

Lecture 36

PHYC 161 Fall 2015

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Maxwell's equations of electromagnetism

- All the relationships between electric and magnetic fields and their sources are summarized by four equations, called **Maxwell's equations**.
- The first Maxwell equation is Gauss's law for electric fields from Chapter 22:

Gauss's law for \vec{E} :

Flux of electric field through a closed surface

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Charge enclosed by surface

Electric constant

- The second Maxwell equation is Gauss's law for magnetic fields from Chapter 27:

Gauss's law for \vec{B} :

Flux of magnetic field through any closed surface ...

$$\oint \vec{B} \cdot d\vec{A} = 0$$

... equals zero.

Maxwell's equations of electromagnetism

- The third Maxwell equation is this chapter's formulation of Faraday's law:

Faraday's law
for a stationary
integration path:

Line integral of electric field around path

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt}$$

Negative of the time
rate of change of
magnetic flux through path

- The fourth Maxwell equation is Ampere's law, including displacement current:

Ampere's law
for a stationary
integration path:

Line integral of magnetic
field around path

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}}$$

Electric
constant

Time rate of change of
electric flux through path

Magnetic
constant

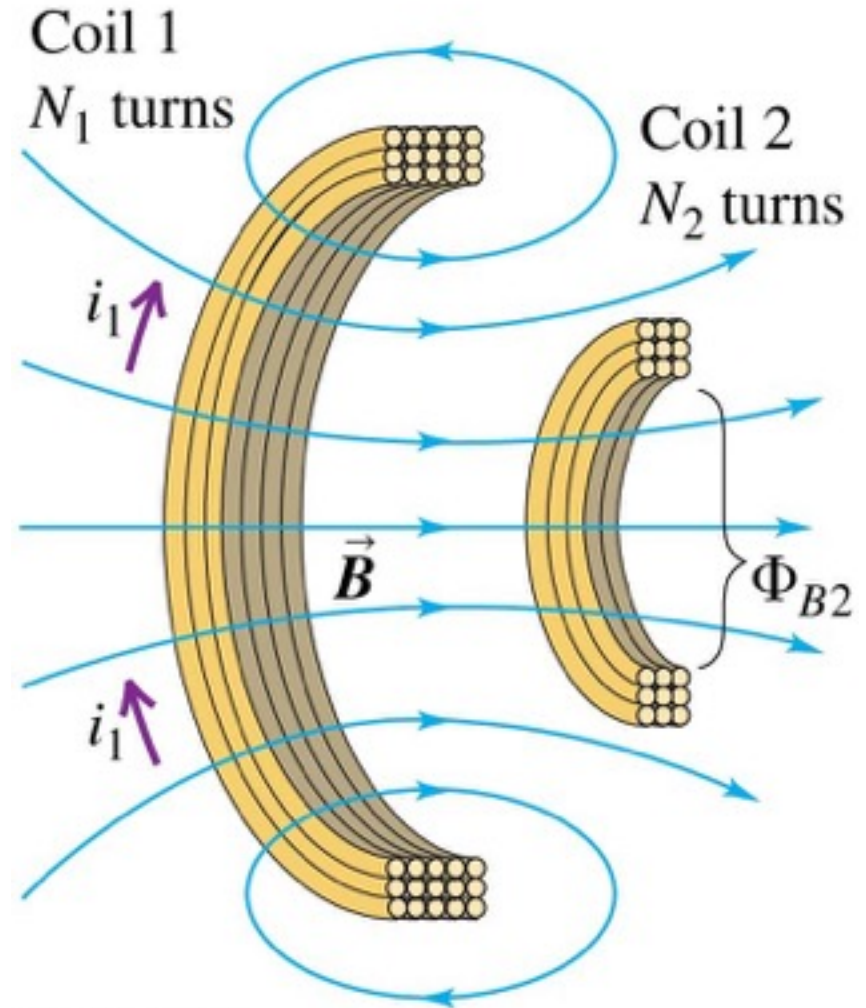
Conduction current
through path

Displacement current
through path

Mutual inductance

- Consider two neighboring coils of wire, as shown.
- If the current in coil 1 changes, this induces an emf in coil 2, and vice versa.
- The proportionality constant for this pair of coils is called the **mutual inductance**, M .

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B2}}{dt}$$



Mutual inductance

- The mutual inductance M is:

The diagram illustrates the definition of mutual inductance M between two coils. It shows two equivalent expressions for M based on the flux linkage in each coil. The first expression is $M = \frac{N_2 \Phi_{B2}}{i_1}$, where N_2 is the number of turns in coil 2, Φ_{B2} is the magnetic flux through each turn of coil 2, and i_1 is the current in coil 1. The second expression is $M = \frac{N_1 \Phi_{B1}}{i_2}$, where N_1 is the number of turns in coil 1, Φ_{B1} is the magnetic flux through each turn of coil 1, and i_2 is the current in coil 2. The text 'Mutual inductance of coils 1 and 2' points to the M in the equations. Dotted arrows connect the labels to the corresponding variables in the equations.

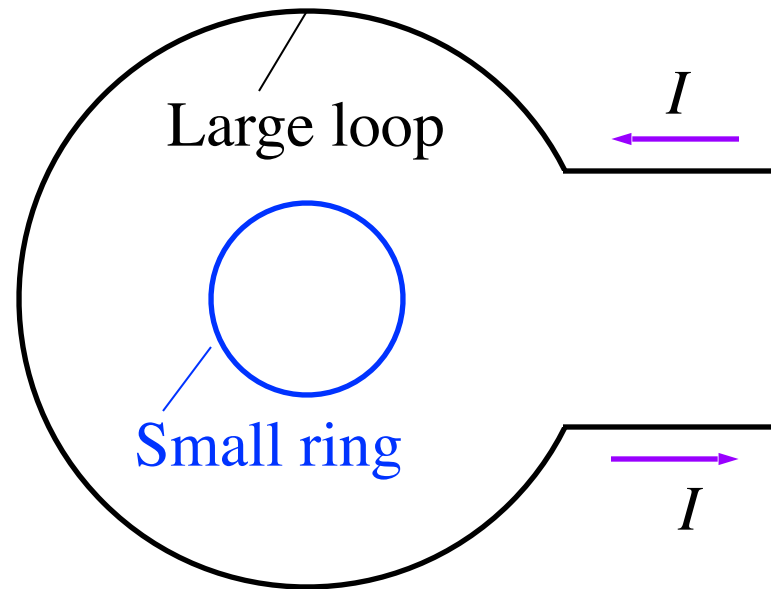
$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

- The SI unit of mutual inductance is called the henry (1 H), in honor of the American physicist Joseph Henry.

$$1 \text{ H} = 1 \text{ Wb/A} = 1 \text{ V} \cdot \text{s/A} = 1 \Omega \cdot \text{s} = 1 \text{ J/A}^2$$

Q30.1

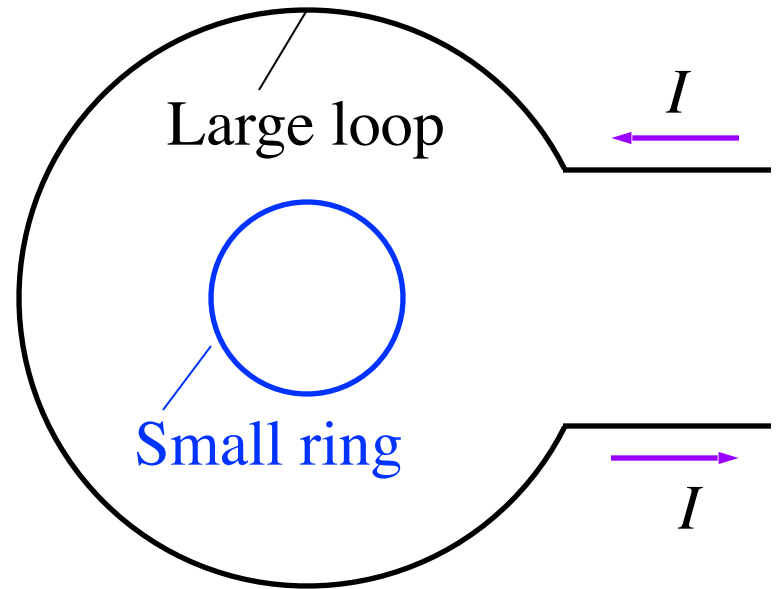
A small, circular ring of wire (shown in blue) is inside a larger loop of wire that carries a current I as shown. The small ring and the larger loop both lie in the same plane. If I increases, the current that flows in the small ring



- A. is clockwise and caused by self-inductance.
- B. is counterclockwise and caused by self-inductance.
- C. is clockwise and caused by mutual inductance.
- D. is counterclockwise and caused by mutual inductance.
- E. is zero, since the two rings of wire are not connected.

A30.1

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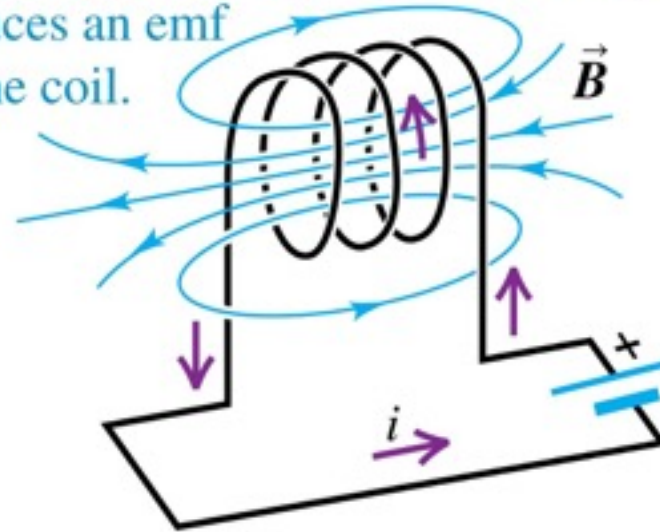


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Self-inductance

- Any circuit with a coil that carries a varying current has a **self-induced emf**.
- We define the self-inductance L of the circuit as:

Self-inductance: If the current i in the coil is changing, the changing flux through the coil induces an emf in the coil.



Self-inductance (or inductance) of a coil

$$L = \frac{N\Phi_B}{i}$$

Number of turns in coil

Flux due to current through each turn of coil

Current in coil

Inductors and lightning strikes

- If lightning strikes part of an electrical power transmission system, it causes a sudden spike in voltage that can damage the components of the system.
- To minimize these effects, large **inductors** are incorporated into the transmission system.
- These use the principle that an inductor opposes and suppresses any rapid changes in the current.



Inductors as circuit elements

- In the circuit shown, the box enables us to control the current i in the circuit.
- The potential difference between the terminals of the inductor L is:

$$V_{ab} = V_a - V_b = L \frac{di}{dt}$$

