Lecture 39

PHYC 161 Fall 2016

Announcements

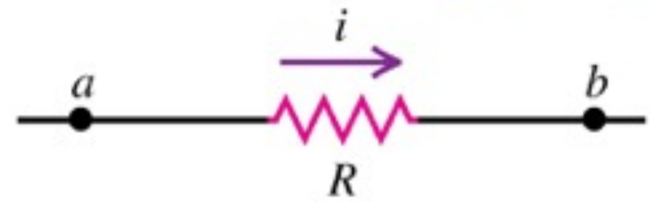
DO THE ONLINE COURSE EVALUATIONS

- response so far is < 28 %

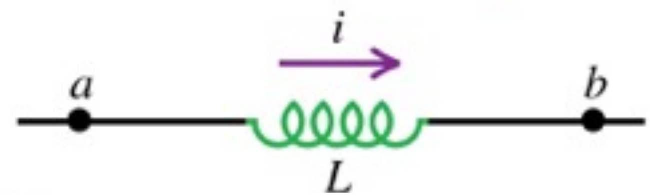
Magnetic field energy

- A resistor is a device in which energy is irrecoverably *dissipated*.
- By contrast, energy stored in a current-carrying inductor can be recovered when the current decreases to zero.

Resistor with current i: energy is *dissipated*.



Inductor with current i: energy is *stored*.



Inductance

Energy stored in an inductor

$$U = L \int_0^I i \, di = \tfrac{1}{2} L I^2$$

Final current

Integral from initial (zero) value of instantaneous current to final value

Q30.3

A steady current flows through an inductor. If the current is doubled while the inductance remains constant, the amount of energy stored in the inductor

A. increases by a factor of $\sqrt{2}$.

B. increases by a factor of 2.

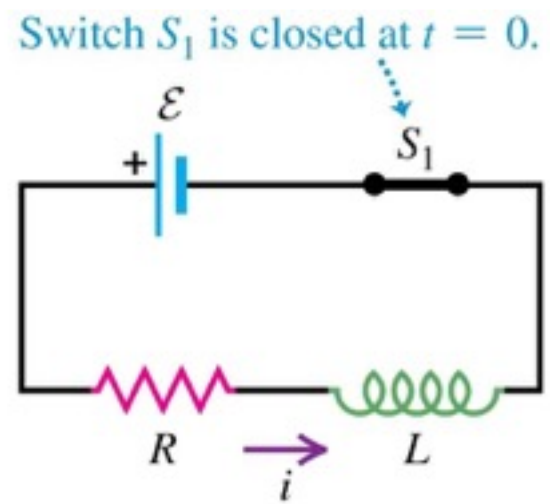
C. increases by a factor of 4.

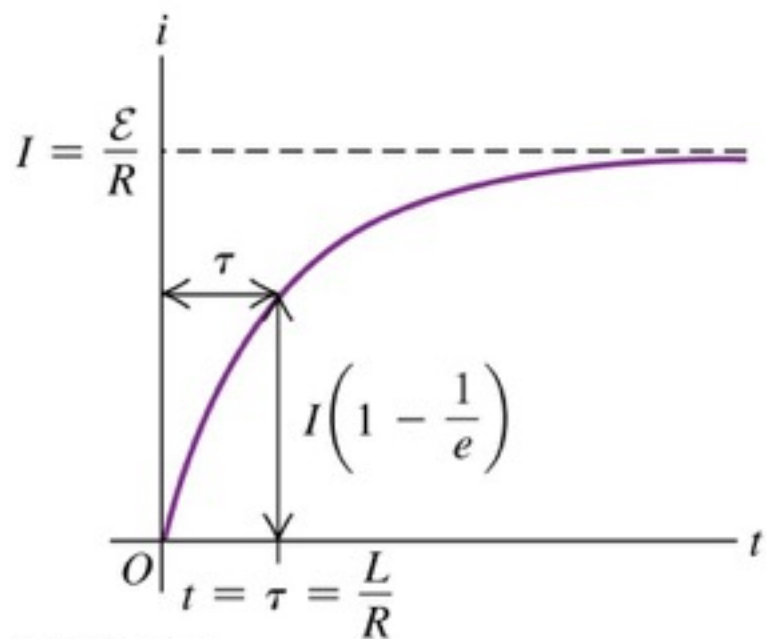
D. increases by a factor that depends on the geometry of the inductor.

E. none of the above

Current growth in an *R-L* circuit

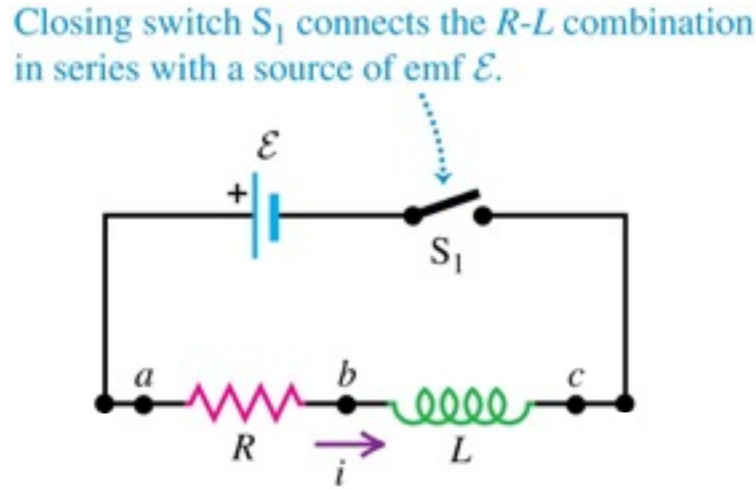
- Suppose that at some initial time $t = 0$ we close switch S_1.
- The current cannot change suddenly from zero to some final value.
- As the current increases, the rate of increase of current given becomes smaller and smaller.
- This means that the current approaches a final, steady-state value I.
- The **time constant** for the circuit is $\tau = L/R$.





Q30.4

An inductance L and a resistance R are connected to a source of emf as shown. When switch S_1 is closed, a current begins to flow. The *final* value of the current is



A. directly proportional to RL.

B. directly proportional to R/L.

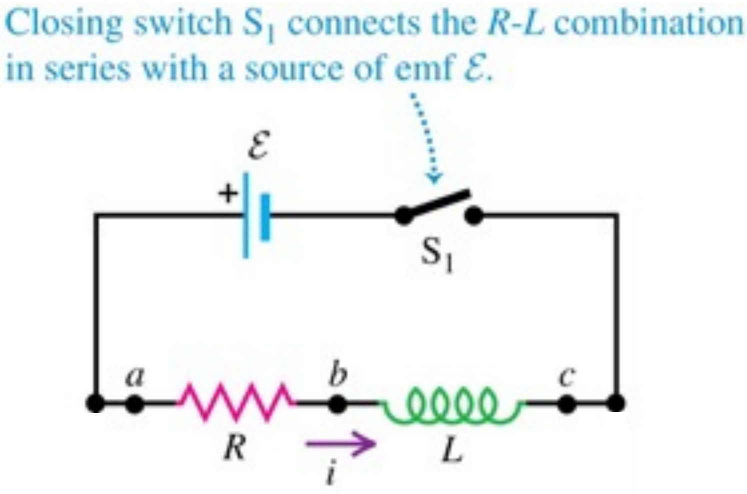
C. directly proportional to L/R.

D. directly proportional to $1/(RL)$.

E. independent of L.

Q30.5

An inductance L and a resistance R are connected to a source of emf as shown. When switch S_1 is closed, a current begins to flow. The *time* required for the current to reach one-half its final value is



A. directly proportional to RL.

B. directly proportional to R/L.

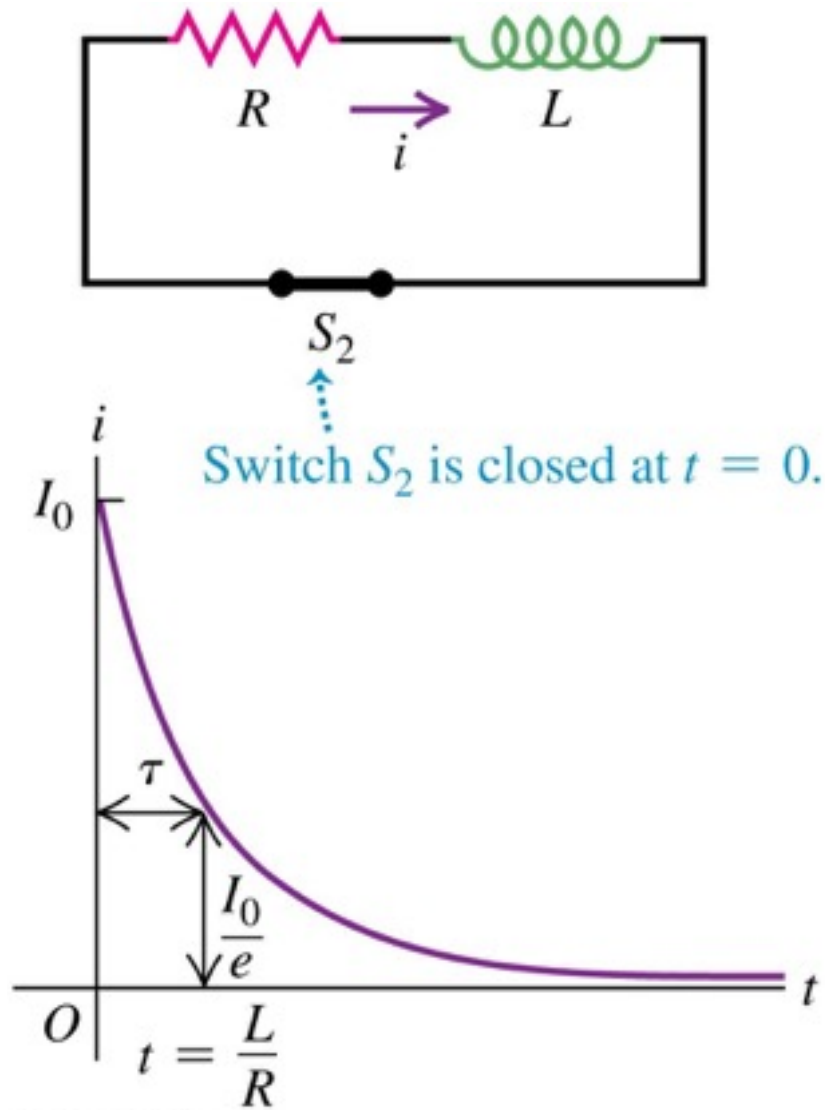
C. directly proportional to L/R.

D. directly proportional to $1/(RL)$.

E. independent of L.

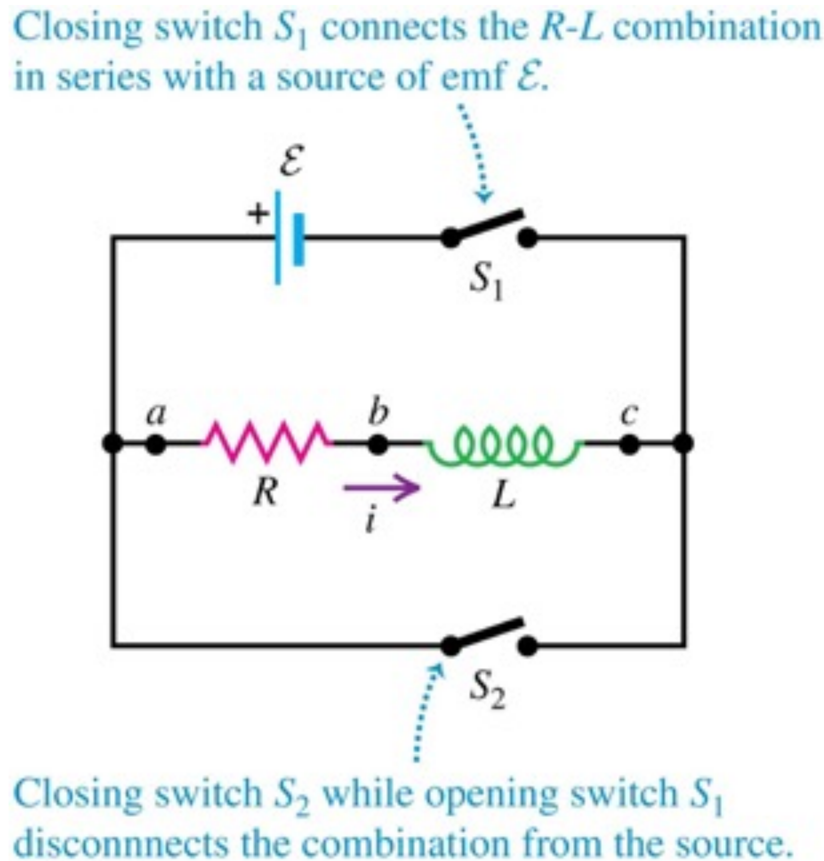
Current decay in an *R-L* circuit

- Suppose there is an initial current I_0 running through the resistor and inductor shown.
- At time $t = 0$ we close the switch S_2, bypassing the battery (not shown).
- The energy stored in the magnetic field of the inductor provides the energy needed to maintain a decaying current.
- The **time constant** for the exponential decay of the current is $\tau = L/R$.



Q30.6

An inductance L and a resistance R are connected to a source of emf as shown. Initially, switch S_1 is closed, switch S_2 is open, and current flows through L and R. When S_1 is opened and S_2 is simultaneously closed, the *rate* at which this current decreases



A. remains constant.

B. increases with time.

C. decreases with time.

D. Any of A, B, or C is possible.

E. Misleading question — the current does not decrease.

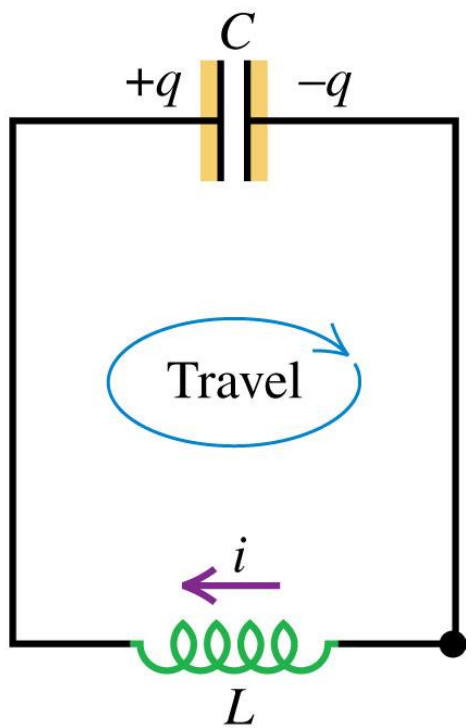
- We start with a charged capacitor in series with a switch and an inductor. $Q_{init}=CV_{init}$
- Let's apply Kirchhoff's loop rule:

$$\sum \Delta V = -\frac{q}{C} - L\frac{di}{dt} = 0 \Rightarrow$$

$$\frac{q}{C} + L\frac{di}{dt} = 0 \Rightarrow$$

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0 \Rightarrow$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$



Capacitor discharges through inductor

LC Circuits

- Do you recognize this equation?

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

- What if I just changed q to x and renamed the constant?

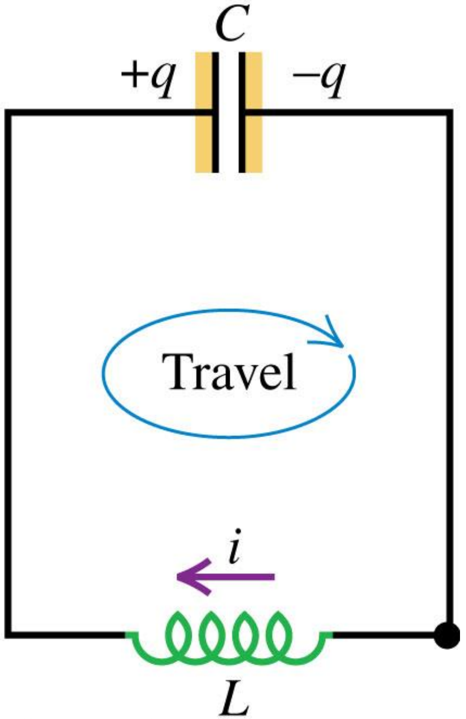
$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

- So the equation just describes oscillations with a frequency:

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \Rightarrow$$

$$q = Q_0 \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{1}{LC}}$$



Analogy to Spring-Mass Oscillations

$$\frac{d^2x}{dt^2}+\frac{k}{m}x=0\Rightarrow$$

$$x=X_0\cos(\omega t+\phi)$$

$$\omega=\sqrt{\frac{k}{m}}$$

$$\frac{d^2q}{dt^2}+\frac{1}{LC}q=0\Rightarrow$$

$$q=Q_0\cos(\omega t+\phi)$$

$$\omega=\sqrt{\frac{1}{LC}}$$

Table 30.1 Oscillation of a Mass-Spring System Compared with Electrical Oscillation in an *L-C* Circuit

Mass-Spring System

Kinetic energy $=\frac{1}{2}mv_x^2$

Potential energy $=\frac{1}{2}kx^2$

$\frac{1}{2}mv_x^2+\frac{1}{2}kx^2=\frac{1}{2}kA^2$

$v_x=\pm\sqrt{k/m}\sqrt{A^2-x^2}$

$v_x=dx/dt$

$\omega=\sqrt{\frac{k}{m}}$

$x=A\cos(\omega t+\phi)$

Inductor-Capacitor Circuit

Magnetic energy $=\frac{1}{2}Li^2$

Electric energy $=q^2/2C$

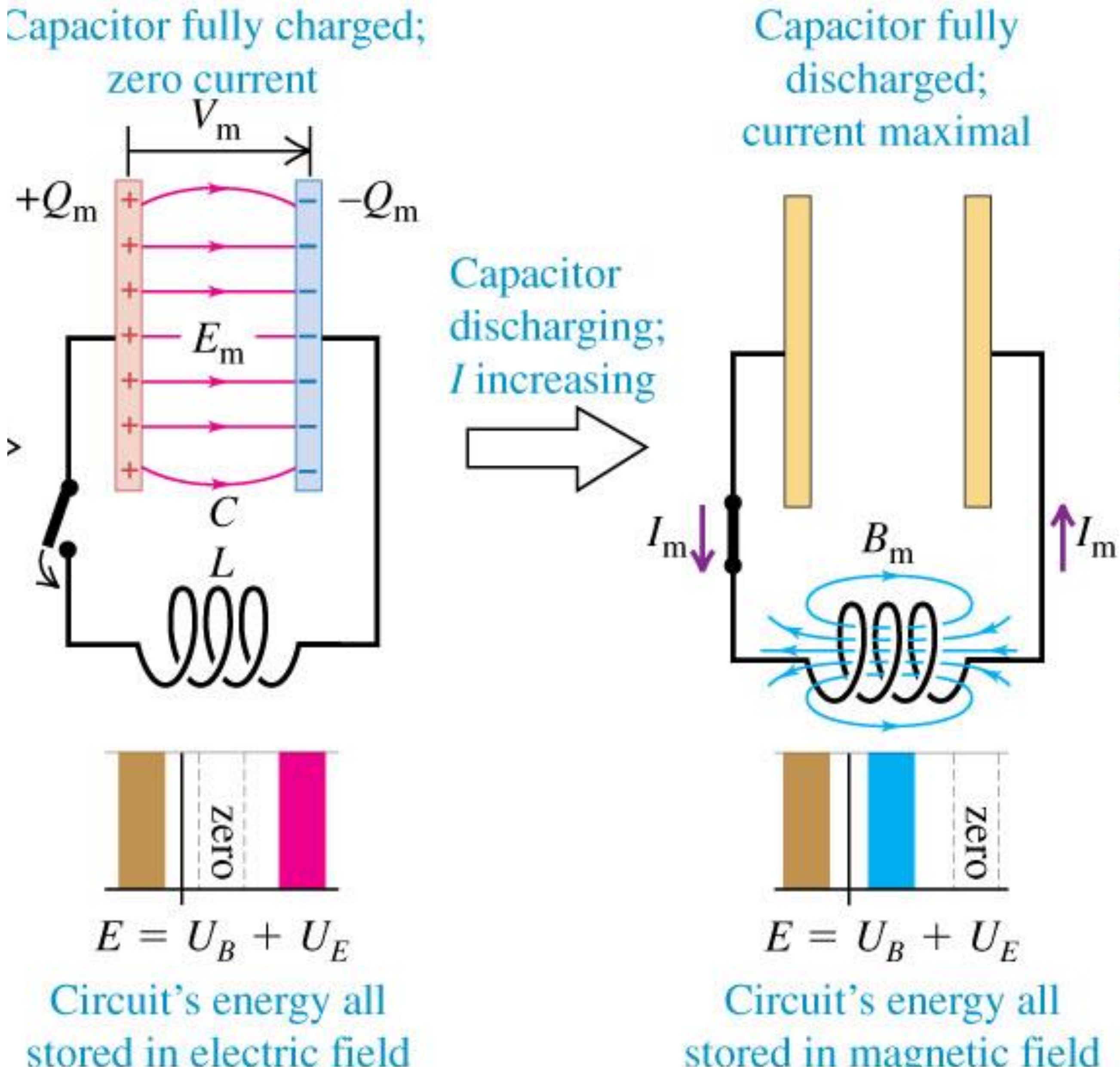
$\frac{1}{2}Li^2+q^2/2C=Q^2/2C$

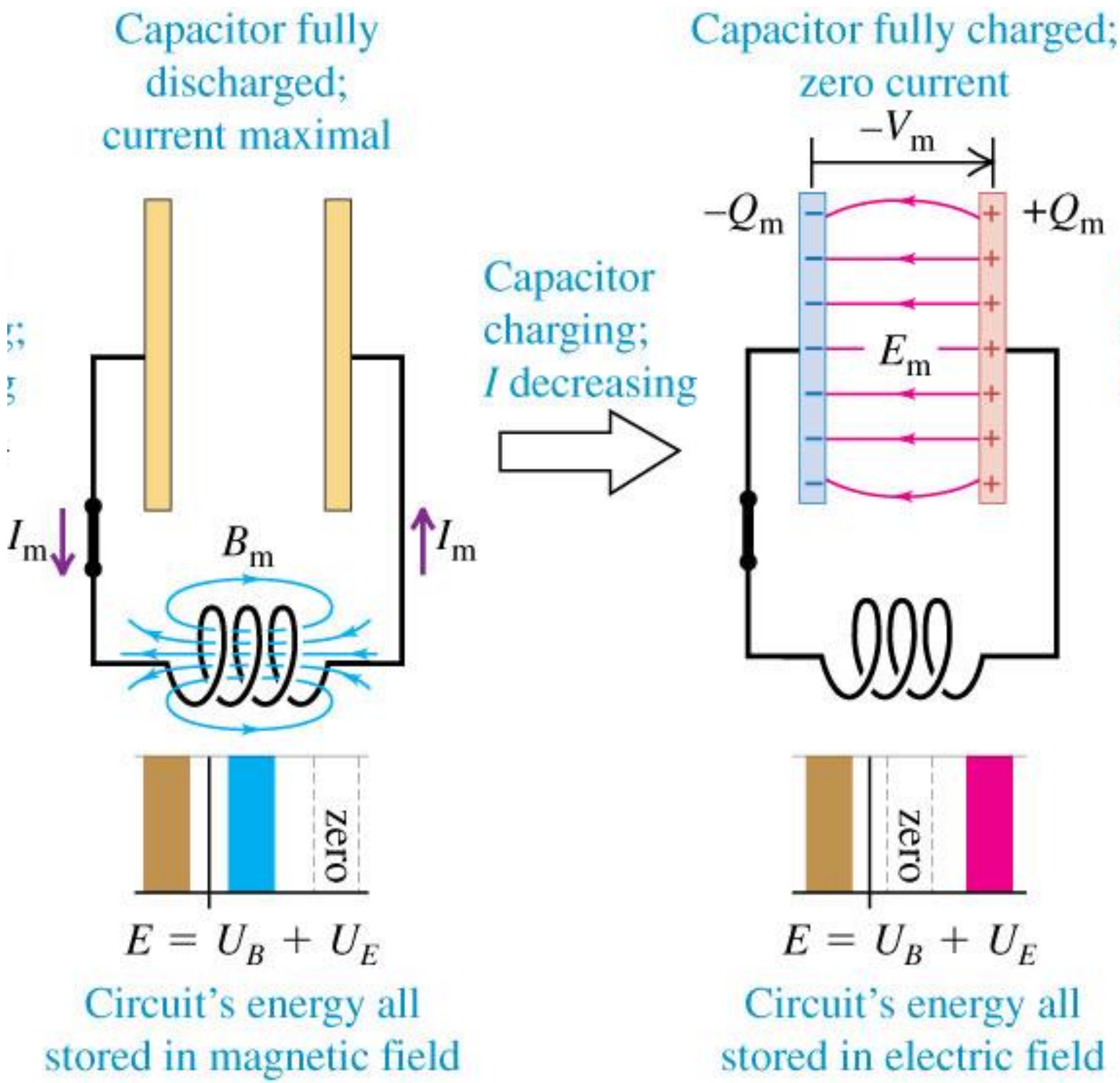
$i=\pm\sqrt{1/LC}\sqrt{Q^2-q^2}$

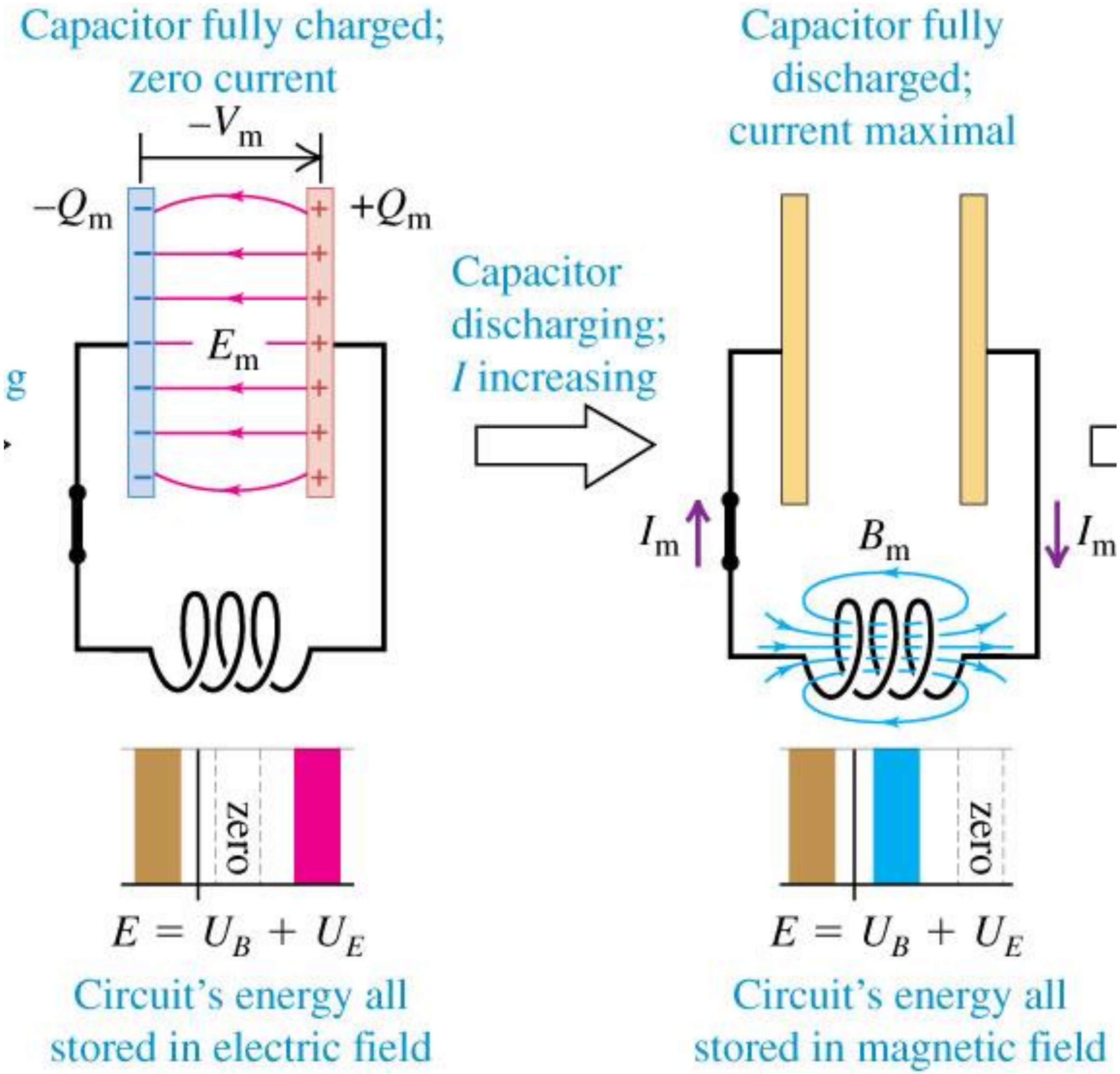
$i=dq/dt$

$\omega=\sqrt{\frac{1}{LC}}$

$q=Q\cos(\omega t+\phi)$


Capacitor fully charged; zero current
V_m
$+Q_m$
$-Q_m$
E_m
C
L
Capacitor discharging; I increasing
Capacitor fully discharged; current maximal
I_m
B_m
I_m
zero
$E = U_B + U_E$
Circuit's energy all stored in electric field
zero
$E = U_B + U_E$
Circuit's energy all stored in magnetic field

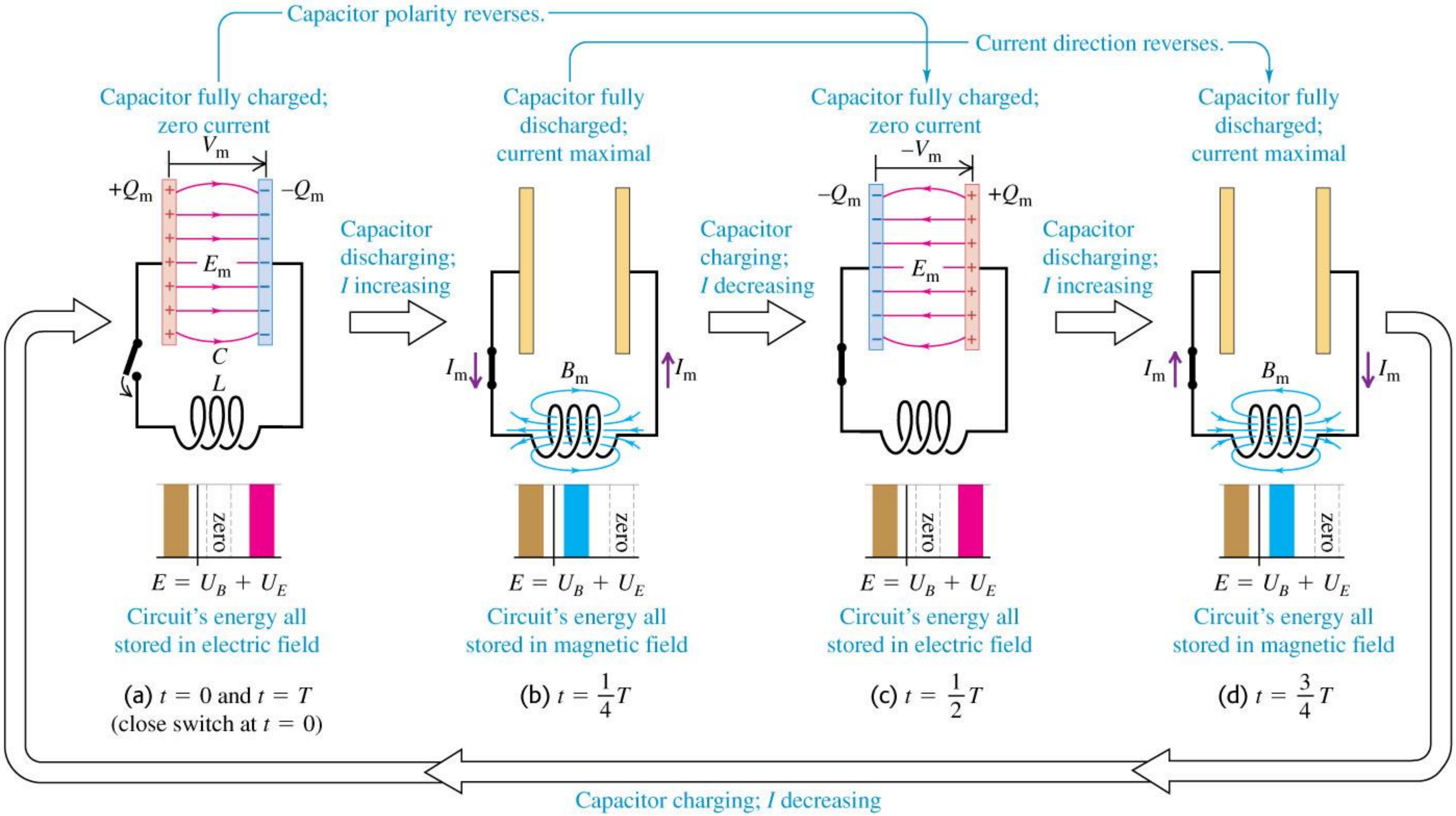

Capacitor fully discharged; current maximal
I_m
B_m
I_m
zero
$E = U_B + U_E$
Circuit's energy all stored in magnetic field
Capacitor charging; I decreasing
Capacitor fully charged; zero current
$-V_m$
$-Q_m$
$+Q_m$
E_m
zero
$E = U_B + U_E$
Circuit's energy all stored in electric field



Capacitor fully charged; zero current
$-V_m$
$-Q_m$
$+Q_m$
E_m
zero
$E = U_B + U_E$
Circuit's energy all stored in electric field
Capacitor discharging; I increasing
Capacitor fully discharged; current maximal
I_m
B_m
I_m
zero
$E = U_B + U_E$
Circuit's energy all stored in magnetic field

Energy/Current/Charge Oscillations

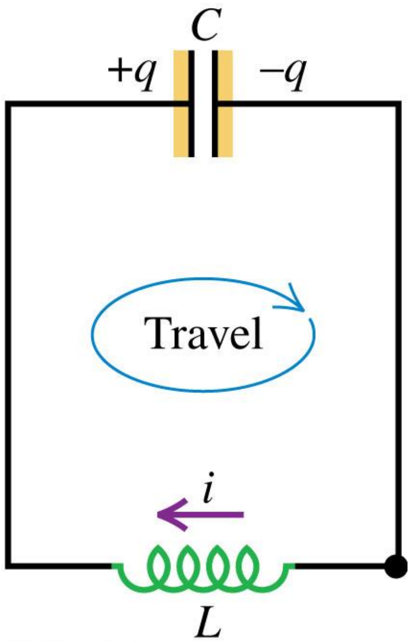
$$E_{\text{Total}} = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \text{Constant} = \frac{1}{2}\frac{Q_{\max}^2}{C} = \frac{1}{2}LI_{\max}^2$$



CPS 37-1

An inductor (inductance *L*) and a capacitor (capacitance *C*) are connected as shown.

If the values of both *L* and *C* are doubled, what happens to the *time* required for the capacitor charge to oscillate through a complete cycle?



A. It becomes 4 times longer.

B. It becomes twice as long.

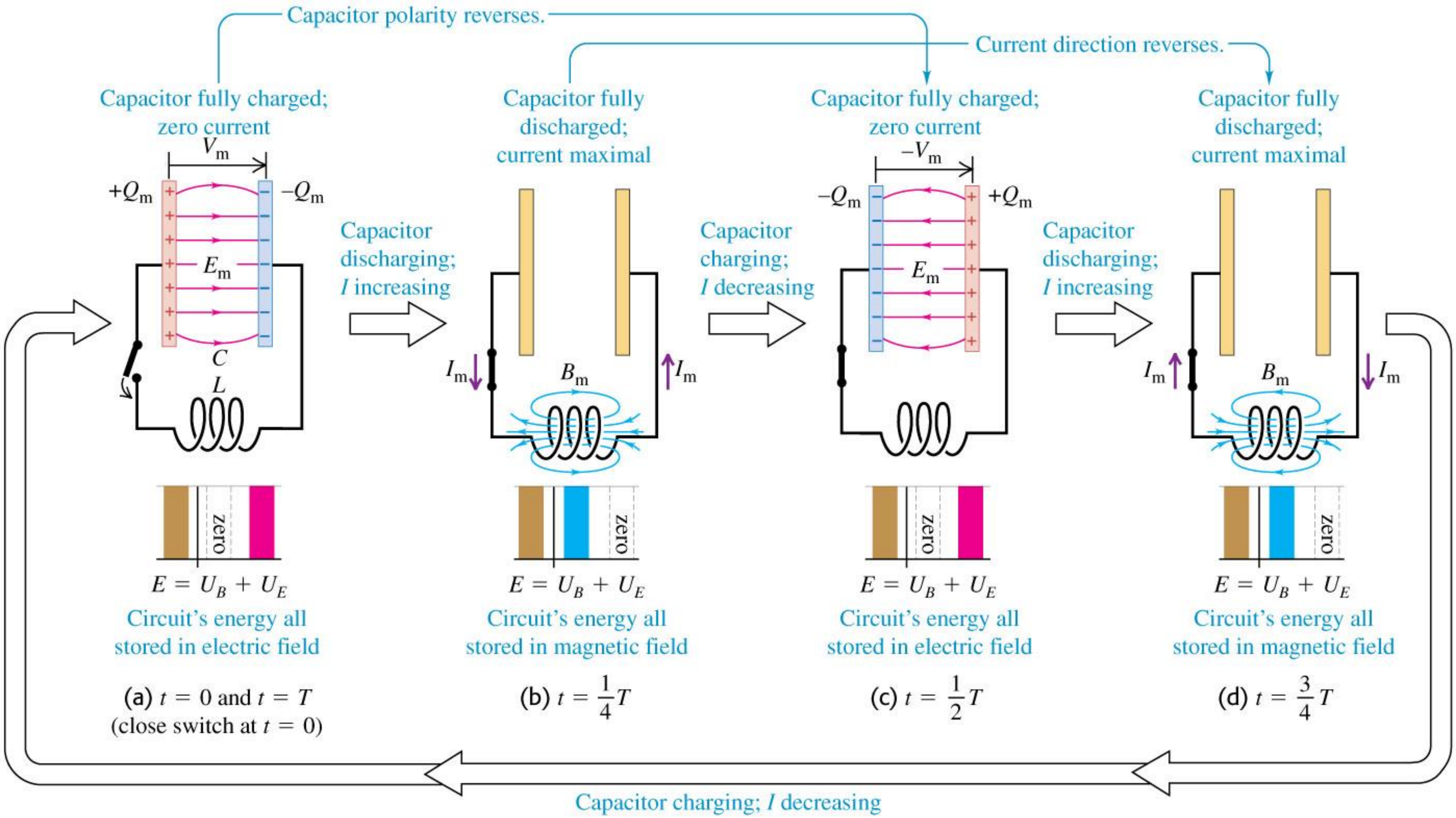
C. It is unchanged.

D. It becomes 1/2 as long.

E. It becomes 1/4 as long.

Energy/Current/Charge Oscillations

$$E_{\text{Total}} = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \text{Constant} = \frac{1}{2}\frac{Q_{\max}^2}{C} = \frac{1}{2}LI_{\max}^2$$

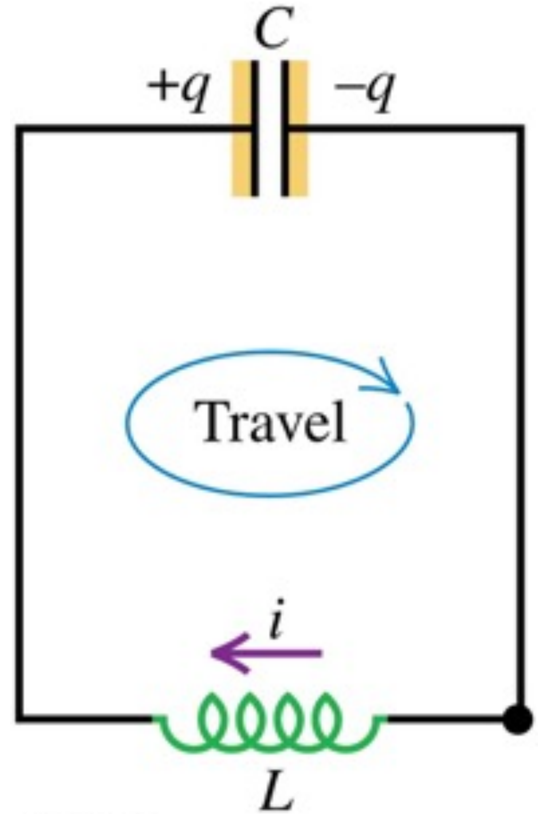


Electrical and mechanical oscillations: analogies

Mass-Spring System	Inductor-Capacitor Circuit
Kinetic energy $= \frac{1}{2}mv_x^2$	Magnetic energy $= \frac{1}{2}Li^2$
Potential energy $= \frac{1}{2}kx^2$	Electrical energy $= q^2/2C$
$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$	$\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$
$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$	$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$
$v_x = dx/dt$	$i = dq/dt$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{1}{LC}}$
$x = A\cos(\omega t + \phi)$	$q = Q\cos(\omega t + \phi)$

Electrical oscillations in an *L-C* circuit

- We can apply Kirchhoff's loop rule to the circuit shown.
- This leads to an equation with the same form as that for simple harmonic motion studied in Chapter 14.
- The charge on the capacitor and current through the circuit are functions of time:



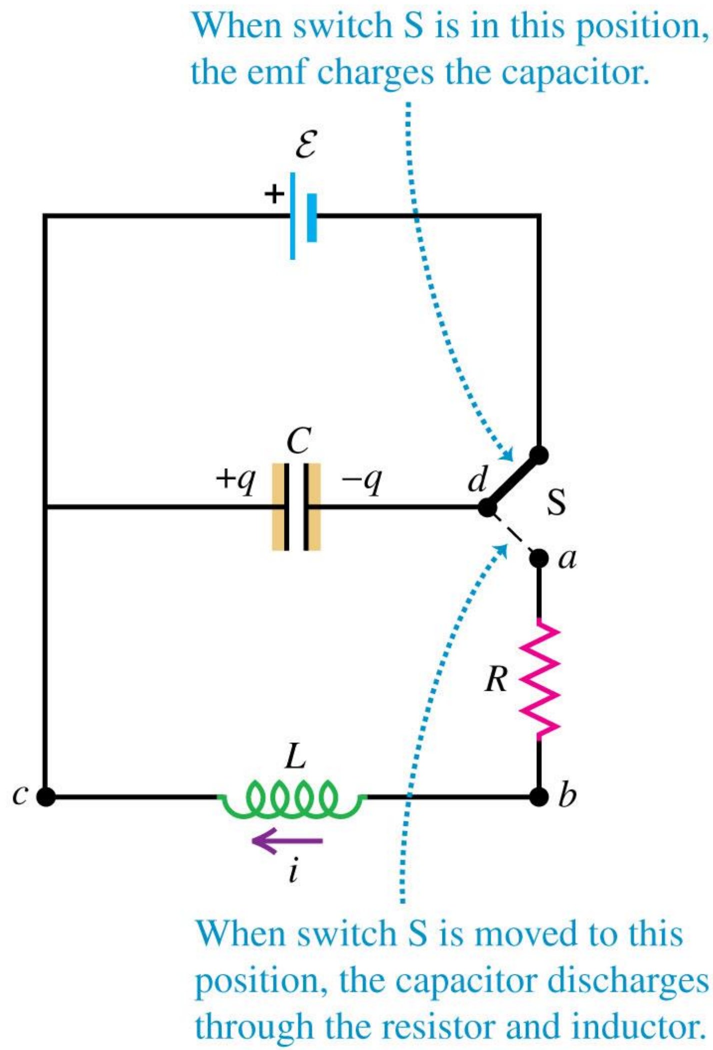
$$q = Q\cos(\omega t + \phi)$$

$$i = -\omega Q\sin(\omega t + \phi)$$

Angular frequency of oscillation in an *L-C* circuit $\omega = \sqrt{\dfrac{1}{LC}}$ Capacitance Inductance

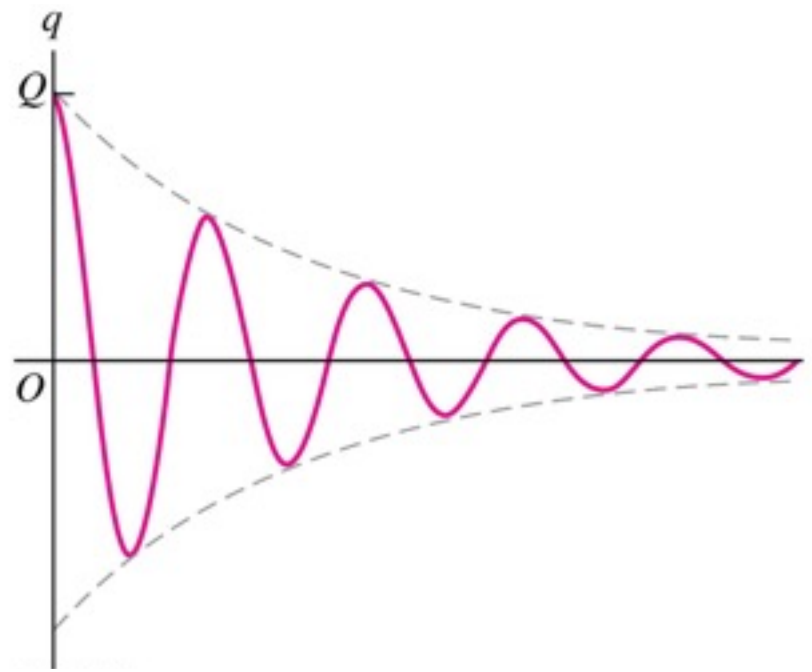
Energy/Current/Charge Oscillations

- Now that you have the spring-mass system in your head as an analogy, what do you think happens if we put a resistor in series with the capacitor and inductor?
- Remember that with the resistor, the total energy will decrease due to the power dissipated in the resistor.



The *L-R-C* series circuit

- An *L*-*R*-*C* circuit exhibits **damped harmonic motion** if the resistance is not too large.
- The charge as a function of time is sinusoidal oscillation with an exponentially decaying amplitude, and angular frequency:



Angular frequency of underdamped oscillations in an *L*-*R*-*C* series circuit

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

Resistance

Inductance

Inductance

Capacitance