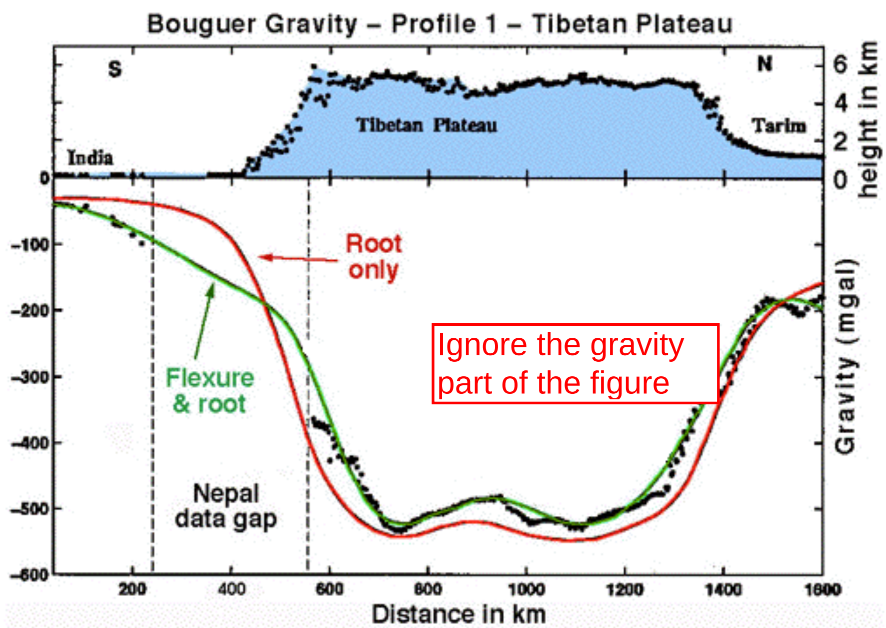
PHYC 480/581 **EARTH DEFORMATION**
HOMEWORK 3 DUE FRIDAY, DECEMBER 5TH

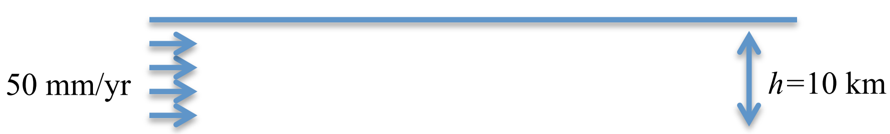
1. Incompressible fluid flow on an inclined plane. Problem 6-4 in T&S.

2. In many laboratory experiments, we see that crustal minerals (dominated by quartz and feldspar) undergo ductile flow at lower-crustal temperatures and pressures. This observation raises the possibility that deformation in the lower crust may be treated as a simple Newtonian viscous flow in a 1D channel.

   (a) Using the N-S topographic profile below from the western Himalaya, estimate the pressure as a function of position due to the weight of the mountains. (You can digitize or make an approximated profile in Matlab and calculate the pressure along the profile.



   (b) The weight of the mountains provides a pressure gradient that acts within a lower crustal channel (channel thickness h=10 km) occupied by a Newtonian fluid.



   On the south side (left) assume that material flows *in* from the south at a uniform rate of 50 mm/yr (approximate convergence rate between India and Tibet). Using the relation between velocity and pressure-gradient in a 1D channel (eqn 6-10 in the book):

$$\eta \frac{d^2u}{dx^2} = \frac{dp}{dx}$$

solve for the predicted flow within the channel due to the weight of the mountains if the lower crustal viscosity is 10^{19} Pa s. For this, you can assume that the topography far away from the Tibetan plateau is flat (no pressure gradient) and that the walls of the channel are stationary. Plot the expected velocity profile as a function of depth within the channel at x= 600, 1000, and 1400 km. (HINT: Re-use your 1D finite-difference code for the heat equation – now the constant of proportionality is the viscosity and the "heat source" term corresponds to the pressure gradient. The 50 mm/yr incoming velocity is a boundary condition at x= 0 km.)

(c) How would your predicted flow rate change if the lower crustal viscosity is 10 times larger? Or ten times smaller?

3. Corner Flow. (See section 6-11).

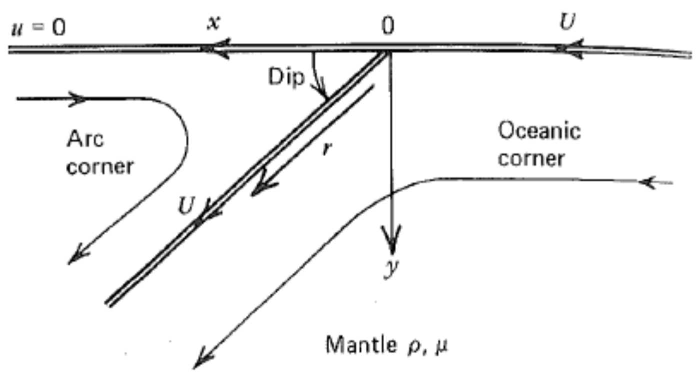


Figure 6.18 Viscous corner flow model for calculating induced flow pressures on a descending lithosphere.

a. First, using the code attached for the analytic solution for corner flow, calculate the velocity and streamlines for the arc and ocean corners of a subduction zone as shown in the figure below, where the subducting slab enters the mantle at a dip of 45 degrees.

b. From the velocity fields you found in (a), use the equation of motion for an incompressible fluid (section 6-8) to solve for the pressure-gradients in the problem. (These are the pressure-gradients due to flow, not due to the weight of the rocks above a given point.)

c. Assuming a mantle density of 3300 kg/m^3, how do your pressure gradients in (b) compare to the "lithostatic" gradient due to the weight of rocks above?

```
% Analytic Corner Flow Solution
% from Bachelor Ch 4
% Mousumi Roy, UNM

clear all;
clf

% corner parameters
thet0 = pi/4; % in radians
thet2 = thet0*thet0;
st0   = sin(thet0);
st2   = st0*st0;
ct0   = cos(thet0);
denom = [thet2 - st2];

u1    = 1; % in some units
u2    = 5;

A = - [u1*thet2 - u2*thet0*st0]/denom;
B = 0;
C = [u1*thet0 + u2*thet0*ct0 - (u2 + u1*ct0)*st0]/denom;
D = st0*(u1*st0 - u2*thet0)/denom;

%define domain in cylindrical coordinates
th = thet0*[0:0.1:1];
r = [0.001:5:100]; % avoid singularity at origin if we want pressures

[TH, R] = meshgrid(th, r);

% find cartesian domain
[x y] = pol2cart(TH,R);

f    = A*sin(TH) + C*TH.*sin(TH) + D*TH.*cos(TH);
psi  = R.*f;

ur = A*cos(TH) + C*(sin(TH) + TH.*cos(TH)) + D*(cos(TH) - TH.*sin(TH));
ut = -f;

vx = [ur.*cos(TH)-ut.*sin(TH)];
vy = [ur.*sin(TH)+ut.*cos(TH)];

quiver(x, y, vx, vy); hold on
plot(x,y,'b.');
axis equal
contour(x,y,psi);
```