PHYC 480/581 **EARTH DEFORMATION**
HOMEWORK 1 DUE TUESDAY, OCTOBER 7TH

For this homework, unless otherwise specified, use the following constants:

$$\rho_c = \rho_s = 2600 \text{ kg/m}^3 \text{ and } \rho_m = 3300 \text{ kg/m}^3,\ E = 70 \text{ GPa, and } \nu = 0.25.$$

1. Isostasy and sedimentary basins: T & S, Q. 2-5. (5)

2. A tensor, represented by a 3 x 3 matrix A_{ij}, may be rotated using a transformation matrix R_{ij}, where the rotated tensor is given by multiplying the original tensor on the left:

$$A' = R\ A\ R^{-1}$$

   a) Write down the rotation matrices that represent a 90^o counter clockwise rotation about the x-axis, y-axis, and z-axis. (6)
   b) If I told you that the tensor A_{ij} describes the elastic properties of a crystal which has a symmetry so that A_{ij} is unchanged upon rotation by 90 degrees around any of the axes, x, y, or z, then how many independent components does A_{ij} have? (4)

3. Stress transformation for a 2D stress-state: T & S, Q. 2-13 (5)

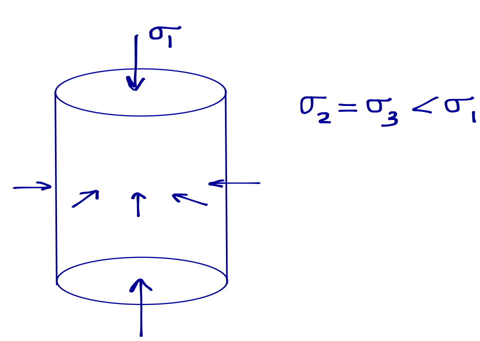
4. For the 3D stress tensor given, find:

$$\sigma_{ij} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

a) The isotropic part. (5)
b) The deviatoric stress tensor. (5)
c) The three principal stresses and the components of the three vectors representing the principal directions. (5)

If you use Matlab to answer (c), show that the principal stress components and axes are correct.

5. In a laboratory experiment, samples of Navajo sandstone were subjected to uniaxial compression (see diagram). The orientation of the sample and the stresses were kept fixed, but the magnitudes of the applied stresses were varied with time. The experiments found that the samples failed at the following conditions:



Failure occurs at (all are in MPa):

Test #	Confining Pressure, σ_2	Uniaxial Compression, σ_1
1	20	1100
2	40	1700

Assume that the rock samples are all initially intact, with a cohesion of C_0, and fail according to a Mohr-Coulomb type of failure criterion, $\tau = C_0 + \mu\sigma$ where τ and σ are the shear and normal stresses on the failure plane. Using the above, find (you should use graph paper and the Mohr circle):

a) The coefficient of friction and the cohesion (5)

b) The orientation of the failure planes in both tests relative to σ_1. (5)

c) The maximum shear stress in each of the two tests above. (5)

d) In a third test, the sample was found to fail on a plane that is at a completely different orientation than tests 1 and 2. You suspect that the sample had a pre-existing crack in it. What possible orientations could the pre-existing crack have so that the sample fails on it (without making a new fracture)? Remember, on a pre-existing crack the cohesion will be zero. (10)

6. Strain accumulation at the San Andreas fault. Just NE of Los Angeles, the San Andreas fault trends approximately N65°W - S65°E. To within observational error, the displacement gradient there is observed to be (annual average):

$$\begin{pmatrix} 0.15 & 0.24 \\ 0.00 & -0.15 \end{pmatrix}$$

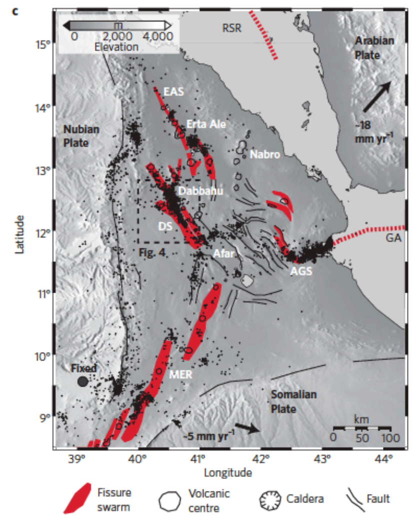
Where the x-axis points East and y is North and the units are in 10^{-6} strain.

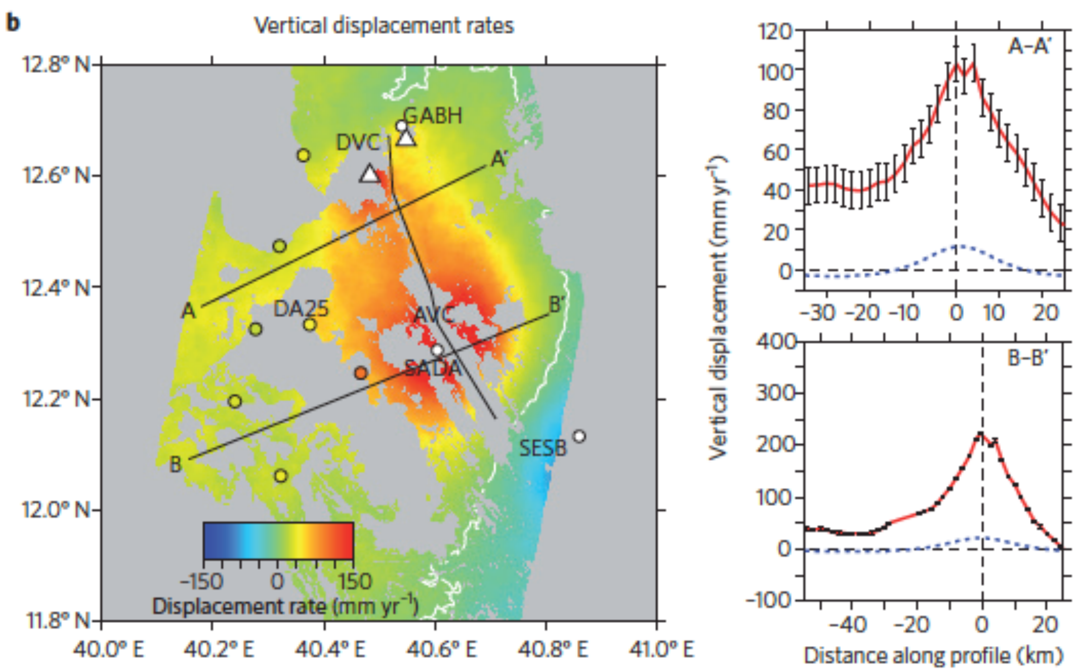
a) Write down the 2D strain tensor, the rotation tensor, and the dilatation. Is this what you expect if the San Andreas is a strike-slip fault? (5)

7. Flexure of oceanic plates: problem 3-19 in T&S. (5)

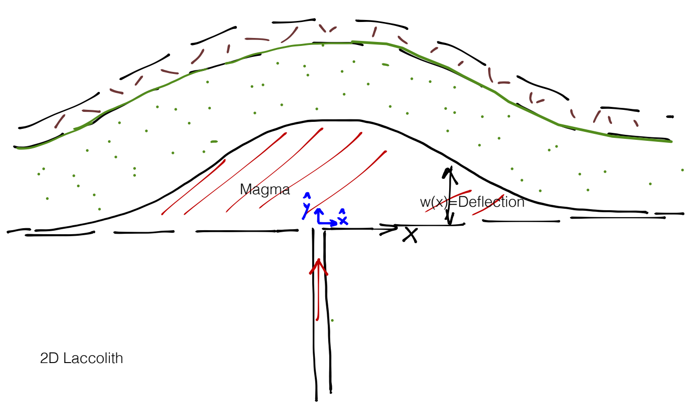
8. Flexure on continents: Problem 3-22 in T&S. (5)

9. <u>Flexure above a laccolithic magma body.</u> The figure below shows inferred rate of elevation change due to injection of magma beneath the Dabbahu rift zone in Afar (location map shows faults and seismicity in black dots). The large uplift observed since 2006 is believed to be the result of the inflation of a magma body beneath the region. You may wish to take a look at the paper by Wright et al. in Nature Geoscience in our class webpage for more info.





Lets assume that the injection of magma occurs into a 2D “laccolith” which is shown here in a cartoon cross-section:



a. Using the flexure equation, assuming that the magma pressure holds up the deformed rock layers above the laccolith, find an analytic expression for the deflection, w(x). The parameters you will need to include are the depth of the floor of the laccolith, and the magma pressure, P. (Hint: assume that the rock above the magma body is simply bent and not eroded/removed, etc. Also, take dw/dx = 0 at the edges of the structure and look for solutions in the form of a polynomial expansion in x.) (20)

b. If the magma body in Dabbahu can be approximated as a 2D laccolith, find its depth and overpressure. To do this, fit the annual elevation change along B-B' assuming a 2D laccolith (we assume here that a rate of 100 mm/yr will average to 100 mm of uplift in one year). Use the relation between the laccolith width L and flexural rigidity D given below (typical magma densities are 2200 kg/m^3 for granitic magmas):

$$D = gL^4 (\rho_c - \rho_{magma})/256$$

(This is a rearrangement of equation 3-127 relating flexural parameter and rigidity.)

You will need to decide on a measure of "goodness of fit", e.g., the root-mean-square error. You can also use the canned Matlab routine 'polyfit' to generate a best fit using the functional form of the deflection we derived in class. Be careful about units!! (10)

(b) Does your inferred laccolith depth and width seem reasonable given the tectonic setting of the area? (5)

(c) Assume that dikes represent magma-filled cracks that nucleate in regions of tensional stress. Assuming that dikes initiate at the roof of the magma chamber, calculate the location(s) at which it is most likely that a dike will form above the laccolith. (10)

10. Consider a broken plate without topographic loading; assume that the plate end is located at x=0 and that the effective elastic plate thickness is T_e = 20 km. The equation for one-sided plate deflection subject to an end load V_0 and bending moment M_0 (both acting at x=0) is given by:

$$w(x) = e^{-x/\alpha}\left\{B\sin\frac{x}{\alpha} + A\cos\frac{x}{\alpha}\right\}$$

where $A = (V_0\alpha + M_0)\dfrac{\alpha^2}{2D}$ (Eqn. 3-151 in T&S)

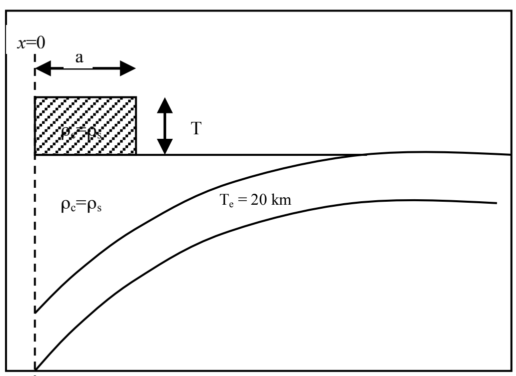
$$B = -M_0\frac{\alpha^2}{2D}$$

(a) Use the Matlab program *notopo.m* to plot *w(x)* for the case where V_0 = -1.5x10^{12} N/m and M_0 = -5x10^{17} N.

(b) How does the deflection change if you try setting
 (i) V_0 = 0 and M_0 = -5x10^{17} N, and
 (ii) V_0 = -1.5x10^{12} N/m and M_0 = 0?
 (Note: To put multiple plots on the same axes, use the command 'hold on'. Then run the program again to make a new plot. Either label the plots by hand, or use the 'gtext' function in Matlab.)

(c) Try using different values of Te. What is the general relationship between flexural response and the effective elastic plate thickness?

11. Now if we do have some topographic load on the plate, say a square plateau of width a, and height T, as shown in the sketch below, the deflection is given by the equations below:



$$w(x) = e^{-x/\alpha}\left\{B\sin\frac{x}{\alpha} + A\cos\frac{x}{\alpha}\right\} + \frac{\rho_c T}{2(\rho_m - \rho_c)}\left[2 - \cos\left(\frac{a+x}{\alpha}\right)e^{-\left(\frac{a+x}{\alpha}\right)} - \cos\left(\frac{x-a}{\alpha}\right)e^{\left(\frac{x-a}{\alpha}\right)}\right] \quad \text{for } |x| <= a$$

(inside plateau)

$$w(x) = e^{-x/\alpha}\left\{B\sin\frac{x}{\alpha} + A\cos\frac{x}{\alpha}\right\} + \frac{\rho_c T}{2(\rho_m - \rho_c)}\left[\cos\left(\frac{a-x}{\alpha}\right)e^{\left(\frac{a-x}{\alpha}\right)} - \cos\left(\frac{x+a}{\alpha}\right)e^{-\left(\frac{x+a}{\alpha}\right)}\right] \quad \text{for } |x| > a$$

(outside plateau)

(a) For the case where $V_0 = M_0 = 0$ (i.e. no end load or bending moment acting at the plate end), and the topography is T with width a, then the constants A and B above are:

$$A = \frac{\rho_c T}{(\rho_m - \rho_c)} \sin\left(\frac{a}{\alpha}\right) e^{-\left(\frac{a}{\alpha}\right)}$$
$$B = -A$$

Use this information and the Matlab code *withtopo.m* to plot the deflection *w(x)* for the case where $V_0 = M_0 = 0$, but the plateau has height $T = 1$ km and width $a = 30$ km. You will need to go in and modify the code withtopo.m and put in the equations above for A and B.

(b) Because the flexure equation is linear, the solution for 1(a) and 2(a) above can be simply added to give the full solution for loads $V_0 = -1.5\text{x}10^{12}$ N/m and $M_0 = -5\text{x}10^{17}$ N, **and** topography $T = 1$ km and $a = 30$ km. This represents loading with topography and the specified end load and bending moment. Plot this solution on the same graph as for 2(a).

(c) Comparing the two graphs for 2(a) and 2(b), we see that the effect of adding the end load and bending moment is quite important to the shape of the basin. To investigate this further, repeat question (2a&b) but for a plateau with $T = 4$ km and $a = 500$ km (half the size of Tibet). How important are the end load and bending moment in this case? Therefore, could one use the shape of the Ganges foredeep basin south of the Tibetan Plateau to study the effects of slab pull on the Indian plate?