Lecture 38

PHYC 161 Fall 2016

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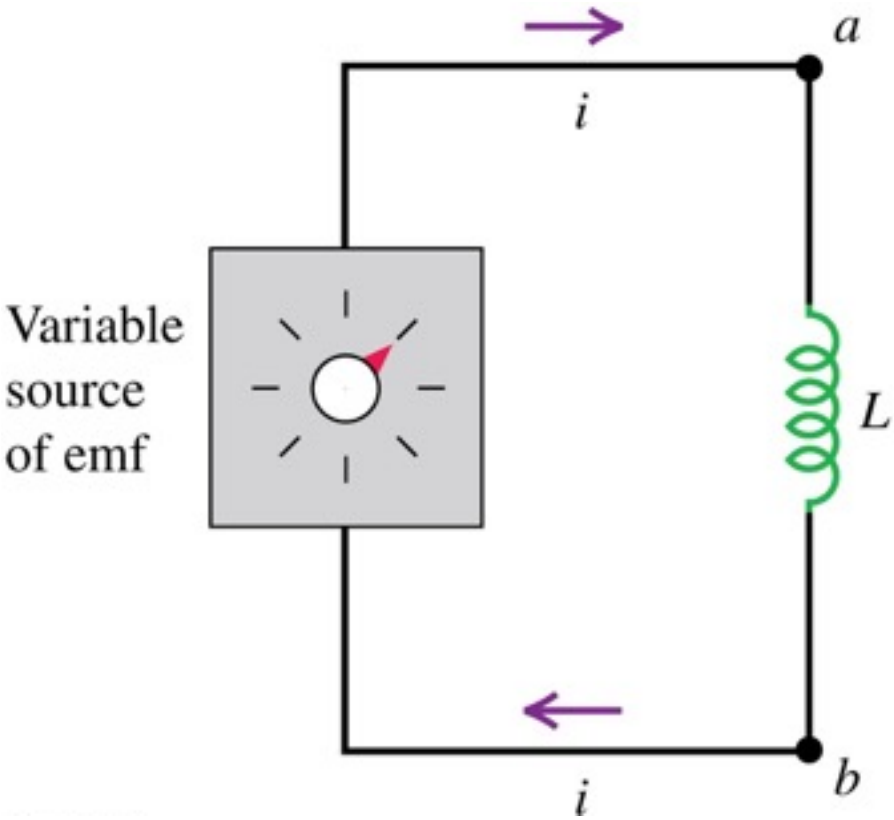
Inductors as circuit elements

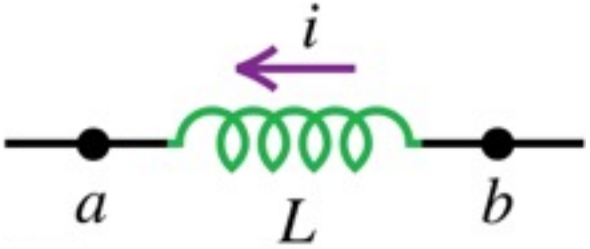
- In the circuit shown, the box enables us to control the current *i* in the circuit.
- The potential difference between the terminals of the inductor *L* is:

$$V_{ab} = V_a - V_b = L\frac{di}{dt}$$



Q30.2

A current i flows through an inductor L in the direction from point b toward point a. There is zero resistance in the wires of the inductor. If the current is *decreasing*,



A. the potential is greater at point a than at point b.

B. the potential is less at point a than at point b.

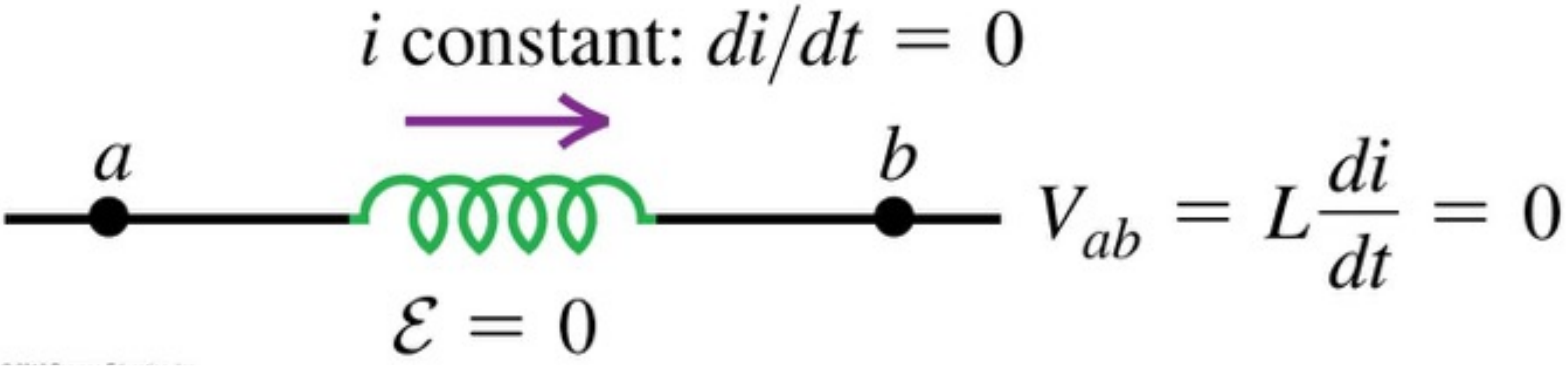
C. the answer depends on the magnitude of di/dt compared to the magnitude of i.

D. the answer depends on the value of the inductance L.

E. both C and D are correct.

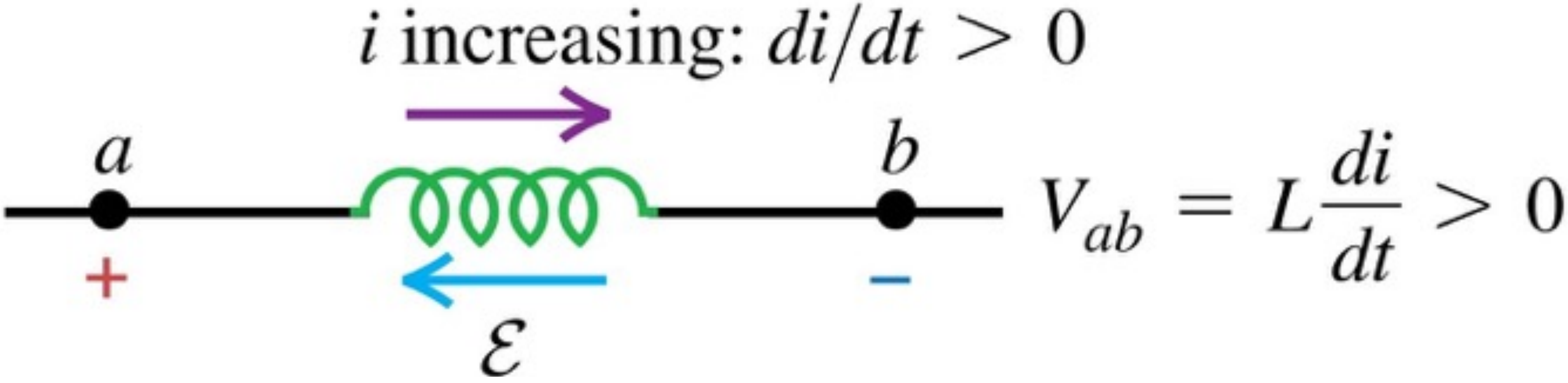
Potential across an inductor with constant current

- The potential difference across an inductor depends on the rate of change of the current.
- When you have an inductor with *constant* current i flowing from a to b, there is *no* potential difference.



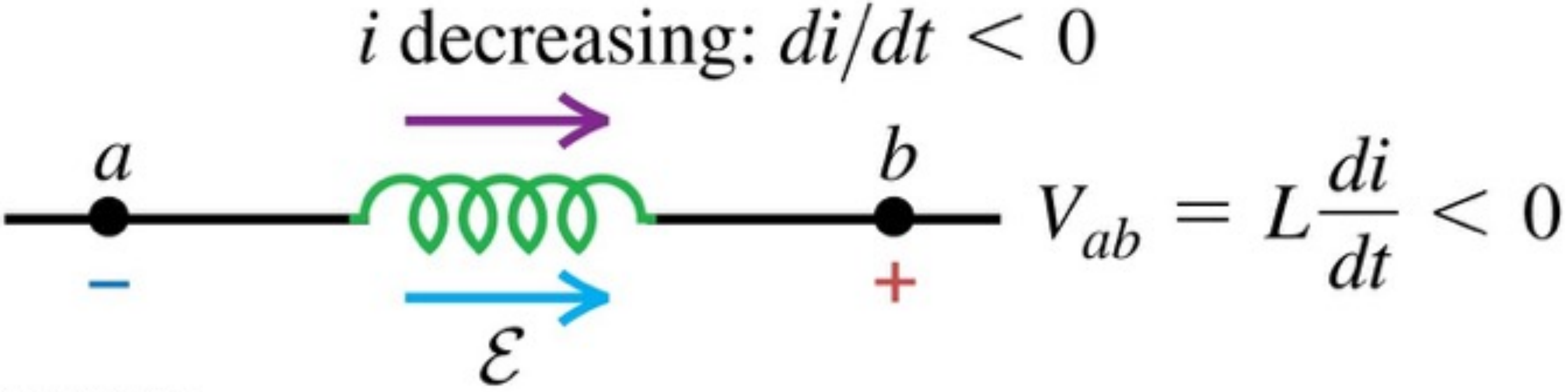
Potential across an inductor with increasing current

- The potential difference across an inductor depends on the rate of change of the current.
- When you have an inductor with *increasing* current i flowing from a to b, the potential *drops* from a to b.



Potential across an inductor with decreasing current

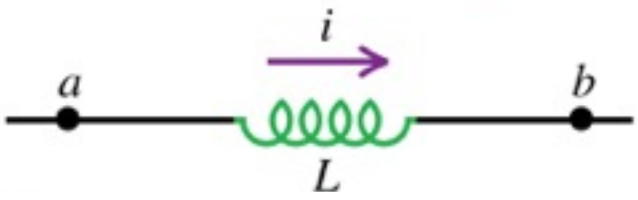
- The potential difference across an inductor depends on the rate of change of the current.
- When you have an inductor with *decreasing* current i flowing from a to b, the potential *increases* from a to b.



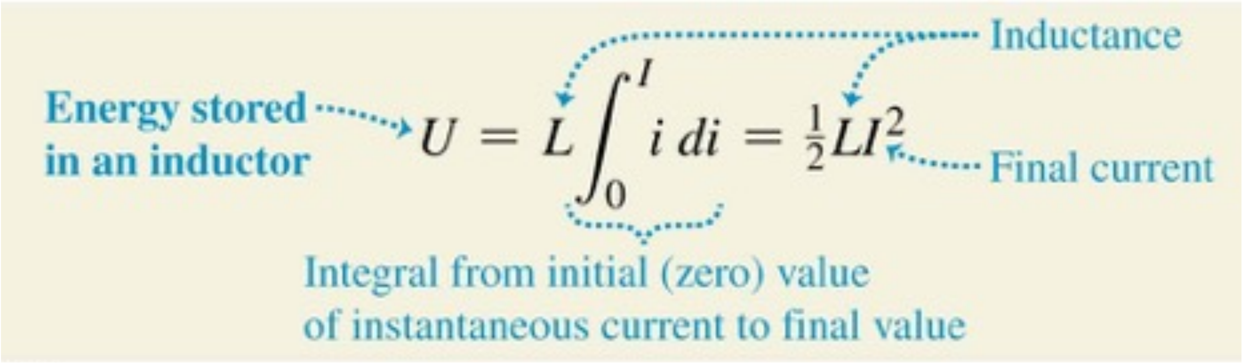
Magnetic field energy

- A resistor is a device in which energy is irrecoverably *dissipated*.
- By contrast, energy stored in a current-carrying inductor can be recovered when the current decreases to zero.

Resistor with current i: energy is *dissipated*.



Inductor with current i: energy is *stored*.



Energy stored in an inductor

$$U = L \int_0^I i \, di = \tfrac{1}{2} L I^2$$

Inductance

Final current

Integral from initial (zero) value of instantaneous current to final value

Magnetic energy density

- The energy in an inductor is actually stored in the magnetic field of the coil, just as the energy of a capacitor is stored in the electric field between its plates.

- In a vacuum, the energy per unit volume, or **magnetic energy density**, is:

Magnetic energy density in vacuum

$$u = \frac{B^2}{2\mu_0}$$

Magnetic-field magnitude

Magnetic constant

- When the magnetic field is located within a material with (constant) magnetic permeability $\mu = K_m \mu_0$, we replace μ_0 by μ in the above equation:

Magnetic energy density in a material

$$u = \frac{B^2}{2\mu}$$

Magnetic-field magnitude

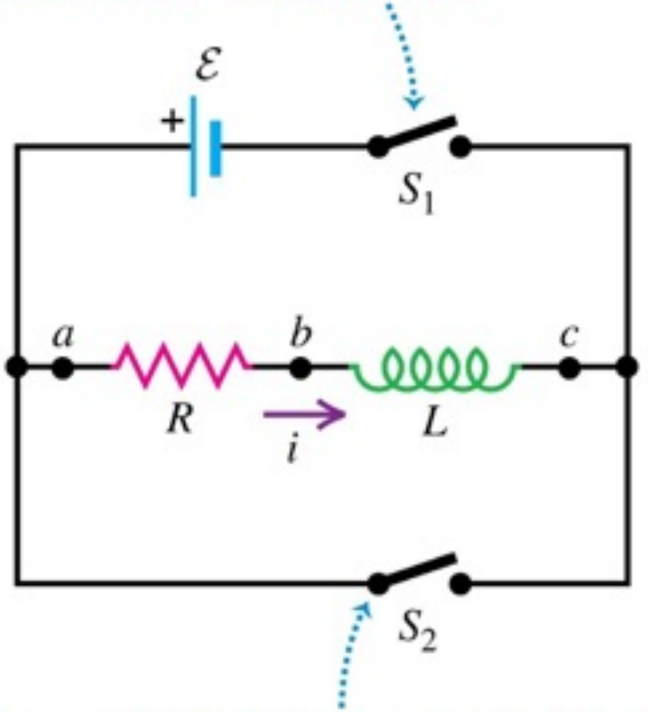
Permeability of material

The *R-L* circuit

- An *R-L circuit* contains a resistor and inductor and possibly an emf source.
- Shown is a typical *R-L* circuit.

Start with S1, S2 both open
At t=0, close S1

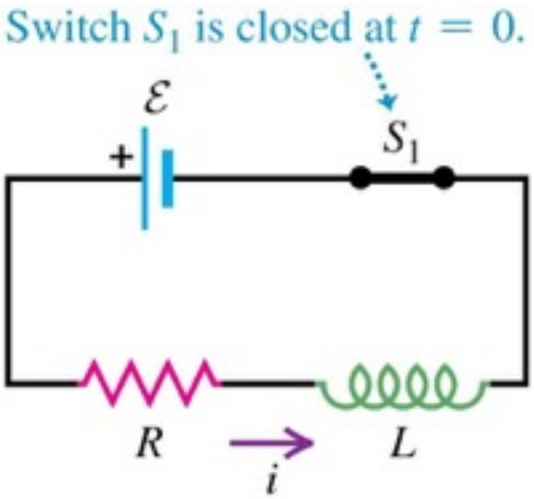
Closing switch S_1 connects the R-L combination in series with a source of emf $\mathcal{E}$.

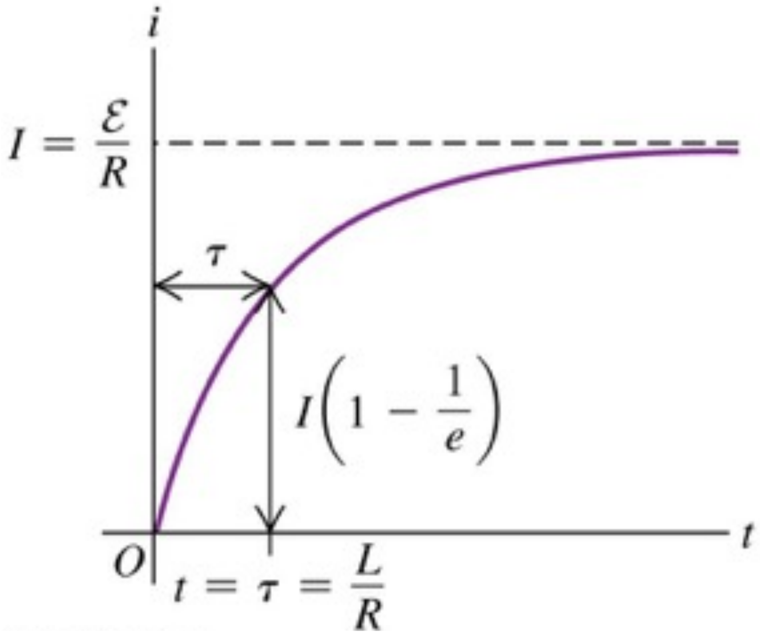


Closing switch S_2 while opening switch S_1 disconnnects the combination from the source.

Current growth in an *R-L* circuit

- Suppose that at some initial time $t = 0$ we close switch S_1.
- The current cannot change suddenly from zero to some final value.
- As the current increases, the rate of increase of current given becomes smaller and smaller.
- This means that the current approaches a final, steady-state value I.
- The **time constant** for the circuit is $\tau = L/R$.

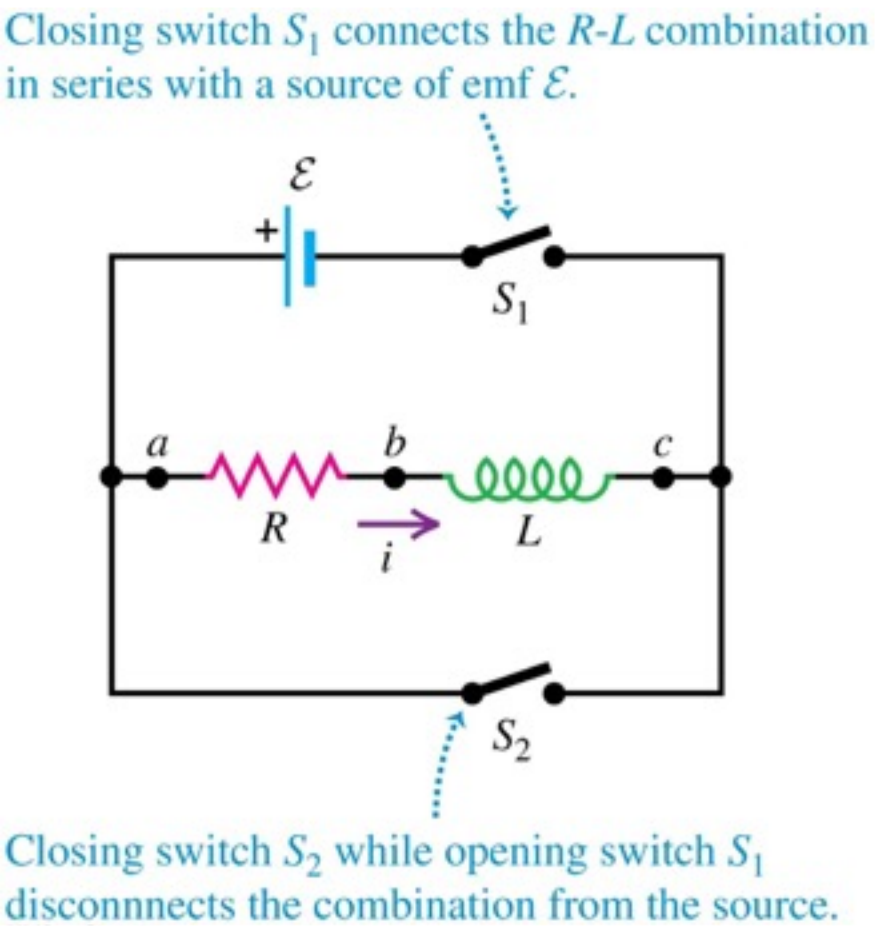

Switch S_1 is closed at $t = 0$.



The *R-L* circuit

- An *R-L circuit* contains a resistor and inductor and possibly an emf source.
- Shown is a typical *R-L* circuit.

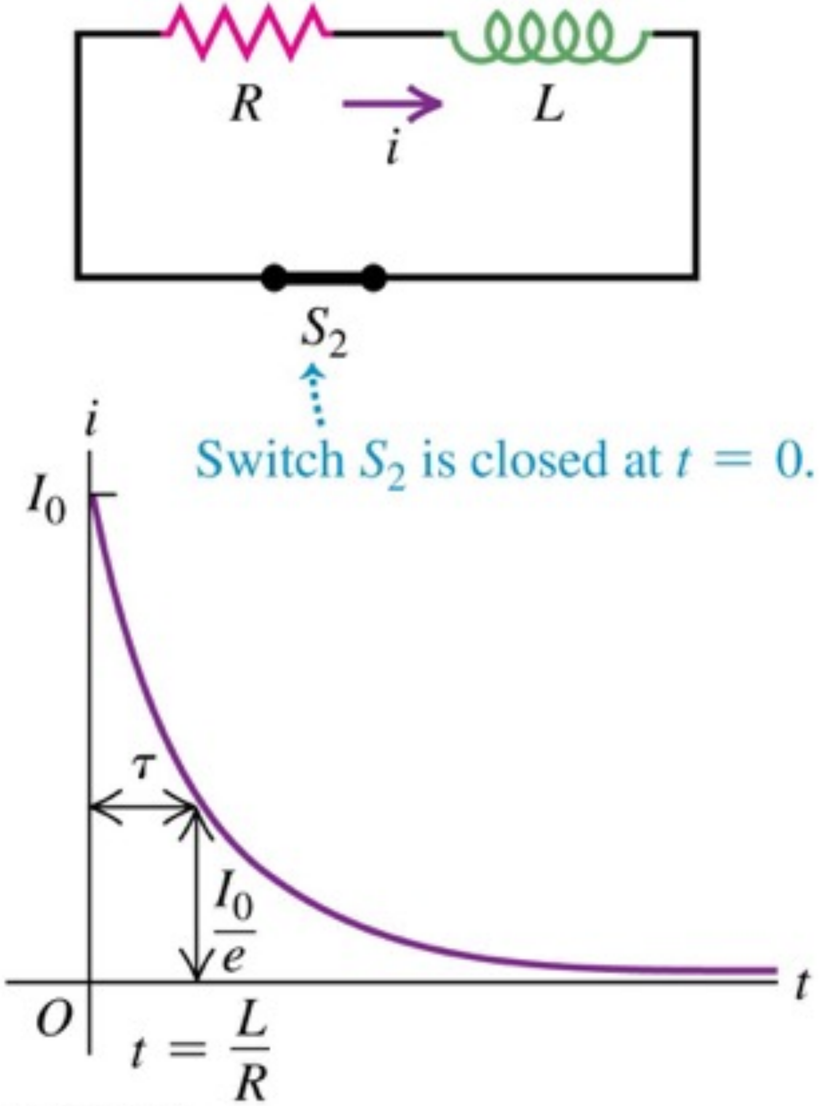
Start with S1 closed for a long time, S2 open
At *t*=0, close S2, open S1


Closing switch S_1 connects the *R-L* combination in series with a source of emf $\mathcal{E}$.

Closing switch S_2 while opening switch S_1 disconnnects the combination from the source.

Current decay in an *R-L* circuit

- Suppose there is an initial current I_0 running through the resistor and inductor shown.
- At time $t = 0$ we close the switch S_2, bypassing the battery (not shown).
- The energy stored in the magnetic field of the inductor provides the energy needed to maintain a decaying current.
- The **time constant** for the exponential decay of the current is $\tau = L/R$.



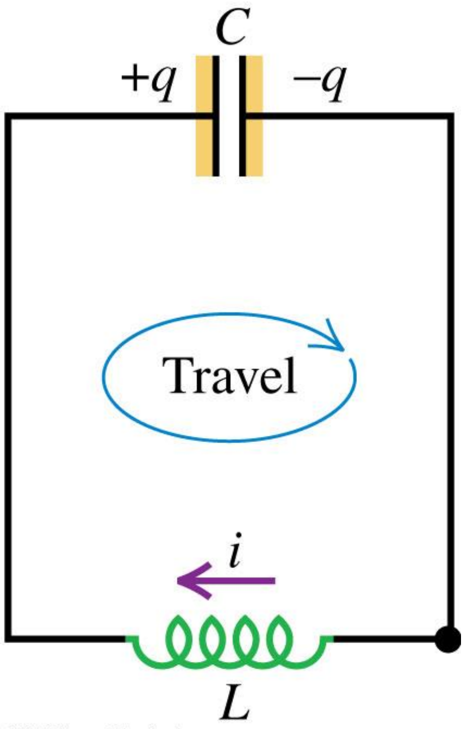
- We start with a charged capacitor in series with a switch and an inductor. $Q_{init}=CV_{init}$
- Let's apply Kirchhoff's loop rule:

$$\sum \Delta V = -\frac{q}{C} - L\frac{di}{dt} = 0 \Rightarrow$$

$$\frac{q}{C} + L\frac{di}{dt} = 0 \Rightarrow$$

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0 \Rightarrow$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$



Capacitor discharges through inductor

LC Circuits

- Do you recognize this equation?

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

- What if I just changed q to x and renamed the constant?

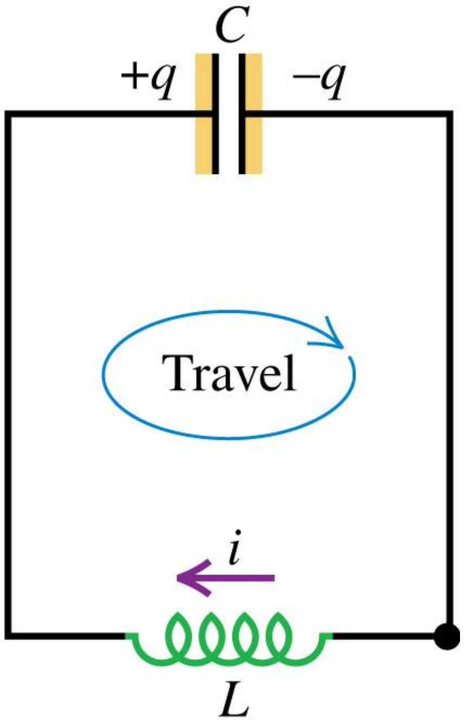
$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

- So the equation just describes oscillations with a frequency:

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \Rightarrow$$

$$q = Q_0 \cos\left(\omega t + \phi\right)$$

$$\omega = \sqrt{\frac{1}{LC}}$$



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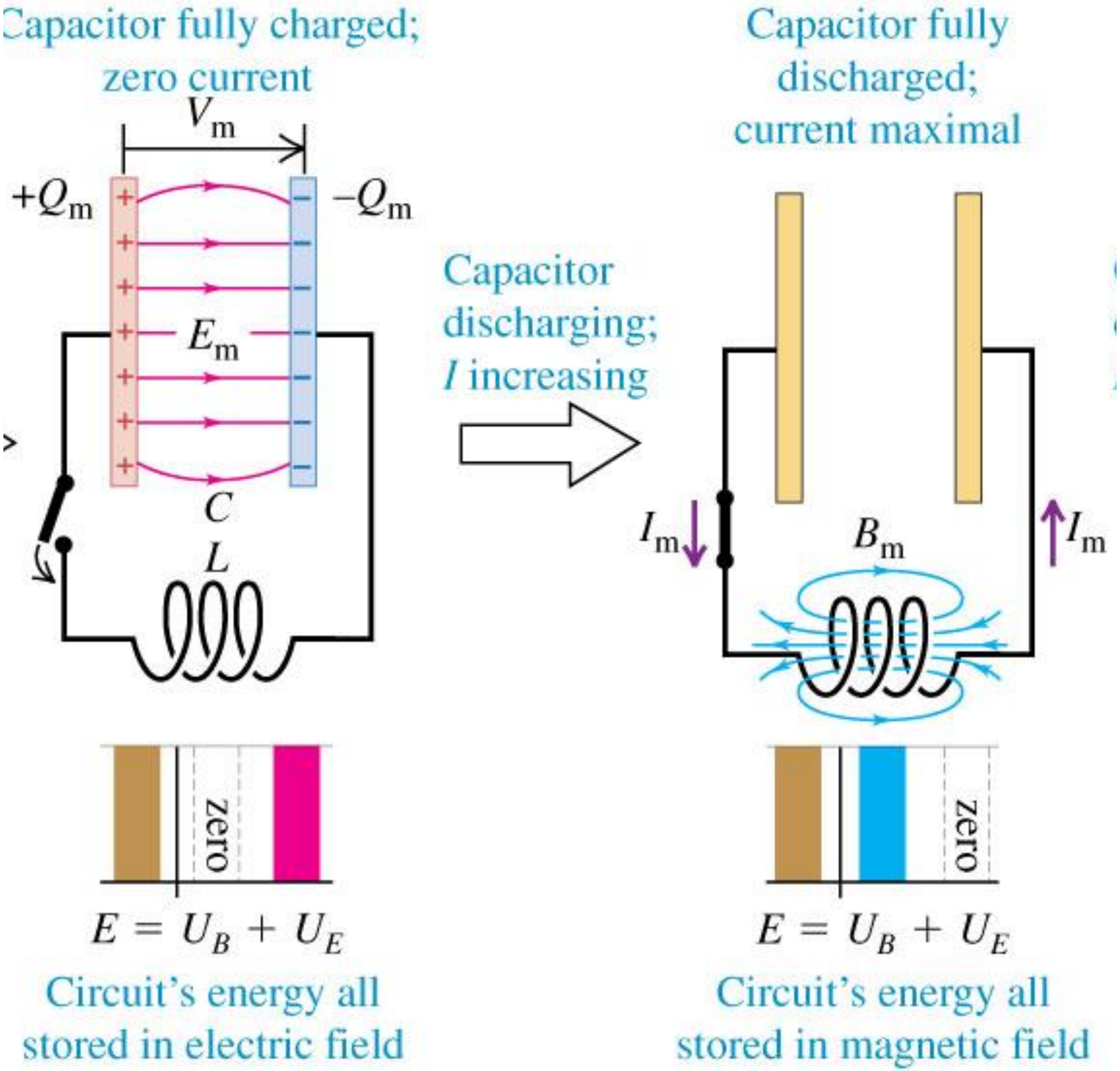
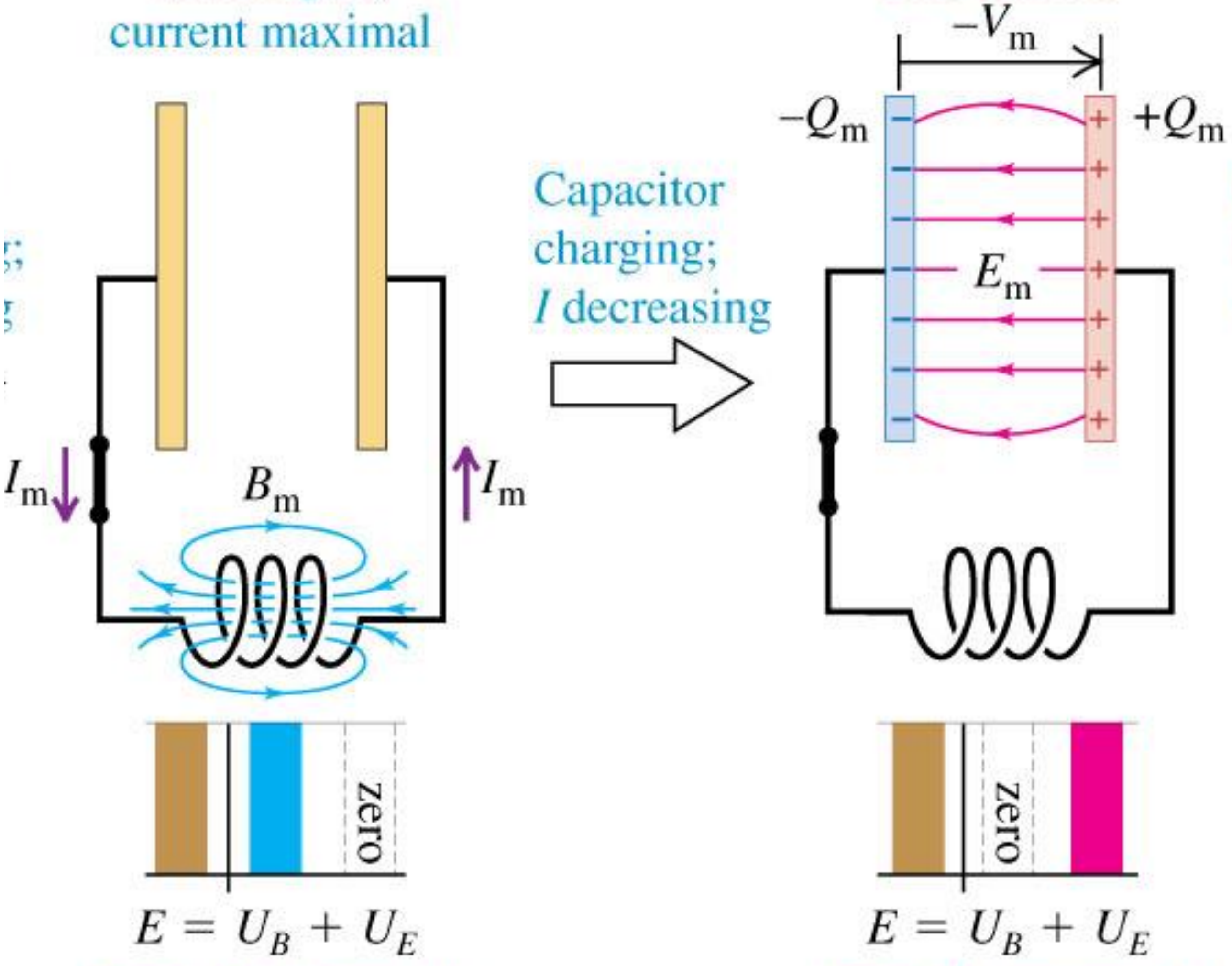

Capacitor fully charged; zero current
V_m
$+Q_m$
$-Q_m$
E_m
C
L
zero
$E = U_B + U_E$
Circuit's energy all stored in electric field
Capacitor discharging; I increasing
Capacitor fully discharged; current maximal
I_m
B_m
I_m
zero
$E = U_B + U_E$
Circuit's energy all stored in magnetic field



Capacitor fully discharged; current maximal

Capacitor charging; I decreasing

Capacitor fully charged; zero current

$E = U_B + U_E$

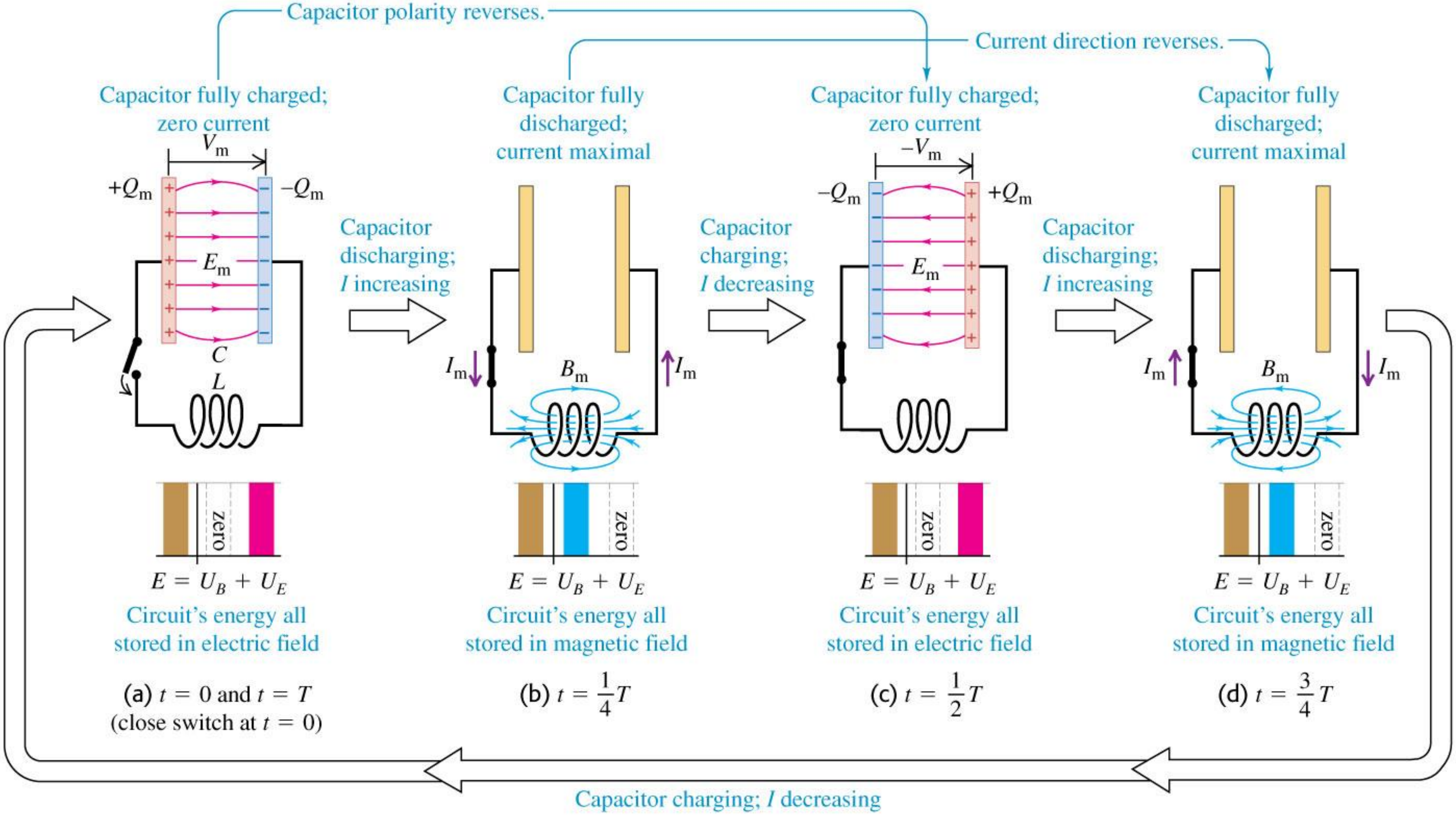
Circuit's energy all stored in magnetic field

$E = U_B + U_E$

Circuit's energy all stored in electric field

Energy/Current/Charge Oscillations

$$E_{\text{Total}} = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \text{Constant} = \frac{1}{2}\frac{Q_{\max}^2}{C} = \frac{1}{2}LI_{\max}^2$$



Analogy to Spring-Mass Oscillations

$$\frac{d^2x}{dt^2}+\frac{k}{m}x=0\Rightarrow$$

$$x=X_0\cos(\omega t+\phi)$$

$$\omega=\sqrt{\frac{k}{m}}$$

$$\frac{d^2q}{dt^2}+\frac{1}{LC}q=0\Rightarrow$$

$$q=Q_0\cos(\omega t+\phi)$$

$$\omega=\sqrt{\frac{1}{LC}}$$

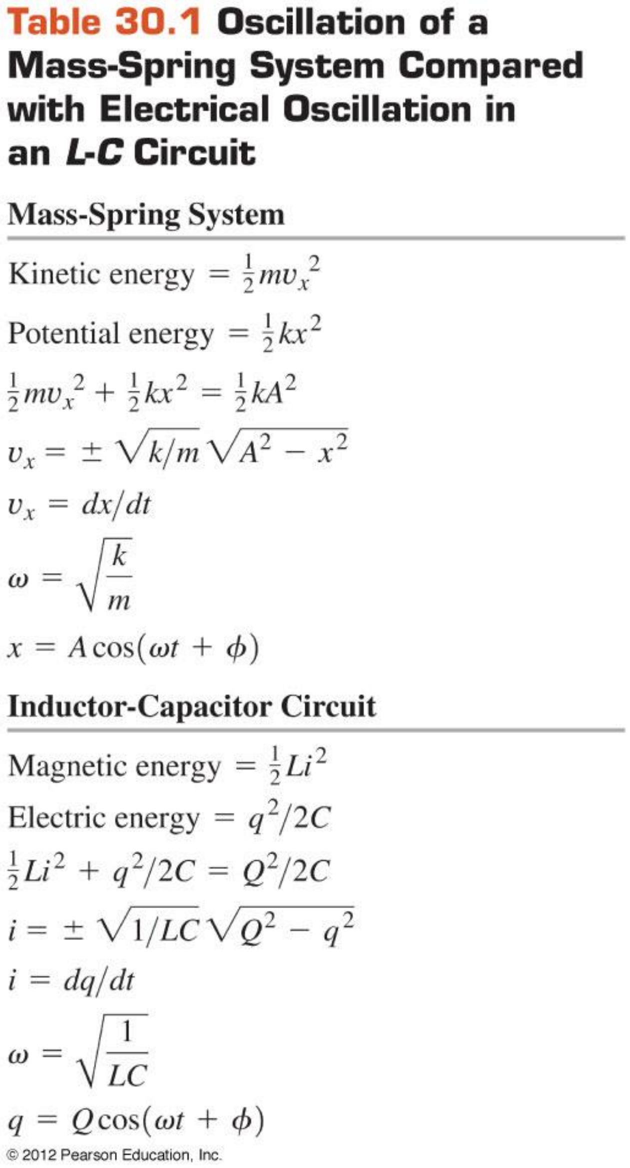


Table 30.1 Oscillation of a Mass-Spring System Compared with Electrical Oscillation in an *L-C* Circuit

Mass-Spring System

Kinetic energy $= \frac{1}{2}mv_x{}^2$

Potential energy $= \frac{1}{2}kx^2$

$\frac{1}{2}mv_x{}^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$

$v_x = dx/dt$

$\omega = \sqrt{\frac{k}{m}}$

$x = A\cos(\omega t + \phi)$

Inductor-Capacitor Circuit

Magnetic energy $= \frac{1}{2}Li^2$

Electric energy $= q^2/2C$

$\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$

$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$

$i = dq/dt$

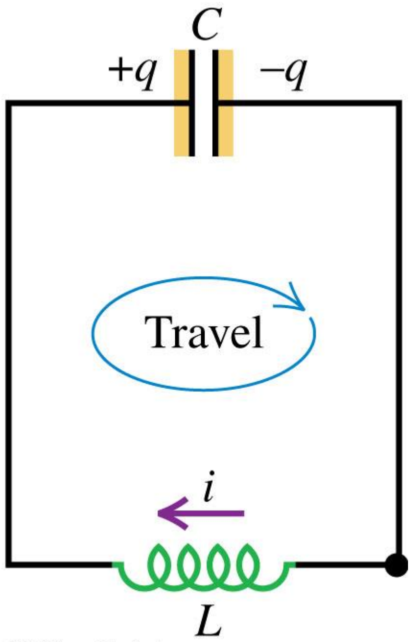
$\omega = \sqrt{\frac{1}{LC}}$

$q = Q\cos(\omega t + \phi)$

CPS 37-1

An inductor (inductance *L*) and a capacitor (capacitance *C*) are connected as shown.

If the values of both *L* and *C* are doubled, what happens to the *time* required for the capacitor charge to oscillate through a complete cycle?



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A. It becomes 4 times longer.

B. It becomes twice as long.

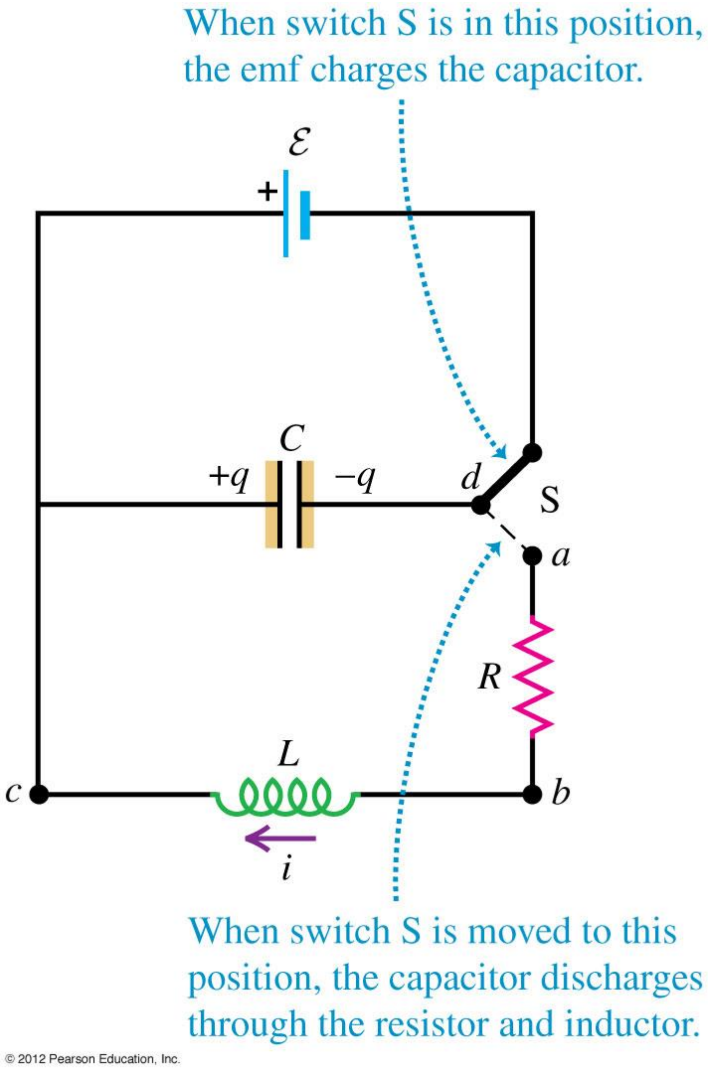
C. It is unchanged.

D. It becomes 1/2 as long.

E. It becomes 1/4 as long.

Energy/Current/Charge Oscillations

- Now that you have the spring-mass system in your head as an analogy, what do you think happens if we put a resistor in series with the capacitor and inductor?
- Remember that with the resistor, the total energy will decrease due to the power dissipated in the resistor.


When switch S is in this position, the emf charges the capacitor.

When switch S is moved to this position, the capacitor discharges through the resistor and inductor.