Lecture 37

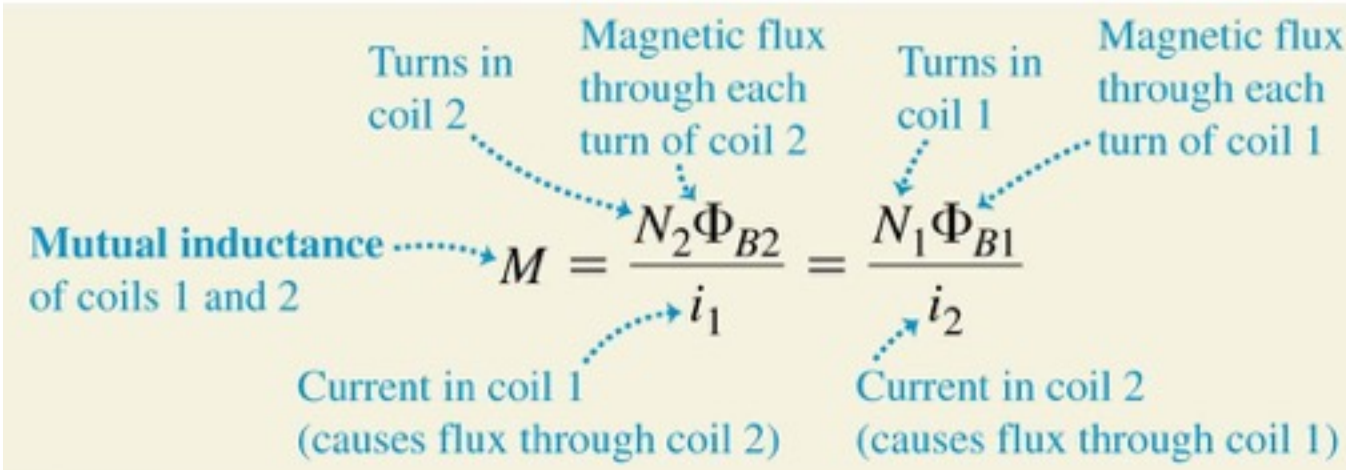
PHYC 161 Fall 2016

Course feedback is OPEN

So far: 12%

Mutual inductance

- The mutual inductance M is:

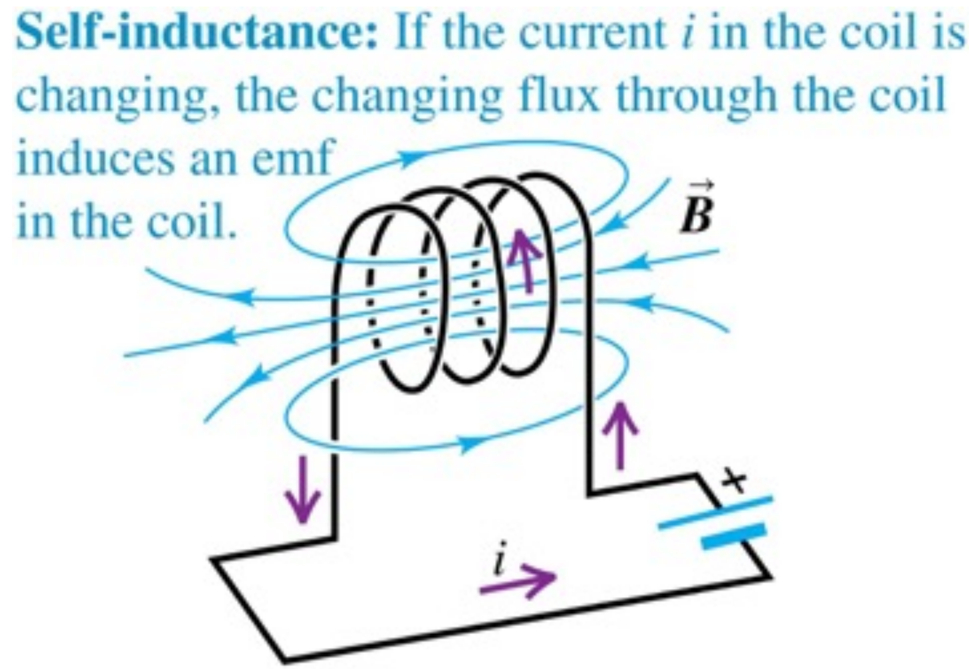


- The SI unit of mutual inductance is called the henry (1 H), in honor of the American physicist Joseph Henry.

$$1 \text{ H} = 1 \text{ Wb/A} = 1 \text{ V} \cdot \text{s/A} = 1\ \Omega \cdot \text{s} = 1 \text{ J/A}^2$$

Self-inductance

- Any circuit with a coil that carries a varying current has a **self-induced emf**.
- We define the self-inductance L of the circuit as:



Self-inductance (or **inductance**) of a coil:

$$L = \frac{N\Phi_B}{i}$$

- Number of turns in coil: N
- Flux due to current through each turn of coil: Φ_B
- Current in coil: i

HW # 40

Exercise 30.6

Description: A toroidal solenoid with mean radius r and cross-sectional area A is wound uniformly with N_1 turns. A second toroidal solenoid with N_2 turns is wound uniformly on top of the first, so that the two solenoids have the same cross-sectional area and...

A toroidal solenoid with mean radius r and cross-sectional area A is wound uniformly with N_1 turns. A second toroidal solenoid with N_2 turns is wound uniformly on top of the first, so that the two solenoids have the same cross-sectional area and mean radius.

Part A

What is the mutual inductance of the two solenoids? Assume that the magnetic field of the first solenoid is uniform across the cross section of the two solenoids.

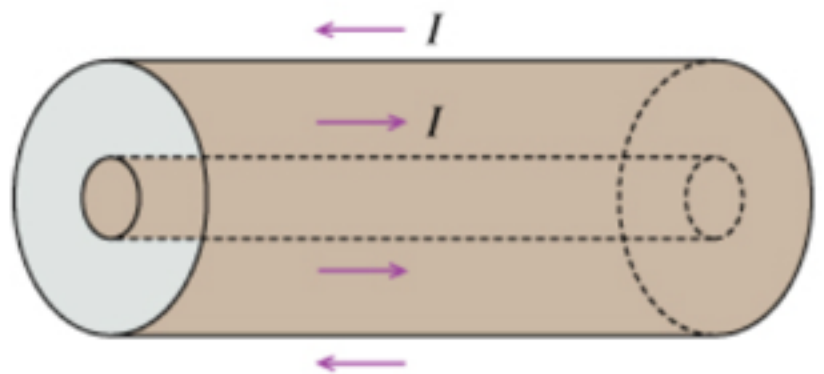
Express your answer in terms of the variables N_1, N_2, A, r, magnetic constant μ_0 and others appropriate constants.

HW #40

Part A

Consider a long coaxial cable made of two coaxial cylindrical conductors that carry equal currents I in opposite directions (see figure). The inner cylinder is a small solid conductor of radius a. The outer cylinder is a thin walled conductor of outer radius b, electrically insulated from the inner conductor. Calculate the self-inductance per unit length $\frac{L}{l}$ of this coaxial cable. (L is the inductance of part of the cable and l is the length of that part.) Due to what is known as the "skin effect", the current I flows down the (outer) surface of the inner conducting cylinder and back along the outer surface of the outer conducting cylinder. However, you may ignore the thickness of the outer cylinder.

Express your answer in terms of some or all the variables I, a, b, and μ_0, the permeability of free space.



HW40

See Example 30.8 in the book!

Exercise 30.8

Description: A toroidal solenoid has N turns, cross-sectional area A, and mean radius r. (a) Calcualte the coil's self-inductance. (b) If the current decreases uniformly from 5.00 A to 2.00 A in 3.00 ms, calculate the self-induced emf in the coil. (c) The...

A toroidal solenoid has 580 turns, cross-sectional area 6.40 cm^2 , and mean radius 4.50 cm .

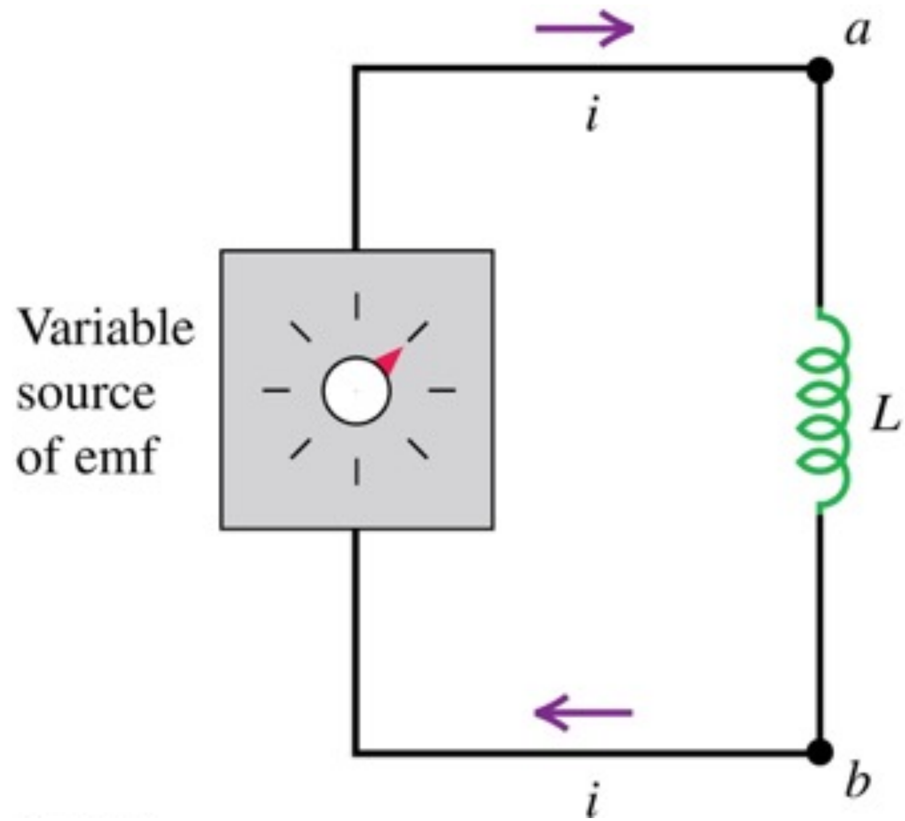
Part A

Calcualte the coil's self-inductance.

Inductors as circuit elements

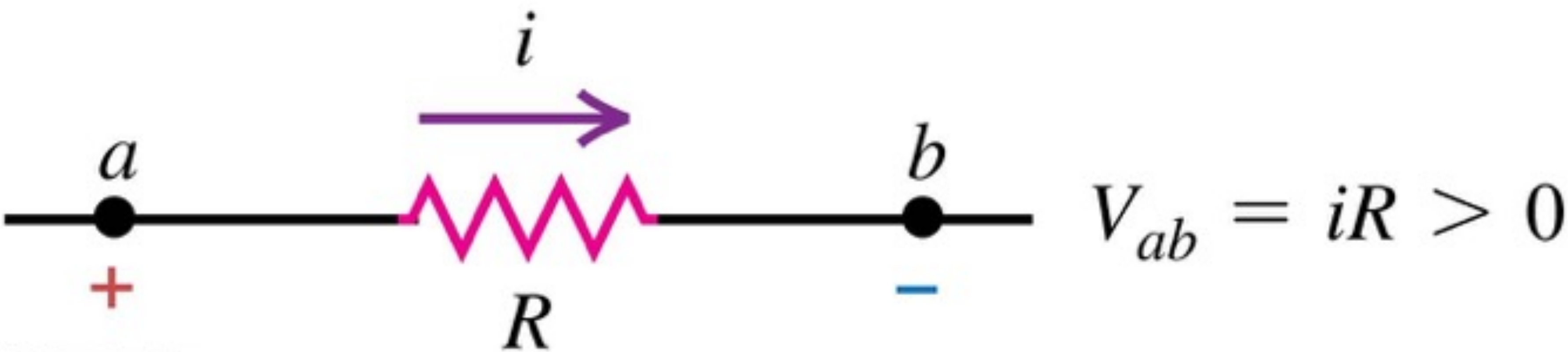
- In the circuit shown, the box enables us to control the current i in the circuit.
- The potential difference between the terminals of the inductor L is:

$$V_{ab} = V_a - V_b = L\frac{di}{dt}$$



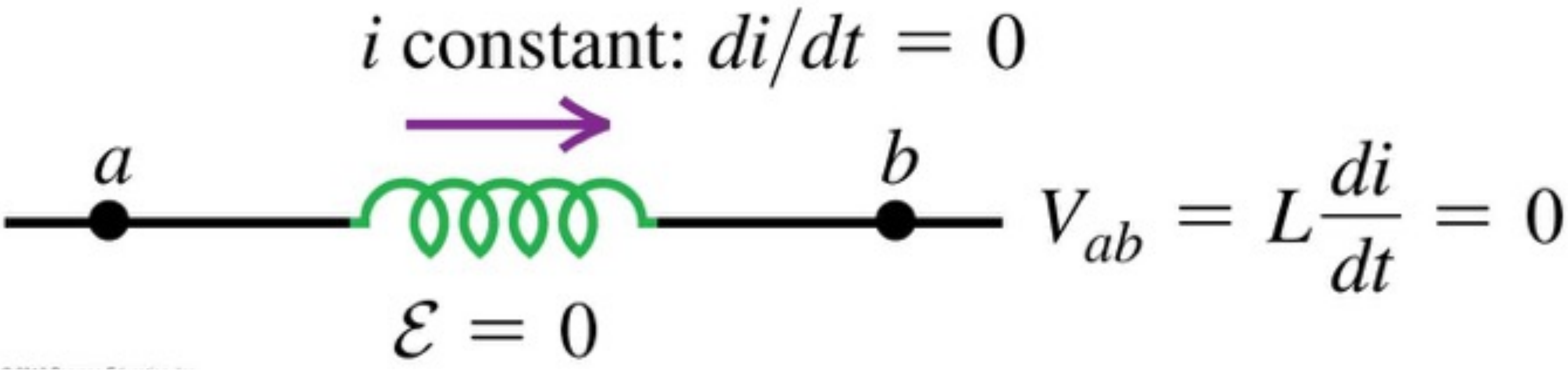
Potential across a resistor

- The potential difference across a resistor depends on the current.
- When you have a resistor with current i flowing from a to b, the potential *drops* from a to b.



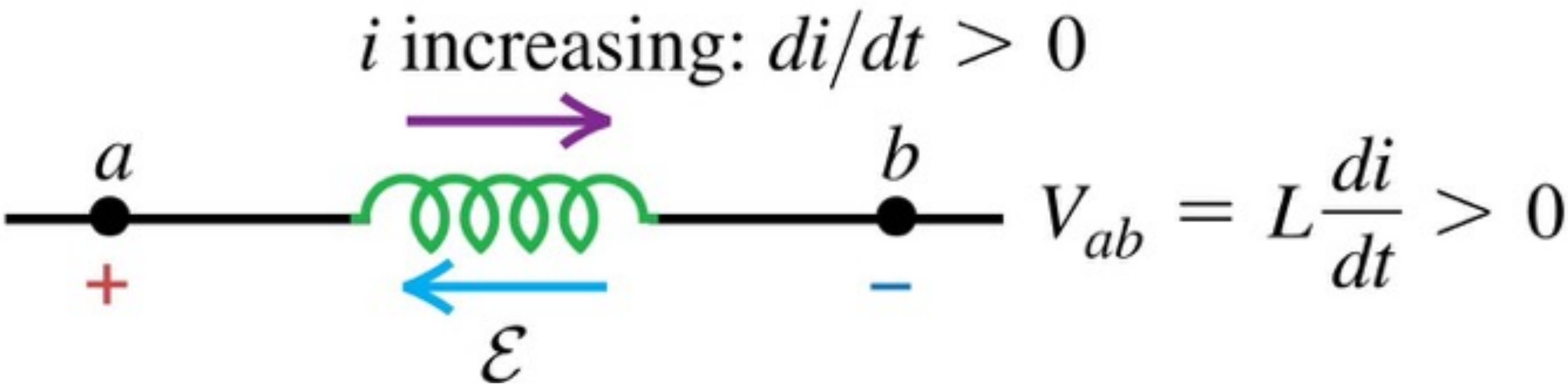
Potential across an inductor with constant current

- The potential difference across an inductor depends on the rate of change of the current.
- When you have an inductor with *constant* current i flowing from a to b, there is *no* potential difference.



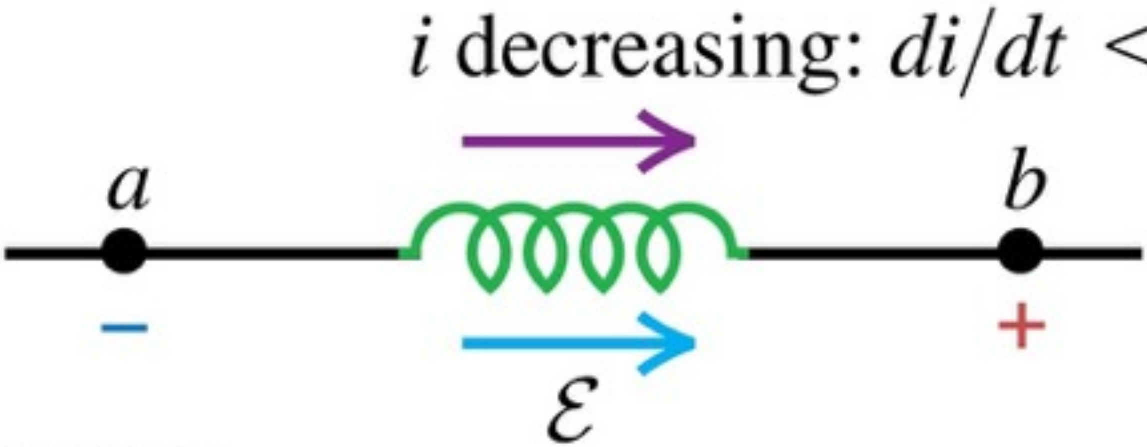
Potential across an inductor with increasing current

- The potential difference across an inductor depends on the rate of change of the current.
- When you have an inductor with *increasing* current i flowing from a to b, the potential *drops* from a to b.



Potential across an inductor with decreasing current

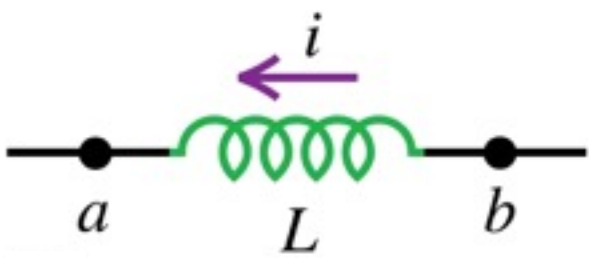
- The potential difference across an inductor depends on the rate of change of the current.
- When you have an inductor with *decreasing* current i flowing from a to b, the potential *increases* from a to b.



$$V_{ab} = L\frac{di}{dt} < 0$$

Q30.2

A current i flows through an inductor L in the direction from point b toward point a. There is zero resistance in the wires of the inductor. If the current is *decreasing*,



A. the potential is greater at point a than at point b.

B. the potential is less at point a than at point b.

C. the answer depends on the magnitude of di/dt compared to the magnitude of i.

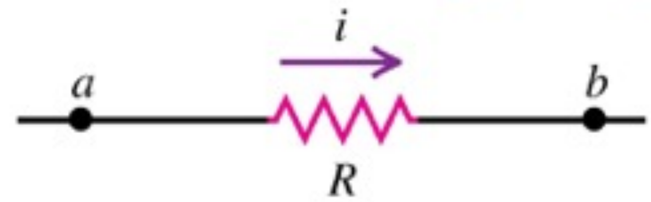
D. the answer depends on the value of the inductance L.

E. both C and D are correct.

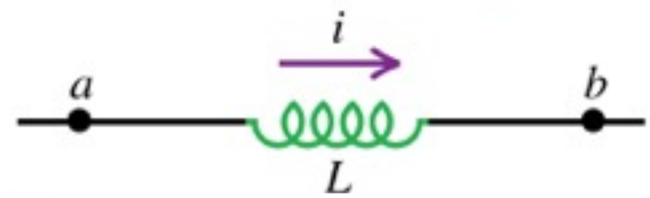
Magnetic field energy

- A resistor is a device in which energy is irrecoverably *dissipated*.
- By contrast, energy stored in a current-carrying inductor can be recovered when the current decreases to zero.

Resistor with current i: energy is *dissipated*.



Inductor with current i: energy is *stored*.



Energy stored in an inductor → $$U = L\int_0^I i\,di = \tfrac{1}{2}LI^2$$

Inductance (L)

Final current (I)

Integral from initial (zero) value of instantaneous current to final value

Magnetic energy density

- The energy in an inductor is actually stored in the magnetic field of the coil, just as the energy of a capacitor is stored in the electric field between its plates.
- In a vacuum, the energy per unit volume, or **magnetic energy density**, is:

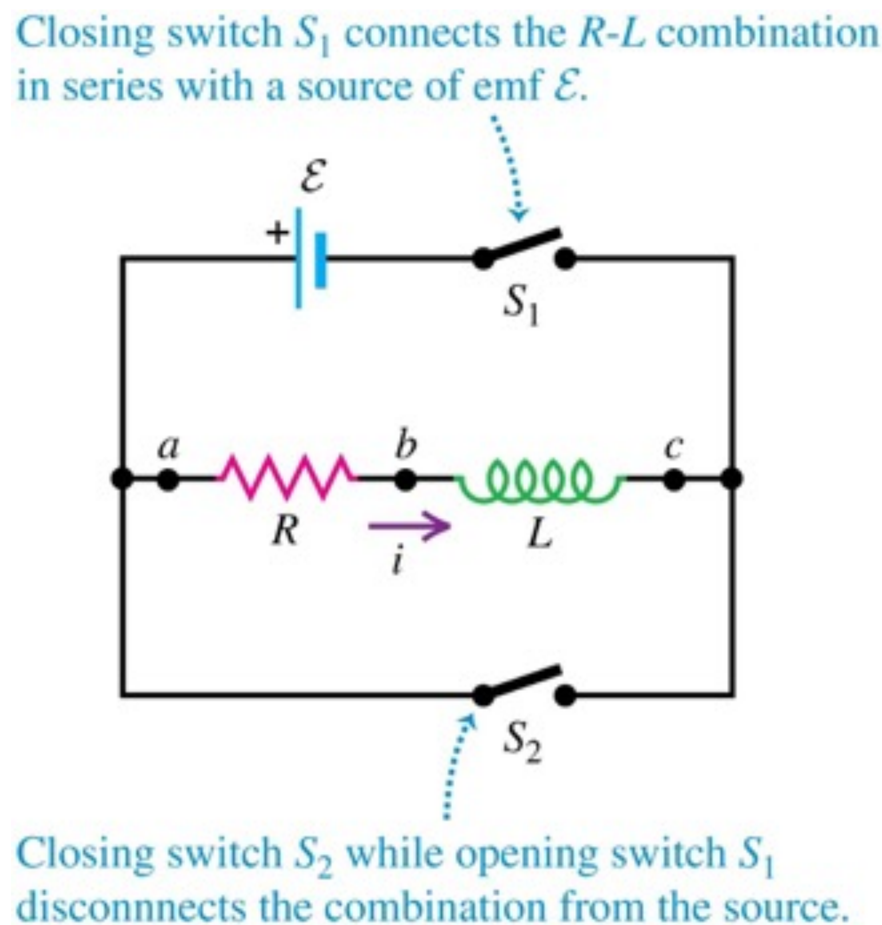
Magnetic energy density in vacuum → $$u = \frac{B^2}{2\mu_0}$$ ← B: Magnetic-field magnitude; μ_0: Magnetic constant

- When the magnetic field is located within a material with (constant) magnetic permeability $\mu = K_m \mu_0$, we replace μ_0 by μ in the above equation:

Magnetic energy density in a material → $$u = \frac{B^2}{2\mu}$$ ← B: Magnetic-field magnitude; μ: Permeability of material

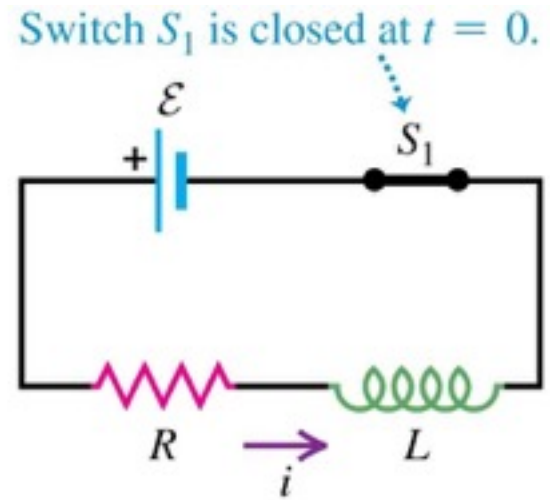
The *R-L* circuit

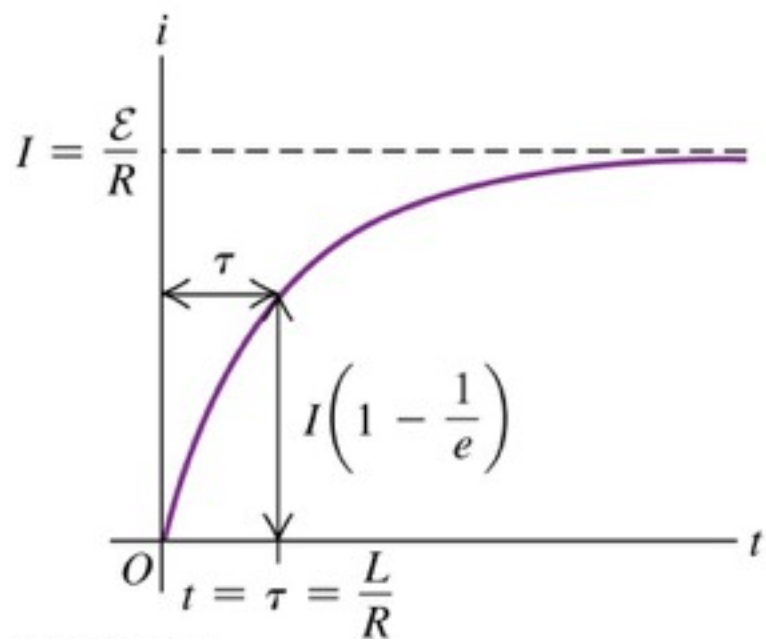
- An *R-L circuit* contains a resistor and inductor and possibly an emf source.
- Shown is a typical *R-L* circuit.



Current growth in an *R-L* circuit

- Suppose that at some initial time $t = 0$ we close switch S_1.
- The current cannot change suddenly from zero to some final value.
- As the current increases, the rate of increase of current given becomes smaller and smaller.
- This means that the current approaches a final, steady-state value I.
- The **time constant** for the circuit is $\tau = L/R$.





Current decay in an *R-L* circuit

- Suppose there is an initial current I_0 running through the resistor and inductor shown.
- At time $t = 0$ we close the switch S_2, bypassing the battery (not shown).
- The energy stored in the magnetic field of the inductor provides the energy needed to maintain a decaying current.
- The **time constant** for the exponential decay of the current is $\tau = L/R$.