Physics 161 Fall 2010 Exam 3 SOLUTIONS

Closed book closed notes calculators OK. Enter 0,0 for 0. Give fields in N/C

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \mathrm{Nm^2/C^2}$$

A thin rod of length L=1 m has charge 0.001 C uniformly distributed.

1] What is the sign of the x-component of the E field at P, which is 1 m from the end of the rod (see sketch)?

a) +   b) -   c) $E_x = 0$ at P.

2&3] What is the magnitude of the x-component of the E field at point P? 4.5×10^6 N/C.

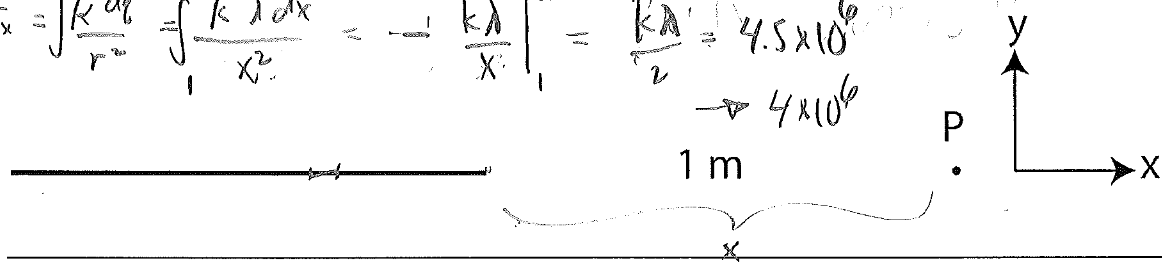
4] What is the sign of the y-component of the E field at P?

a) +   b) -   c) $E_y = 0$ at P.

5&6] What is the magnitude of the y-component of the E field at point P? 0

$$\int dE_x = \int \frac{k\,dq}{r^2} = \int_1^2 \frac{k\lambda\,dx}{x^2} = -\frac{k\lambda}{x}\Big|_1^2 = \frac{k\lambda}{2} = 4.5\times10^6$$

$\rightarrow 4\times10^6$



A quarter ring of radius 1 m has -0.02 C of negative charge uniformly distributed.

7] What is the sign of the x-component of the E field at P on the axis at z=2 m?

a) +   b) -   c) $E_x = 0$ at P.

8&9] What is the magnitude of the x-component of the E field at P?

10] What is the sign of the z-component of the E field at P?

a) +   b) -   c) $E_z = 0$ at P.

11&12] What is the magnitude of the z-component of the E field at P?

$$dE = \frac{k\,dq}{R^2} = \frac{k\lambda a\,d\theta}{R^2}$$

$$\frac{dE_x}{dE} = \frac{x}{R} = \frac{a\cos\theta}{R}$$

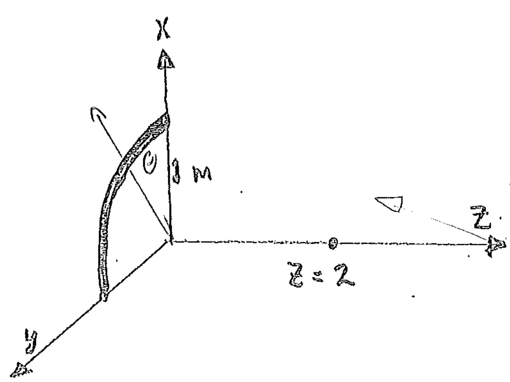
$$E_x = \int dE_x = \frac{k\lambda}{R^3}a^2\int_0^{\pi/2}\cos\theta\,d\theta \quad (=1)$$

$$E_x = \frac{k\cdot Q a^2}{R^3\left(\frac{2\pi a}{4}\right)}$$

$$R = \sqrt{2^2+1^2} = 2.236$$

$$R^3 = 11.18$$

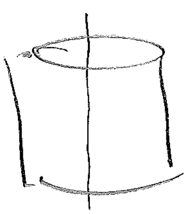
$$= \frac{2kQa}{R^3\cdot\pi} = 1\times10^7 \quad \boxed{8\&9}$$



$$E_z = \int dE_z = \frac{k}{R^3}zQ = 3.22\times10^7 \quad \boxed{11\&12}$$

Applications of Gauss' Law

13&14] An infinite line of charge has a uniform charge density of 0.02 C/m. What is the magnitude of the electric field at a distance of 40 m away from the rod?



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = \frac{\lambda \cdot \cancel{L}}{\epsilon_0} = 2\pi r \cancel{L}\cdot E$$

$$E = \frac{\lambda}{2\pi r \epsilon_0} = \frac{2\lambda k}{r}$$

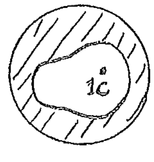
$$= 9\times10^{6}\ N/C$$

15] A hollow conducting shell has a total net charge of 3 C on it. The outer surface of the shell is spherical with a radius of 20 m. In the hollow, off center, there is a point charge of 1 C. What is the sign of the charge on the **inner** surface of the shell?

a) + b) - c) There is no charge on the **inner** surface

16] What is the magnitude of the surface charge on the **inner surface**, to the nearest coulomb? (0-9). 1

17&18] What is the magnitude of the field just outside the outer surface of the shell?



$$Q_{tot} = 4C.$$

$$E = \frac{kQ}{r^2} = 9\times10^{7}\ N/C.$$

Note: $\sigma = \frac{4C}{4\pi r^2}$ and $E = \frac{\sigma}{\epsilon_0}$ → same result

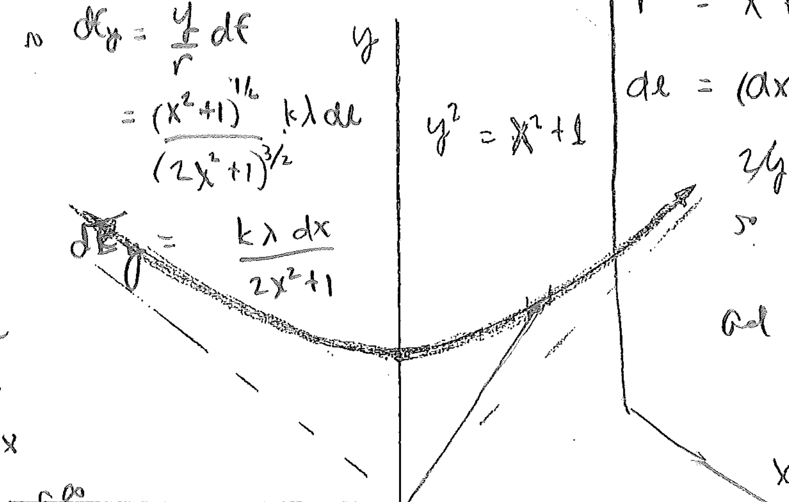
Extra Credit

19&20] What is the magnitude of the electric field at the origin from an infinite insulating wire shaped like a hyperbola ($y^2 = x^2 + 1$; x,y in meters) with uniform charge density 1 μC per meter?

$$\frac{dE_y}{dE} = \frac{y}{r} \quad\text{so}\quad dE_y = \frac{y}{r}\,dE$$

$$= \frac{(x^2+1)^{1/2}}{(2x^2+1)^{3/2}}\,k\lambda\,dl$$

$$dE_y = \frac{k\lambda\,dx}{2x^2+1}$$



$$r^2 = x^2+y^2 = 2x^2+1$$

$$dl = (dx^2+dy^2)^{1/2}$$

$$2y\,dy = 2x\,dx$$

so $dy = \frac{x}{y}dx = \frac{x\,dx}{(x^2+1)^{1/2}}$

and $dl = \left(1+\frac{x^2}{x^2+1}\right)^{1/2}dx$

$$= \left(\frac{2x^2+1}{x^2+1}\right)^{1/2}dx$$

Substitute

$\xi = \sqrt{2}\,x$

$d\xi = \sqrt{2}\,dx$

$$E_y = \int_{-\infty}^{\infty}\frac{k\lambda}{\sqrt{2}}\frac{d\xi}{\xi^2+1} = \frac{k\lambda}{\sqrt{2}}\arctan\xi\Big|_{-\infty}^{+\infty} = \frac{k\lambda\pi}{\sqrt{2}}$$

$$= 2\times10^{4}$$