Lecture 40

PHYC 161 Fall 2016

Announcements

DO THE ONLINE COURSE EVALUATIONS
- response so far is < 28 %

LC Circuits

- Do you recognize this equation?

$$\frac{d^2q}{dt^2}+\frac{1}{LC}q=0$$

- What if I just changed q to x and renamed the constant?

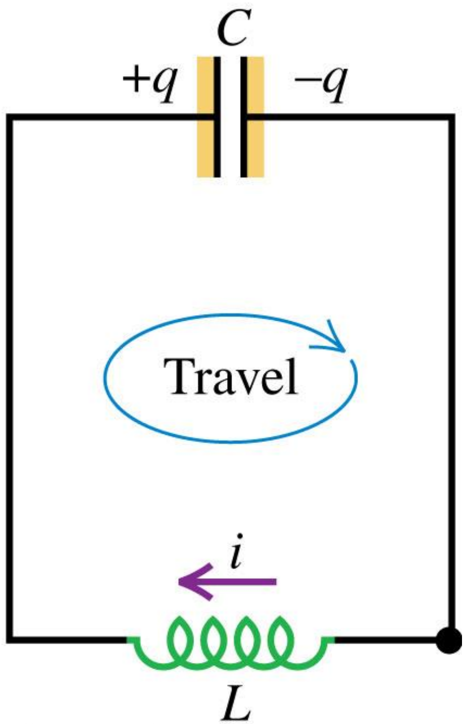
$$\frac{d^2x}{dt^2}+\omega^2x=0$$

- So the equation just describes oscillations with a frequency:

$$\frac{d^2q}{dt^2}+\frac{1}{LC}q=0\Rightarrow$$

$$q=Q_0\cos(\omega t+\phi)$$

$$\omega=\sqrt{\frac{1}{LC}}$$



Analogy to Spring-Mass Oscillations

$$\frac{d^2x}{dt^2}+\frac{k}{m}x=0\Rightarrow$$

$$x=X_0\cos(\omega t+\phi)$$

$$\omega=\sqrt{\frac{k}{m}}$$

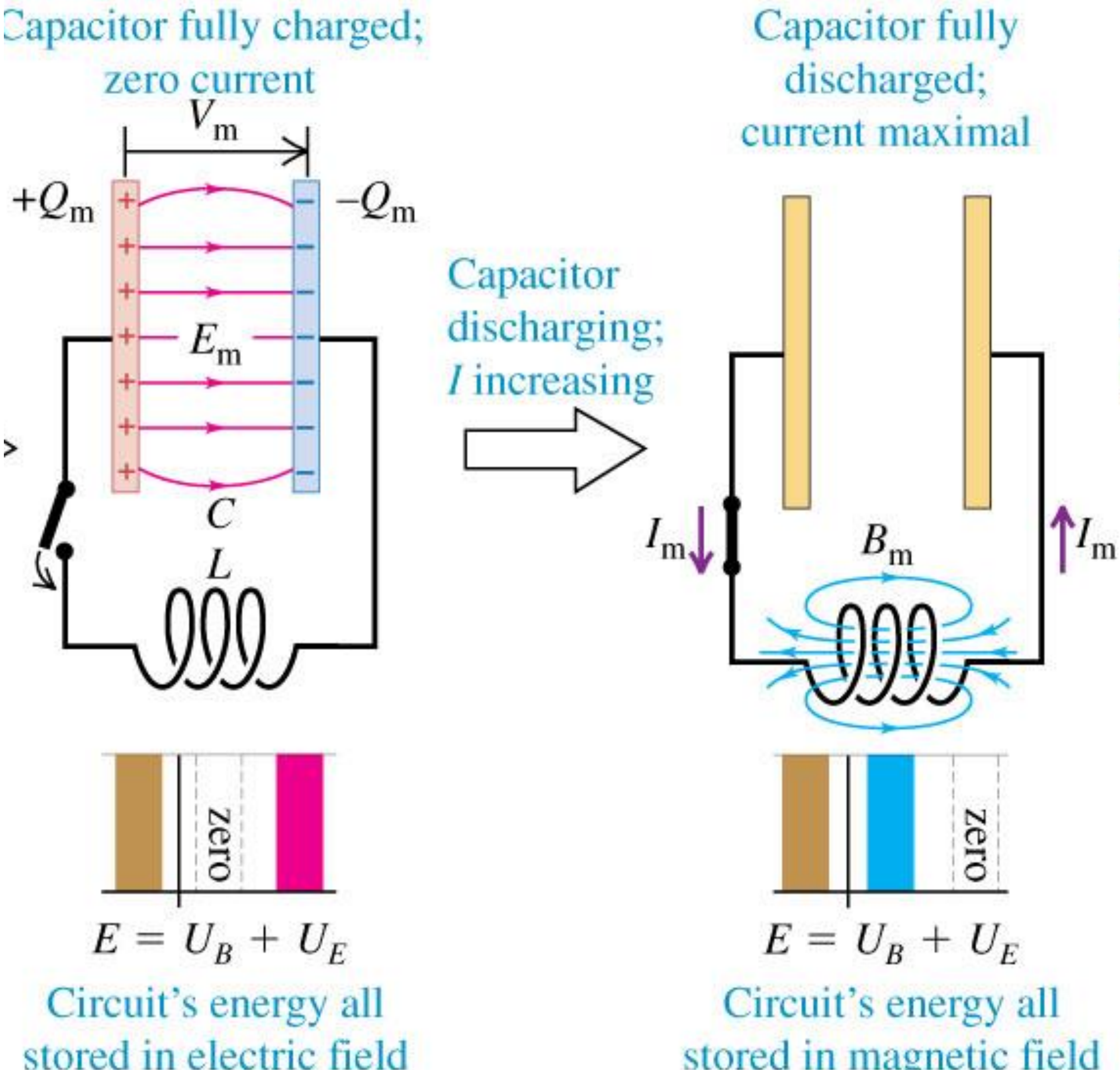
$$\frac{d^2q}{dt^2}+\frac{1}{LC}q=0\Rightarrow$$

$$q=Q_0\cos(\omega t+\phi)$$

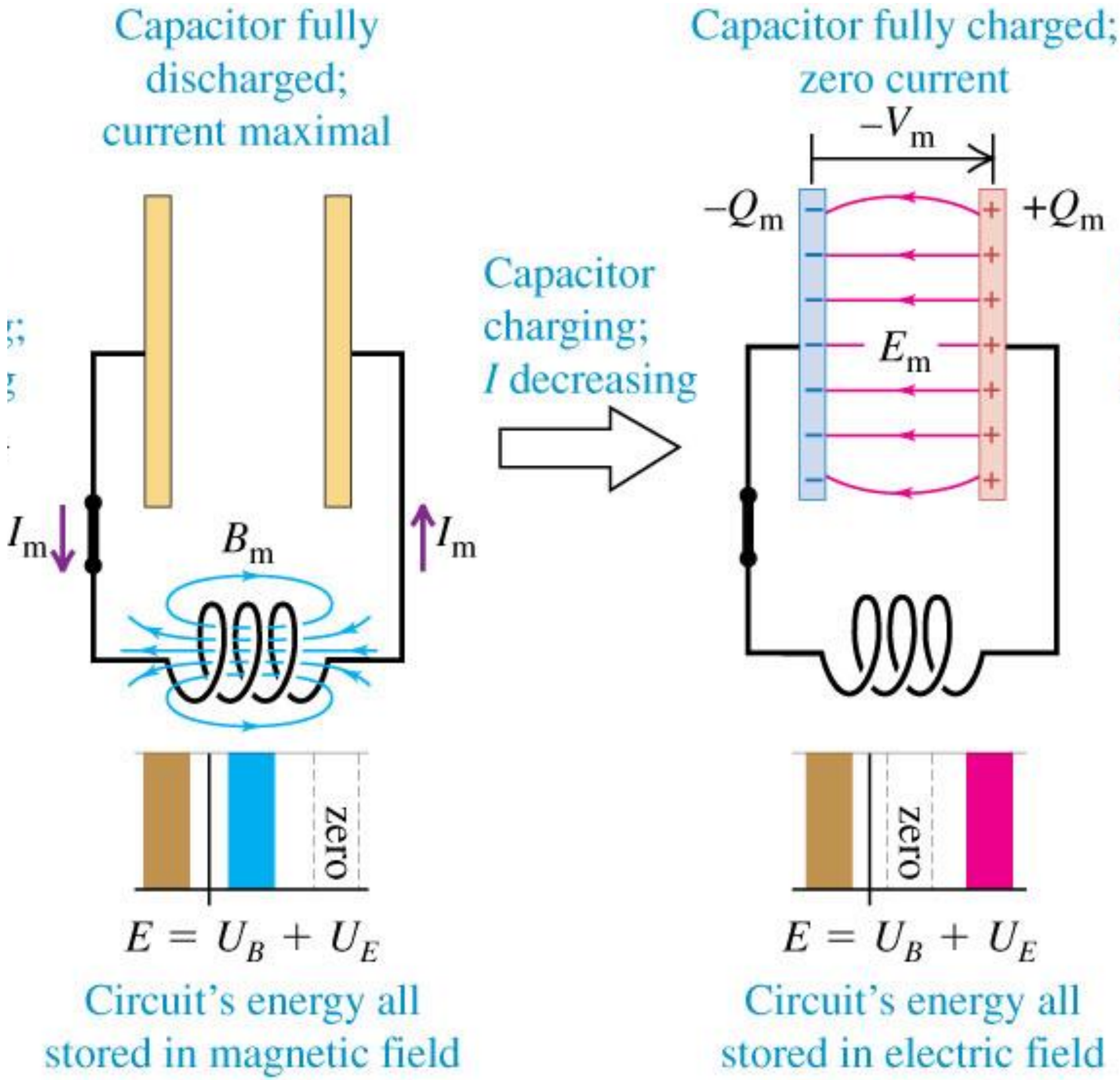
$$\omega=\sqrt{\frac{1}{LC}}$$

Table 30.1 Oscillation of a Mass-Spring System Compared with Electrical Oscillation in an *L-C* Circuit

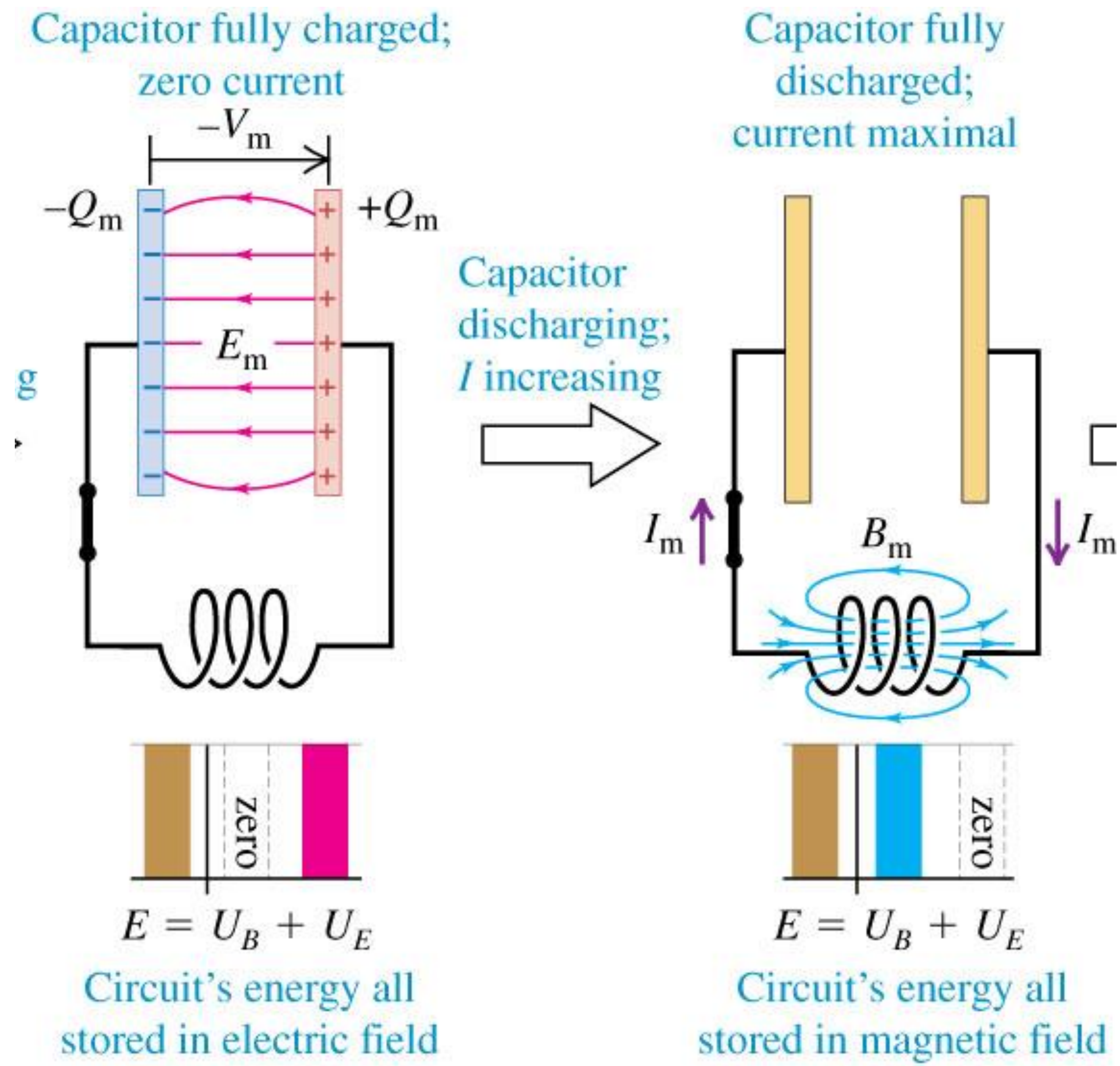
Mass-Spring System
Kinetic energy $= \frac{1}{2}mv_x^2$
Potential energy $= \frac{1}{2}kx^2$
$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$
$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$
$v_x = dx/dt$
$\omega = \sqrt{\frac{k}{m}}$
$x = A\cos(\omega t + \phi)$
Inductor-Capacitor Circuit
Magnetic energy $= \frac{1}{2}Li^2$
Electric energy $= q^2/2C$
$\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$
$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$
$i = dq/dt$
$\omega = \sqrt{\frac{1}{LC}}$
$q = Q\cos(\omega t + \phi)$



Capacitor fully charged;
zero current
V_m
$+Q_m$
$-Q_m$
E_m
C
L
zero
$E = U_B + U_E$
Circuit's energy all
stored in electric field
Capacitor
discharging;
I increasing
Capacitor fully
discharged;
current maximal
I_m
B_m
I_m
zero
$E = U_B + U_E$
Circuit's energy all
stored in magnetic field



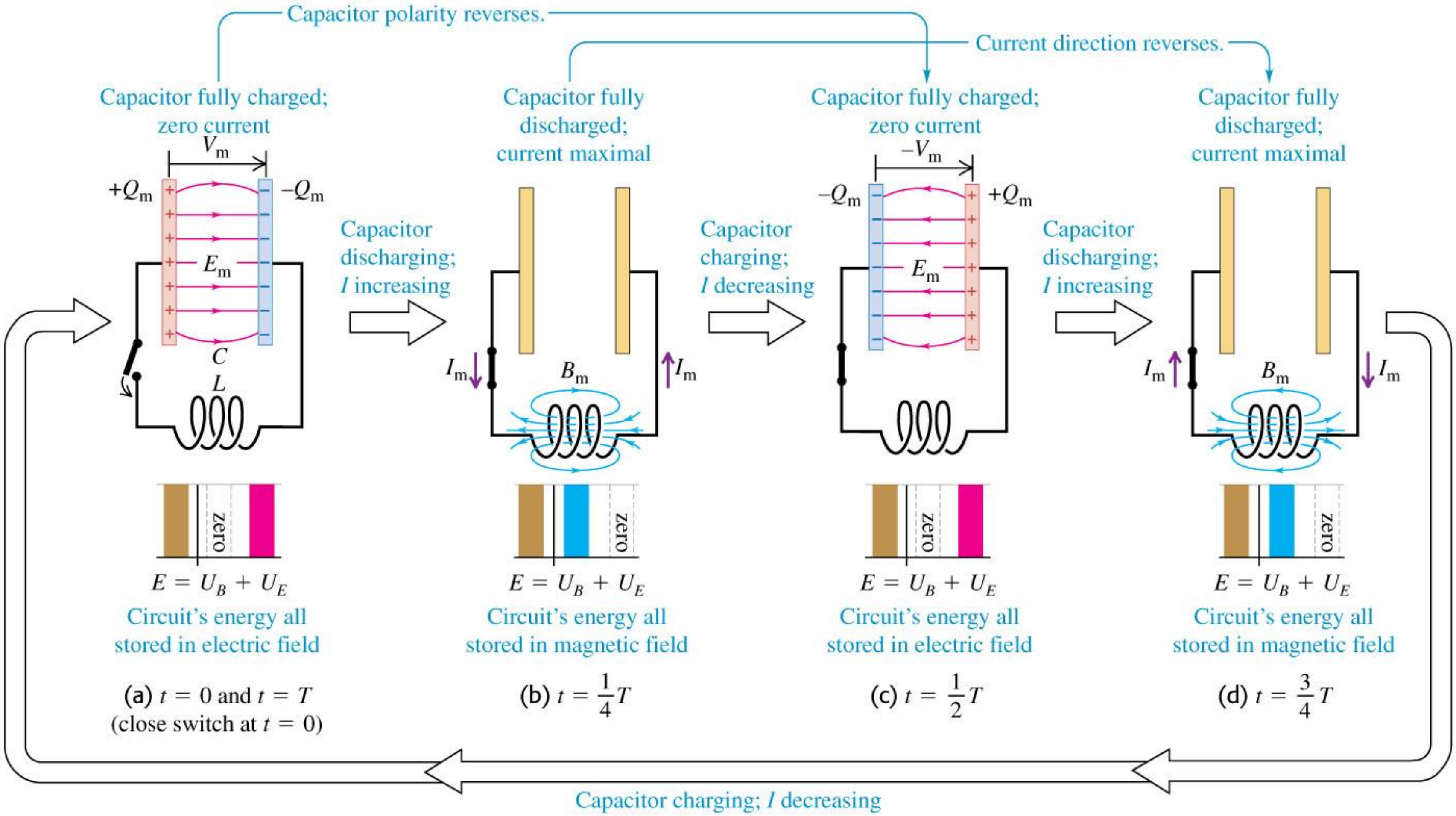
Capacitor fully discharged; current maximal
I_m
B_m
I_m
zero
$E = U_B + U_E$
Circuit's energy all stored in magnetic field
Capacitor charging; I decreasing
Capacitor fully charged; zero current
$-V_m$
$-Q_m$
$+Q_m$
E_m
zero
$E = U_B + U_E$
Circuit's energy all stored in electric field



Capacitor fully charged; zero current
$-V_m$
$-Q_m$
$+Q_m$
E_m
zero
$E = U_B + U_E$
Circuit's energy all stored in electric field
Capacitor discharging; I increasing
Capacitor fully discharged; current maximal
I_m
B_m
I_m
zero
$E = U_B + U_E$
Circuit's energy all stored in magnetic field

Energy/Current/Charge Oscillations

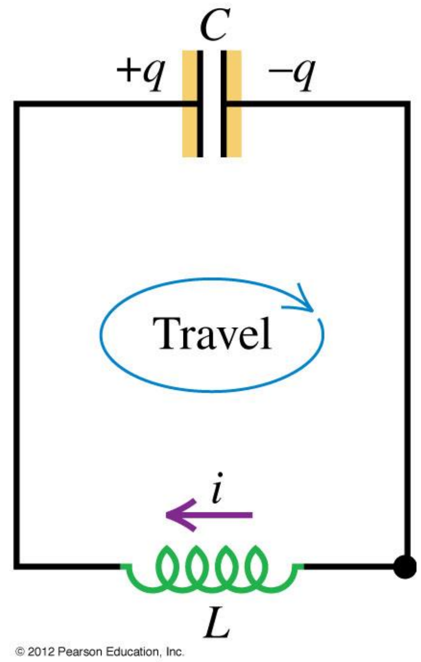
$$E_{\text{Total}} = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \text{Constant} = \frac{1}{2}\frac{Q_{\max}^2}{C} = \frac{1}{2}LI_{\max}^2$$



CPS 37-1

An inductor (inductance *L*) and a capacitor (capacitance *C*) are connected as shown.

If the values of both *L* and *C* are doubled, what happens to the *time* required for the capacitor charge to oscillate through a complete cycle?



A. It becomes 4 times longer.

B. It becomes twice as long.

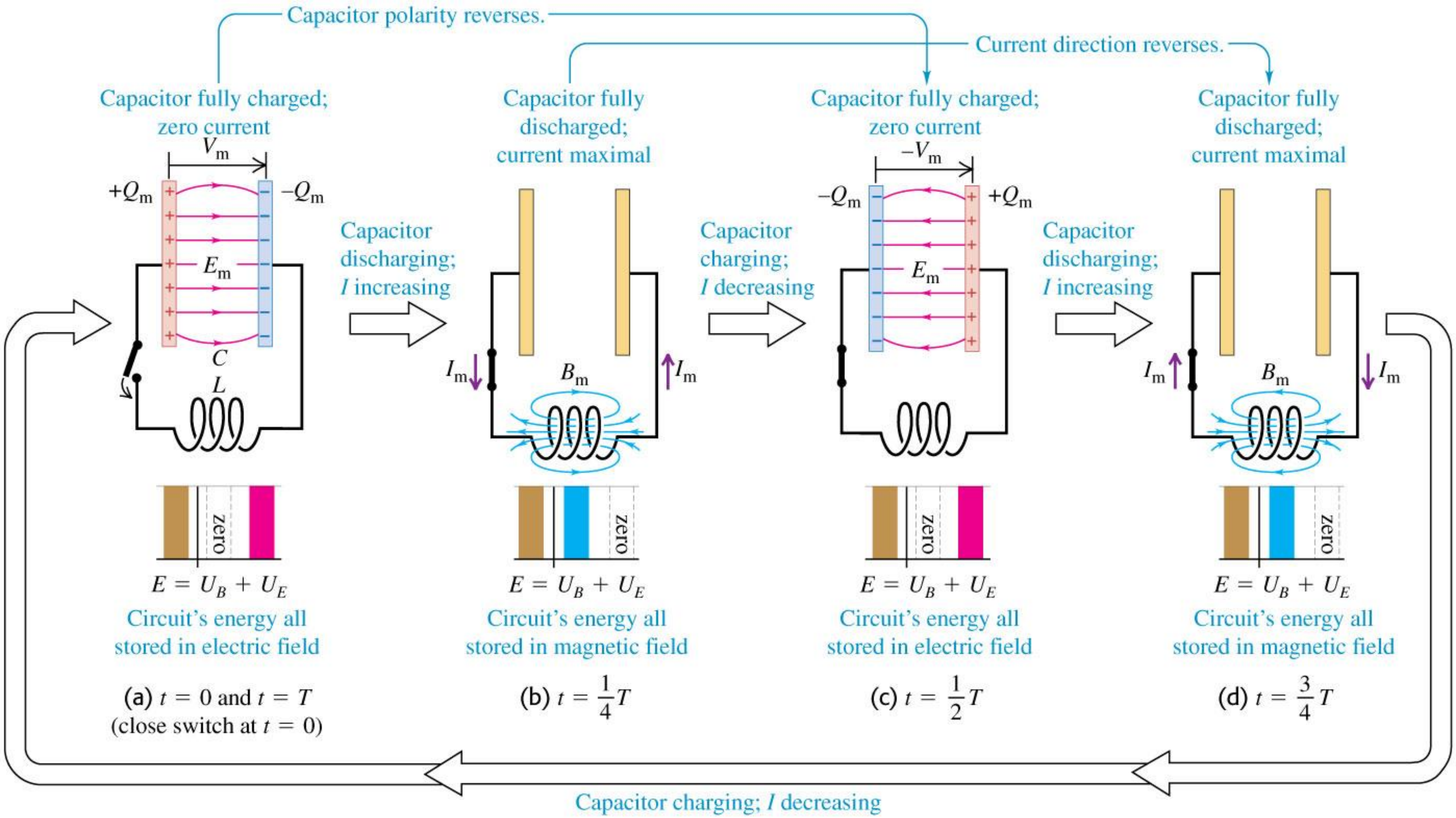
C. It is unchanged.

D. It becomes 1/2 as long.

E. It becomes 1/4 as long.

Energy/Current/Charge Oscillations

$$E_{\text{Total}} = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \text{Constant} = \frac{1}{2}\frac{Q_{\max}^2}{C} = \frac{1}{2}LI_{\max}^2$$

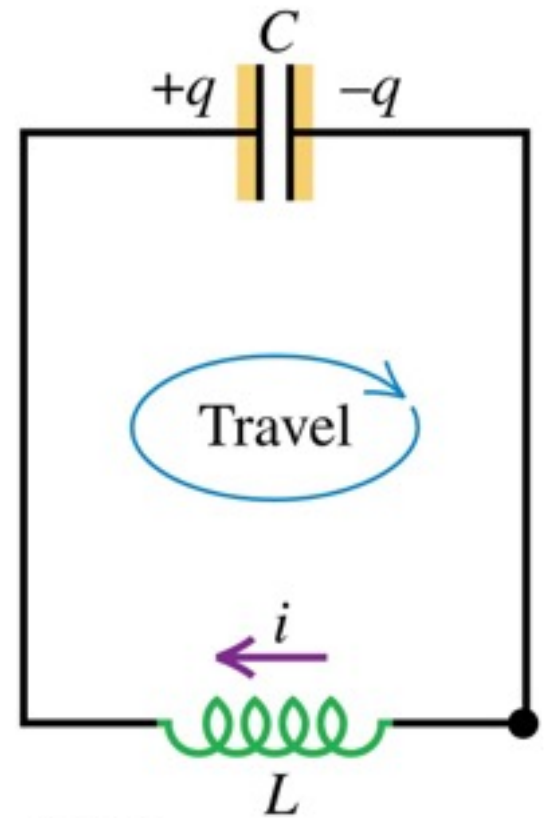


Electrical and mechanical oscillations: analogies

Mass-Spring System	Inductor-Capacitor Circuit
Kinetic energy $= \frac{1}{2}mv_x^2$	Magnetic energy $= \frac{1}{2}Li^2$
Potential energy $= \frac{1}{2}kx^2$	Electrical energy $= q^2/2C$
$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$	$\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$
$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$	$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$
$v_x = dx/dt$	$i = dq/dt$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{1}{LC}}$
$x = A\cos(\omega t + \phi)$	$q = Q\cos(\omega t + \phi)$

Electrical oscillations in an *L-C* circuit

- We can apply Kirchhoff's loop rule to the circuit shown.
- This leads to an equation with the same form as that for simple harmonic motion studied in Chapter 14.
- The charge on the capacitor and current through the circuit are functions of time:



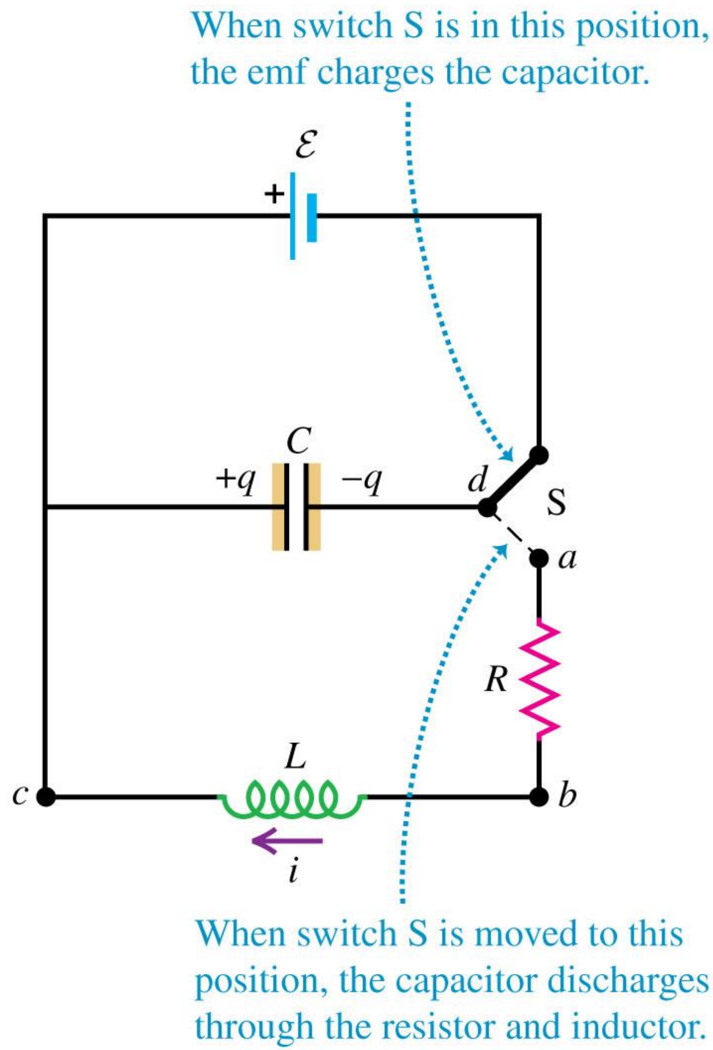
$$q = Q\cos(\omega t + \phi)$$

$$i = -\omega Q\sin(\omega t + \phi)$$

Angular frequency of oscillation in an *L-C* circuit $\omega = \sqrt{\frac{1}{LC}}$ Capacitance, Inductance

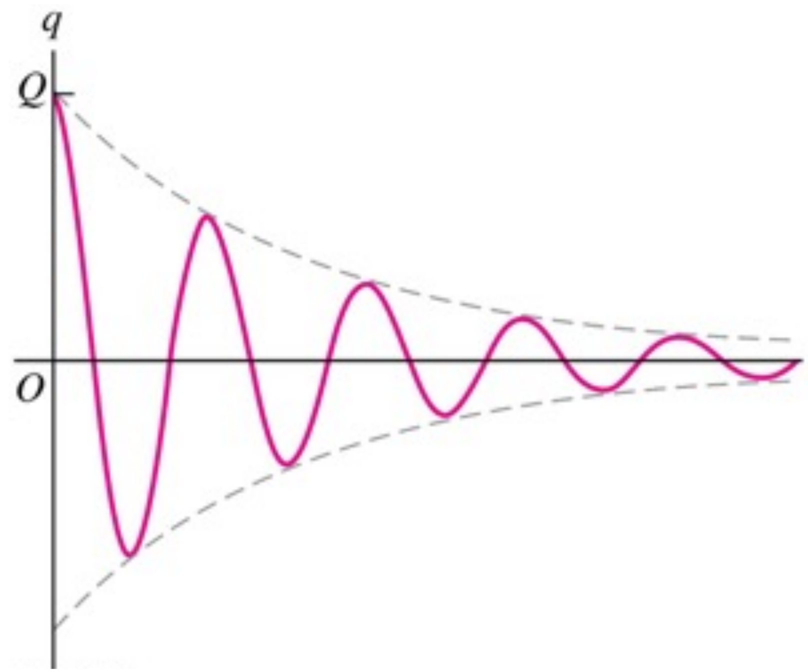
Energy/Current/Charge Oscillations

- Now that you have the spring-mass system in your head as an analogy, what do you think happens if we put a resistor in series with the capacitor and inductor?
- Remember that with the resistor, the total energy will decrease due to the power dissipated in the resistor.



The *L-R-C* series circuit

- An *L-R-C* circuit exhibits **damped harmonic motion** if the resistance is not too large.
- The charge as a function of time is sinusoidal oscillation with an exponentially decaying amplitude, and angular frequency:



Angular frequency of underdamped oscillations in an *L-R-C* series circuit

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

Resistance

Inductance

Inductance

Capacitance