/2} a^2 x\, dx = a^2 \frac{1}{2}\left(\frac{L}{2}\right)^2 = \frac{a^2 L^2}{8} = 0.25 = 25\%$$

5&6]. A particle is in the mixed wavefunction $\psi = a(0.3\psi_1 + 0.1\psi_2)$, where ψ_1 and ψ_2 are properly normalized stationary states of the potential.

What is *a*, for proper normalization of the mixed wave?

$$\sum_i P_i = 1 = (0.3a)^2 + (0.1a)^2 = (0.09 + 0.01)a^2 = 0.1a^2$$

$$a^2 = 10 \qquad a = 3.17$$

7&8] What is the probability that the particle will be observed in state ψ_1 ?

$$P_1 = 0.3^2 a^2 = 0.9 = 90\%$$

9. (3 pts) For the infinite potential well shown (next page), sketch as accurately as you can the n=4 quantum state, with energy 9 eV.

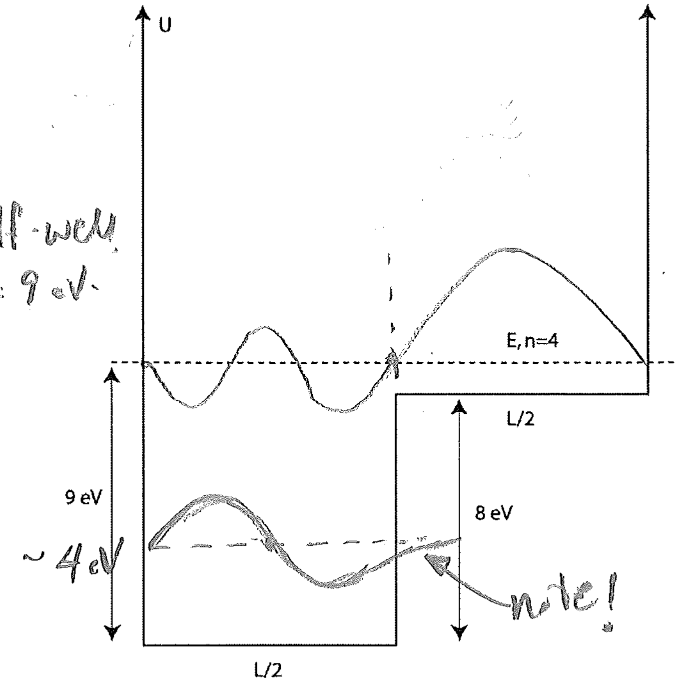
$$\frac{p_L^2}{2m} = 9\,\text{eV} \qquad \frac{p_R^2}{2m} = 1\,\text{eV} \qquad \frac{p_L}{p_R} = \frac{3}{1} = \frac{\lambda_R}{\lambda_L}$$

10. (3 pts) Sketch the n=2 quantum state. What is its energy, approximately?

Note that, in the half-well $n=3$ would be at $E = 9$ eV.

So $n=2 \rightarrow E = 4$ eV

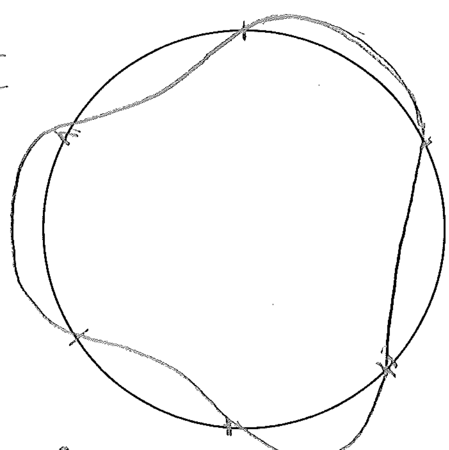
(Actually, a little lower.)



11. (1 pt) Draw the n=3 Bohr wave on the orbit below.

12&13. Suppose h were 10^{-10} Js. What would be the radius of the orbit in angstroms?

Recall $F = \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$



Bohr $mvr = \frac{nh}{2\pi} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{v}$

$$V = \frac{e^2}{2\epsilon_0 nh}$$

So $r = \frac{nh}{2\pi}\frac{1}{mV}$

$$r = \frac{nh}{2\pi m}\cdot\frac{2\epsilon_0 nh}{e^2} = \frac{\epsilon_0 n^2 h^2}{\pi m e^2} = \frac{8.85\times10^{-12}\cdot 3^2\cdot 10^{-20}}{\pi\cdot 9.1\times10^{-31}\,(1.6\times10^{-19})^2} = 1.1\times10^{35}\,\text{m}$$

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