PHYC 480/581 **EARTH DEFORMATION**
HOMEWORK 2 DUE TUESDAY, OCTOBER 7TH

1. A cooling half-space. If we assume the depth x is in km and time, t is in my, and $\kappa = 25$ km^2/my, then the solution for a cooling half-space is

$$T(x) = T_0 + \frac{\Delta T}{2}\left[1 + erf\left(\frac{x}{10\sqrt{t}}\right)\right], \qquad \text{(Eqn. 1)}$$

where T_0 is the initial temperature, ΔT is the thermal perturbation imposed at the surface of the half-space at t=0, and $2\sqrt{\kappa} = 10$ km/my$^{1/2}$.

Program this equation into Matlab and plot the temperature as a function of depth (x from 0-35 km) for T_0=0°C and perturbation ΔT=100°C, at t=0.01, 0.1, 1, 10, 100 my.

2. Now solve this same problem using a finite difference approach in Matlab. First, define a model domain using an array, x, from 0 to 35 km. Choose a discretization dx and a time step, dt.

Initial condition: Start with a temperature array initially, T_{init}, that is zero everywhere (T_{init} is the same length as the x array).

Boundary condition: Set the first element of the temperature array, which corresponds to zero depth, to be $T(1)=\Delta T$=100°C, representing a temperature of ΔT=100°C at the surface.

(a) Show that your solution is stable when $\kappa dt/dx^2 << 1$ and unstable (you get nonsense when $\kappa dt/dx^2 >> 1$.
(b) Using a stable discretization, show that you can match the analytically expected solution (from question 1).

3. In lecture we discussed the linear superposition idea for the cooling of a vertical, planar magma-filled crack (a "dike" in geology jargon).

   a. Use the same type of linear superposition, to *derive* the following analytic solution for the cooling of a *horizontal* planar, magma-filled crack (a "sill" in geology jargon):

$$T(x,t) = T_b + \frac{\Delta T}{2}\left[erf\left(\frac{x - x_1}{10\sqrt{t}}\right) - erf\left(\frac{x - x_2}{10\sqrt{t}}\right)\right] - \frac{\Delta T}{2}\left[erf\left(\frac{x + x_2}{10\sqrt{t}}\right) - erf\left(\frac{x + x_1}{10\sqrt{t}}\right)\right]$$

   Here, ΔT is the excess temperature of the sill, T_b is the initial background geotherm (e.g., linear), and x_1 and x_2 are the depths to the top and bottom of the sill.

REMEMBER that we want the analytic solution to automatically satisfy the boundary condition that the temperature is zero at the surface – this means you will need an "image sill" in the air to counteract the effects of the actual sill.

b. Now program the analytic solution into Matlab, assuming that the background linear geotherm is T_b=25 ^oC/km, ΔT=500 ^oC, and x_1=2 km and x_2=2.5 km (sill is 500 m thick). Plot the temperature as a function of depth (x from 0-35 km) at t=0.01, 0.1, 1, 10, 100 my.

c. Now solve this same problem using a finite difference approach in Matlab. First, define a model domain using an array, x, from 0 to 35 km. Choose a discretization, *dx,* and a time step, *dt.*

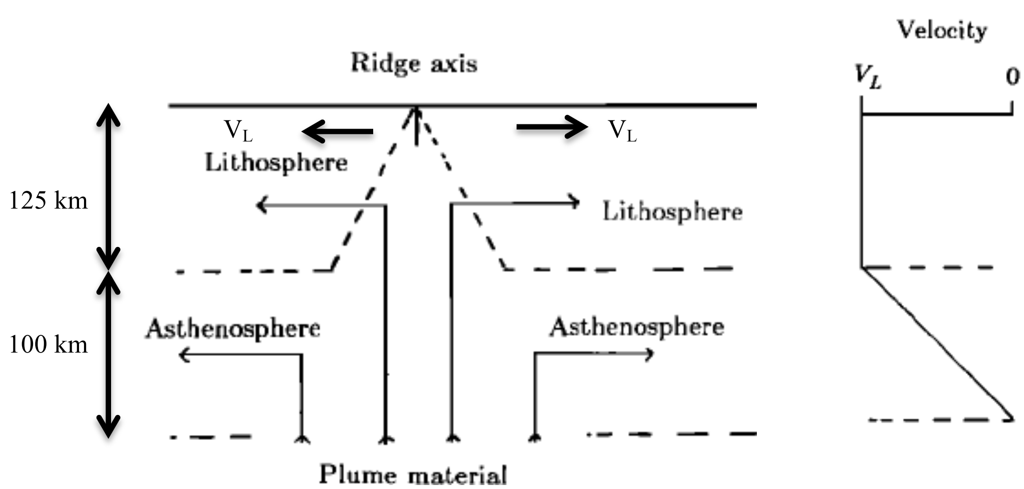
Initial condition: Start with a temperature array initially, T_{init}, that is equal to T_b everywhere (T_b is the same length as the x array and is linearly increasing with x at 25 ^oC/km).

Boundary condition: Set the first element of the temperature array, which corresponds to zero depth, to be $T(1)=0^oC$, representing a fixed surface temperature of 0^oC at the surface.

Plot the temperature as a function of depth (x from 0-35 km) at t=0.01, 0.1, 1, 10, 100 my and compare to your analytic results in (b) for confirmation.

4. Plumes vs. No Plumes. There is an ongoing debate about the role of deep-sourced mantle plumes in the observed volcanism at so-called "hot spots" such as Hawaii and Iceland. You can find out more here: http://www.mantleplumes.org/

First, lets assume that there is a deep-seated mantle plume beneath the mid-ocean ridge in Iceland.



As shown in the diagram, the lithosphere at the mid-ocean ridge must move apart, so we can assume that the material upwelling in the plume replaces the material moved out laterally by plate spreading, at least down to the average depth of the melting region. Assume that the lithosphere and the asthenosphere melting region are 125 km and 100 km thick, respectively, and the velocity profile in the asthenosphere is linear, as shown.

a. Find the volume flux of material in the plume for every km along the ridge axis (think of this 1 km length as distance in/out of the page) as a function of the full spreading rate, V_L.

b. Assuming typical mantle density and specific heat capacity, and assuming that the plume material is hotter than average mantle by ΔT=200 ^{o}C, what is the volumetric heat flux (again per 1 km along the ridge axis) supplied by the plume?

c. If there is no plume beneath Iceland, but instead all the excess melting is just due to anomalously melt-able mantle, then we might expect a normal ridge-like thermal structure beneath Iceland. To get an idea of what that expected thermal structure is, use the *plate-cooling model* parameters for the global model GDH1 (Stein and Stein, 1992; on website), and calculate the temperature as a function of depth and distance from the ridge-axis – this would be easiest to do in Matlab. To do this, you need to decide how many terms to keep in the series expansion and investigate how the solution changes if you keep more/less terms.

d. Use the plate cooling model to plot out the predicted heat flow (per 1 km along the ridge axis) into the *base of the plate* (at $y=y_L$) within a 200 km wide region surrounding the ridge. (You will need to integrate the heat flow going into the base of the plate-cooling model over x=-100 km and x=+100 km.) Comparing this number to your answer in (b), would you say that heatflow data can distinguish between the presence and absence of a plume beneath Iceland?

e. Finally, plot out the predicted *surface* heat flow (per 1 km along the ridge axis) from the GDH1 model. How does this compare to the actual heat flow data from Iceland? (Your comparison may be qualitative here. See the discussion in A&G-2003-Stein-1.8-1.10.pdf on website).