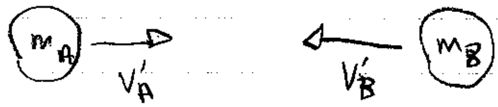
1D Collisions in "center of mass" reference frame (Prime = CM frame)



Since $\vec{F}_{ext} = M_{tot}\,\vec{a}_{cm}$, with no external force $\vec{a}_{cm} = 0$.

So $\vec{v}_{cm}$ = constant

If we move at this velocity, then TO US $\vec{V}_{cm}' = 0$.

$$V_{cm}' = \frac{m_A V_A' + m_B V_B'}{m_A + m_B} = \frac{P_{tot}}{M_{tot}} = 0 \qquad (1D)$$

So

$$\boxed{\frac{m_A}{m_B} = -\frac{V_B'}{V_A'}}$$

both before AND after COLLISION!

If elastic, then $\frac{1}{2} m_A V_{A1}'^2 + \frac{1}{2} m_B V_{B1}'^2 = \frac{1}{2} m_A V_{A2}'^2 + \frac{1}{2} m_B V_{B2}'^2$

Since the RATIO of $\frac{V_B'}{V_A'}$ is FIXED, this equation can only be satisfied if $V_{A2}' = V_{A1}'$, $V_{B2}' = V_{B1}'$ no collision

OR $V_{A2}' = -V_{A1}'$, $V_{B2}' = -V_{B1}'$.

That is, masses bounce off with their same speeds, but directions reversed!

What does this collision look like in the "laboratory frame"?

Consider the special case $V_B = 0$.

Then $V_{cm} = \dfrac{M_A V_A}{M_A + M_B}$

☆ Now, $V_{x/L} = V_{x/c} + V_{c/L}$

$$\begin{array}{ccc} V_{x/L} & V_{x/c} & V_{c/L} \\ \Downarrow & \Downarrow & \Downarrow \\ V & V' & V_{CM} \end{array}$$

X = A or B
L = laboratory frame
C = center of mass frame

So

$$V_{B2} = V'_{B2} + \frac{M_A V_A}{M_A + M_B} = -V'_{B1} + \frac{M_A V_A}{M_A + M_B}. \quad (\text{since } V'_{B2} = -V'_{B1})$$

Use ☆ to find V'_{B1}:

$$V_{B1} = V'_{B1} + \frac{M_A V_A}{M_A + M_B}$$

$$0 =$$

So $V'_{B1} = -\dfrac{M_A V_A}{M_A + M_B}$ and $V_{B2} = \dfrac{2 M_A V_A}{M_A + M_B}.$ [EQN 8.25!]

Exercise: Derive EQN 8.24 in the same way!