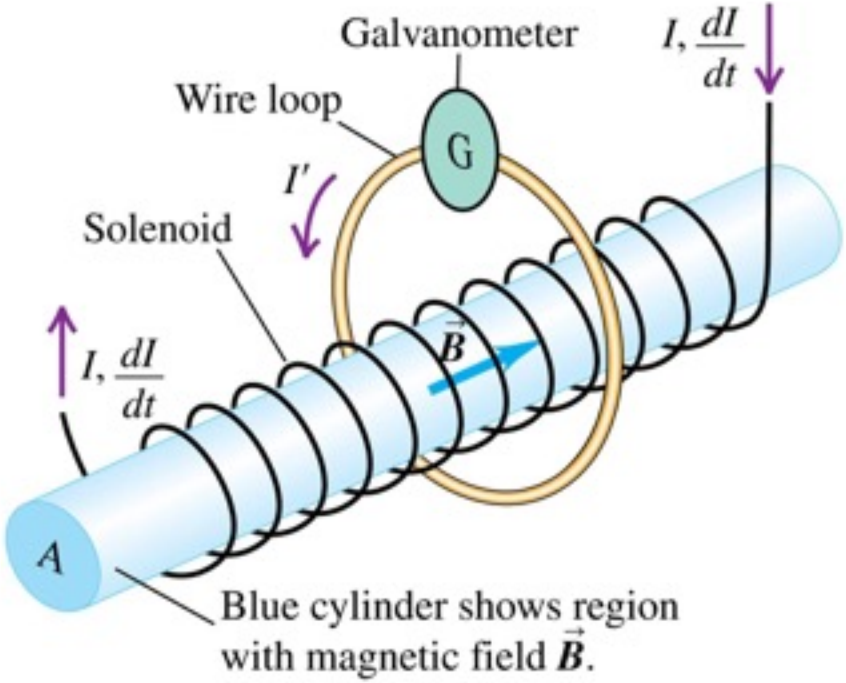
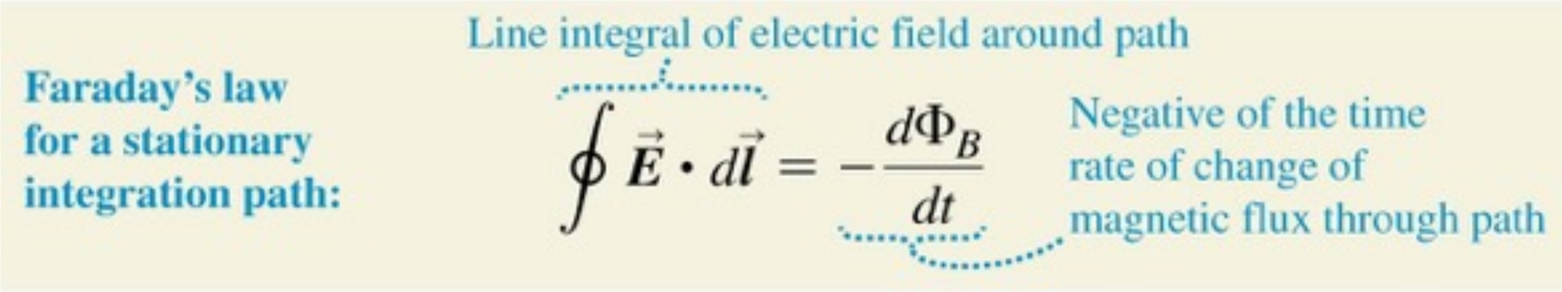
Lecture 35

PHYC 161 Fall 2016

Induced electric fields

- A long, thin solenoid is encircled by a circular conducting loop.
- Electric field in the loop is what must drive the current.
- When the solenoid current I changes with time, the magnetic flux also changes, and the induced emf can be written in terms of **induced electric field**:

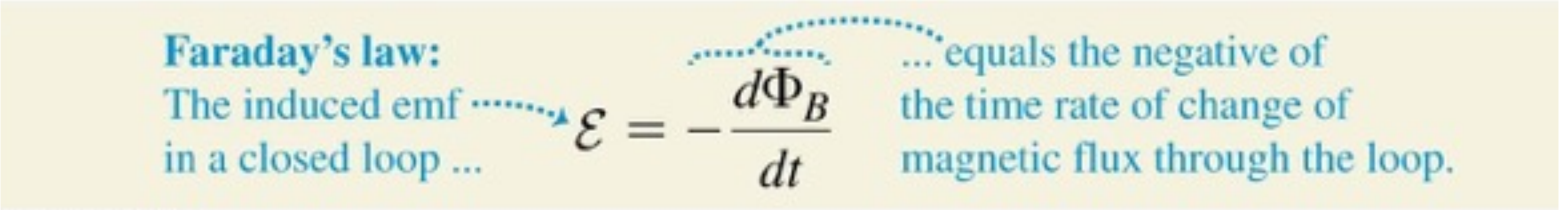




Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E} \cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

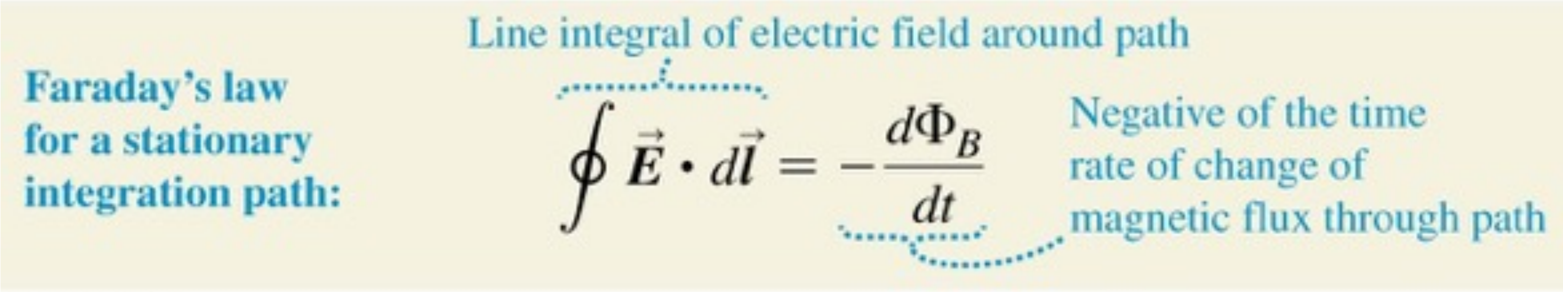
Two equivalent statements:



Faraday's law:
The induced emf in a closed loop ...

$$\mathcal{E} = \overbrace{-\frac{d\Phi_B}{dt}}$$

... equals the negative of the time rate of change of magnetic flux through the loop.



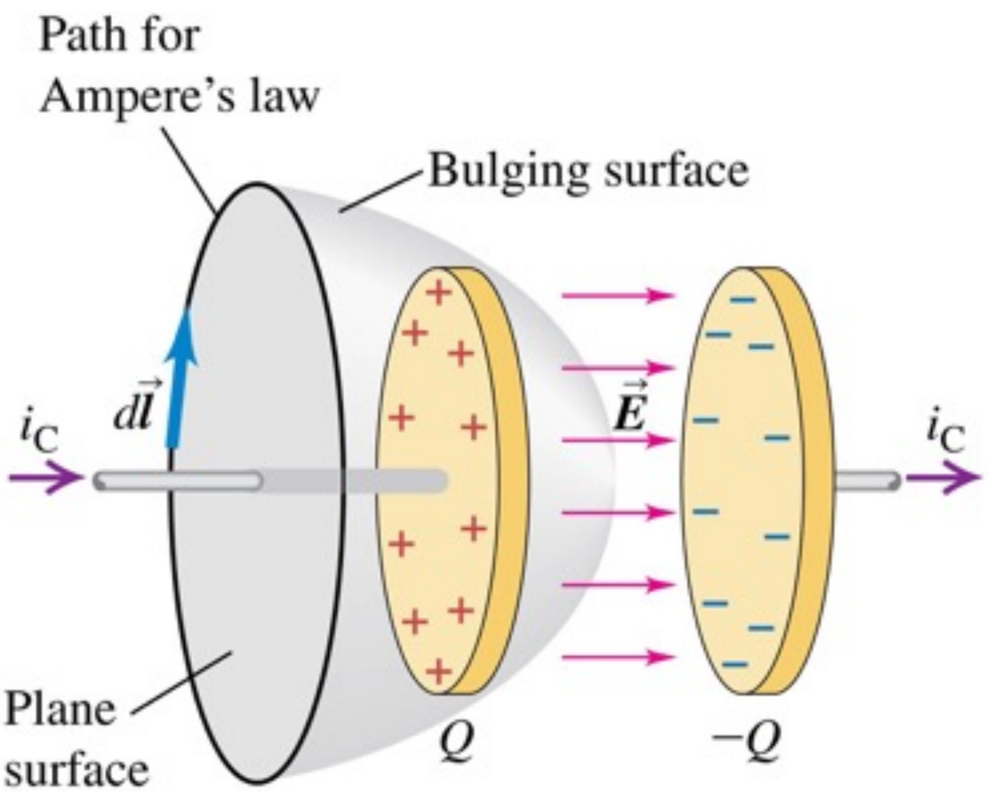
Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E} \cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}$$

Negative of the time rate of change of magnetic flux through path

Displacement current

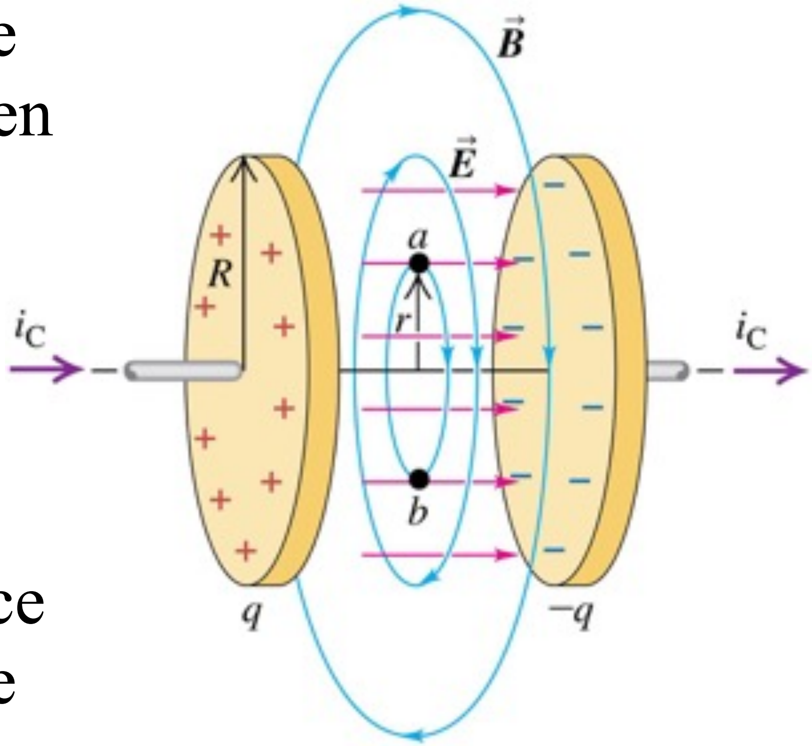
- Ampere’s law is *incomplete*, as can be shown by considering the process of charging a capacitor, as shown.



- For the plane circular area bounded by the circle, I_{encl} is the current i_C in the left conductor.
- But the surface that bulges out to the right is bounded by the same circle, and the current through that surface is zero.
- This leads to a contradiction.

Displacement current

- When a capacitor is charging, the electric field is increasing between the plates.
- We can define a fictitious **displacement current** i_D in the region between the plates.
- This can be regarded as the source of the magnetic field between the plates.



Displacement current through an area

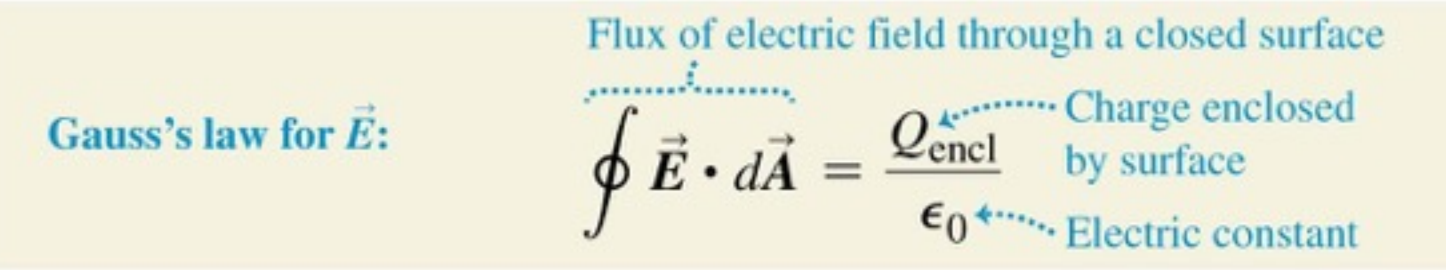
$$i_D = \epsilon \frac{d\Phi_E}{dt}$$

Time rate of change of electric flux through area

Permittivity of material in area

Maxwell’s equations of electromagnetism

- All the relationships between electric and magnetic fields and their sources are summarized by four equations, called **Maxwell’s equations**.
- The first Maxwell equation is Gauss’s law for electric fields from Chapter 22:



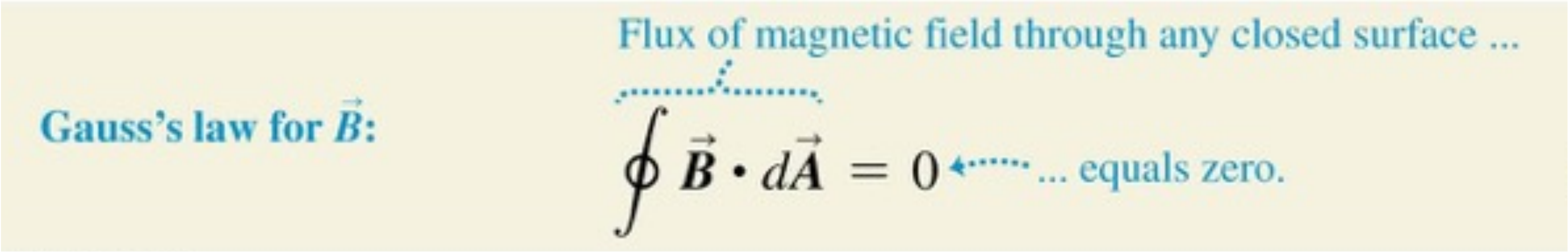
Gauss’s law for $\vec{E}$:

$$\overbrace{\oint \vec{E}\cdot d\vec{A}}^{\text{Flux of electric field through a closed surface}} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Charge enclosed by surface

Electric constant

- The second Maxwell equation is Gauss’s law for magnetic fields from Chapter 27:



Gauss’s law for $\vec{B}$:

$$\overbrace{\oint \vec{B}\cdot d\vec{A}}^{\text{Flux of magnetic field through any closed surface ...}} = 0$$

... equals zero.

Maxwell’s equations of electromagnetism

- The third Maxwell equation is this chapter’s formulation of Faraday’s law:

Faraday’s law for a stationary integration path:

$$\overbrace{\oint \vec{E}\cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

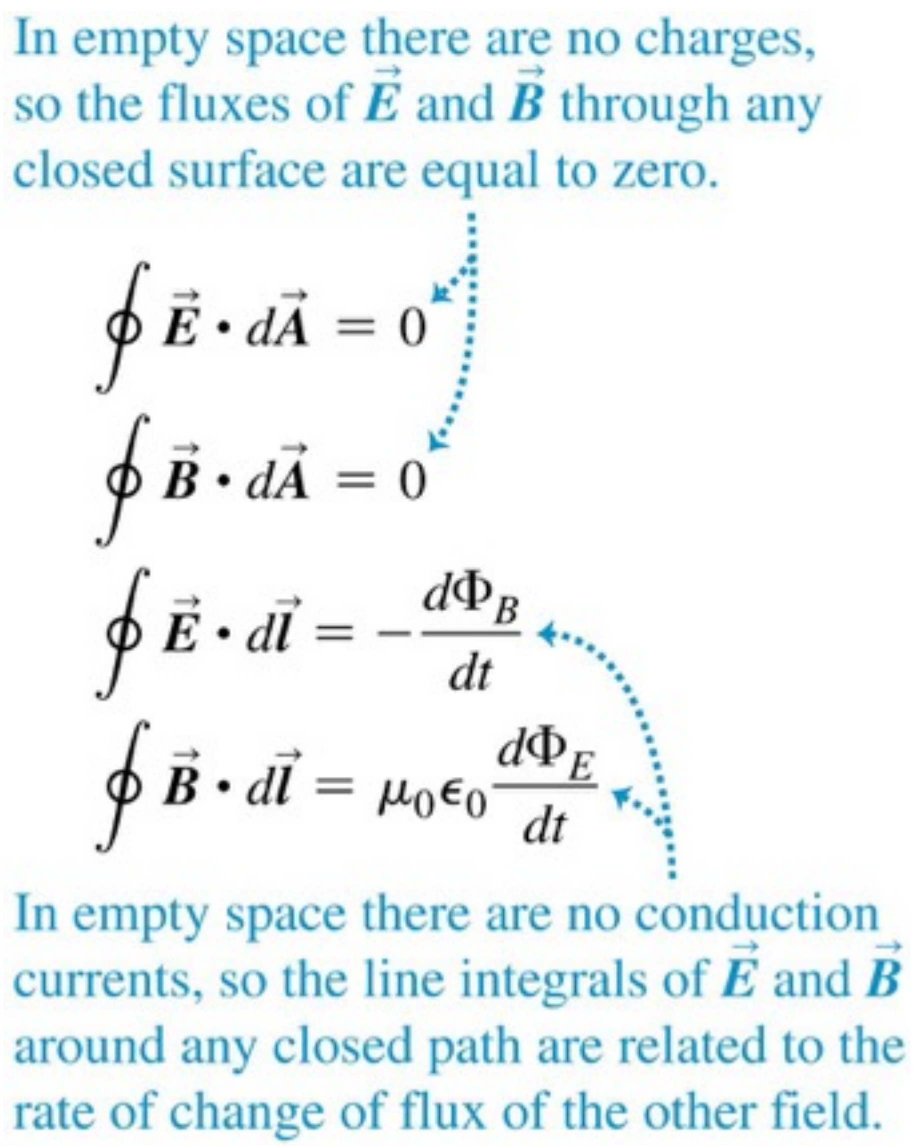
- The fourth Maxwell equation is Ampere’s law, including displacement current:

Ampere’s law for a stationary integration path:

$$\overbrace{\oint \vec{B}\cdot d\vec{l}}^{\text{Line integral of magnetic field around path}} = \underset{\text{Magnetic constant}}{\mu_0}\left(\underset{\text{Conduction current through path}}{i_C} + \underbrace{\overset{\text{Electric constant}}{\epsilon_0}\,\overbrace{\frac{d\Phi_E}{dt}}^{\text{Time rate of change of electric flux through path}}}_{\text{Displacement current through path}}\right)_{\text{encl}}$$

Maxwell’s equations in empty space

- There is a remarkable symmetry in Maxwell’s equations.
- In empty space where there is no charge, the first two equations are identical in form.
- The third equation says that a changing magnetic flux creates an electric field, and the fourth says that a changing electric flux creates a magnetic field.



In empty space there are no charges, so the fluxes of $\vec{E}$ and $\vec{B}$ through any closed surface are equal to zero.

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

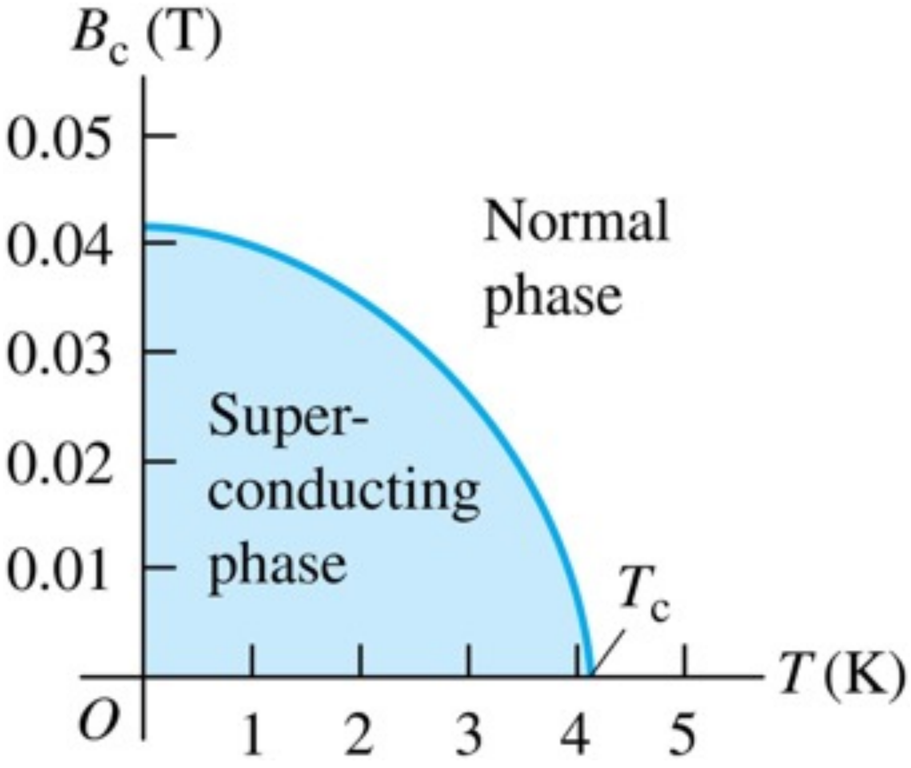
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In empty space there are no conduction currents, so the line integrals of $\vec{E}$ and $\vec{B}$ around any closed path are related to the rate of change of flux of the other field.

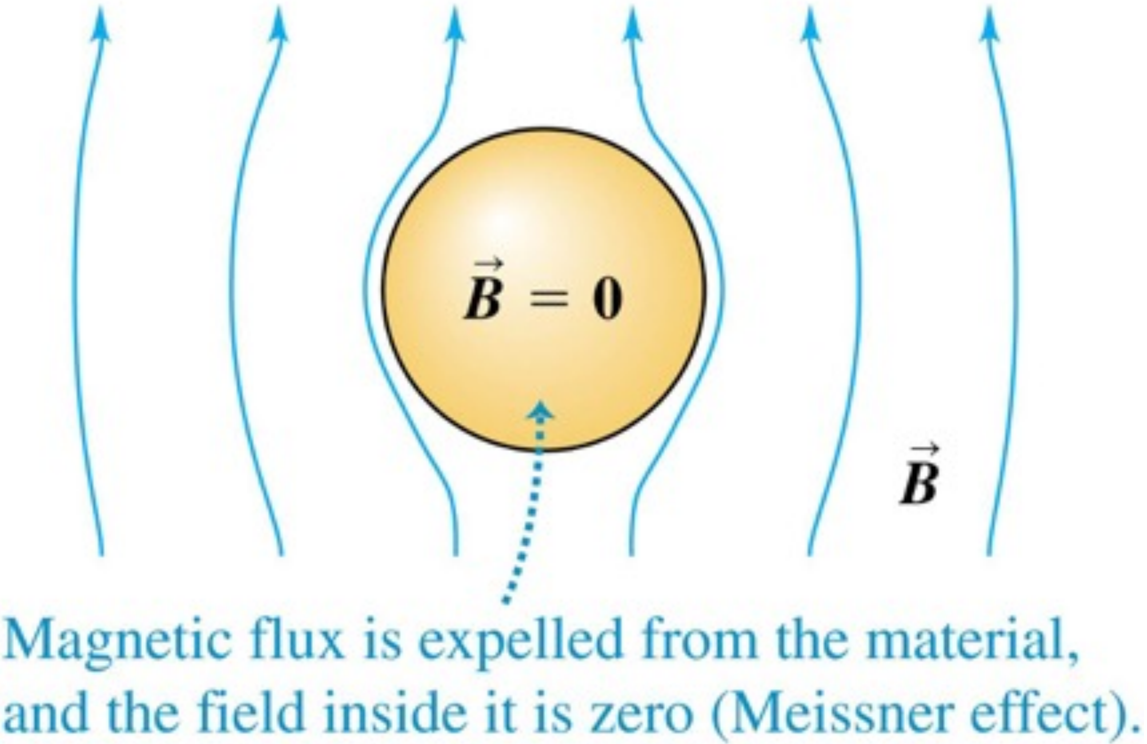
Superconductivity in a magnetic field

- When a superconductor is cooled below its critical temperature T_c, it loses all electrical resistance.
- For any superconducting material the critical temperature T_c changes when the material is placed in an externally produced magnetic field.
- Shown is this dependence for mercury.
- As the external field magnitude increases, the superconducting transition occurs at a lower and lower temperature.



The Meissner effect

- If we place a superconducting material in a uniform applied magnetic field, and then lower the temperature until the superconducting transition occurs, then all of the magnetic flux is expelled from the superconductor.
- The expulsion of magnetic flux is called the **Meissner effect**.

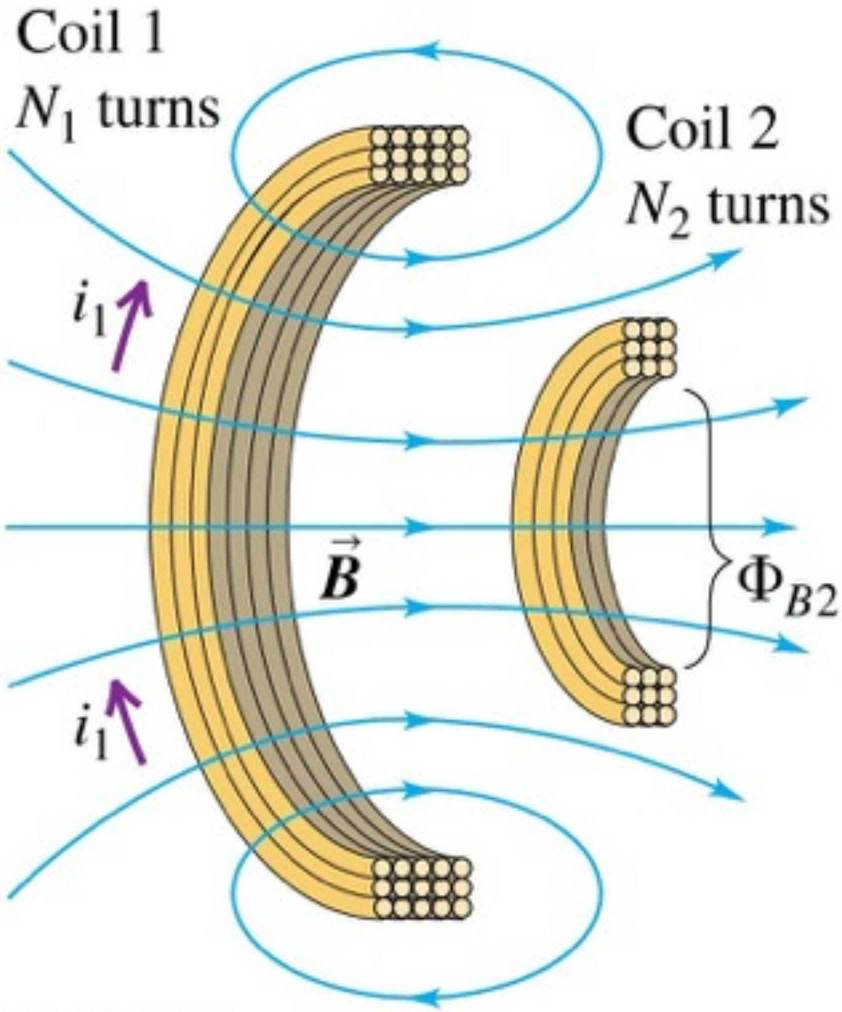


Magnetic flux is expelled from the material, and the field inside it is zero (Meissner effect).

Mutual inductance

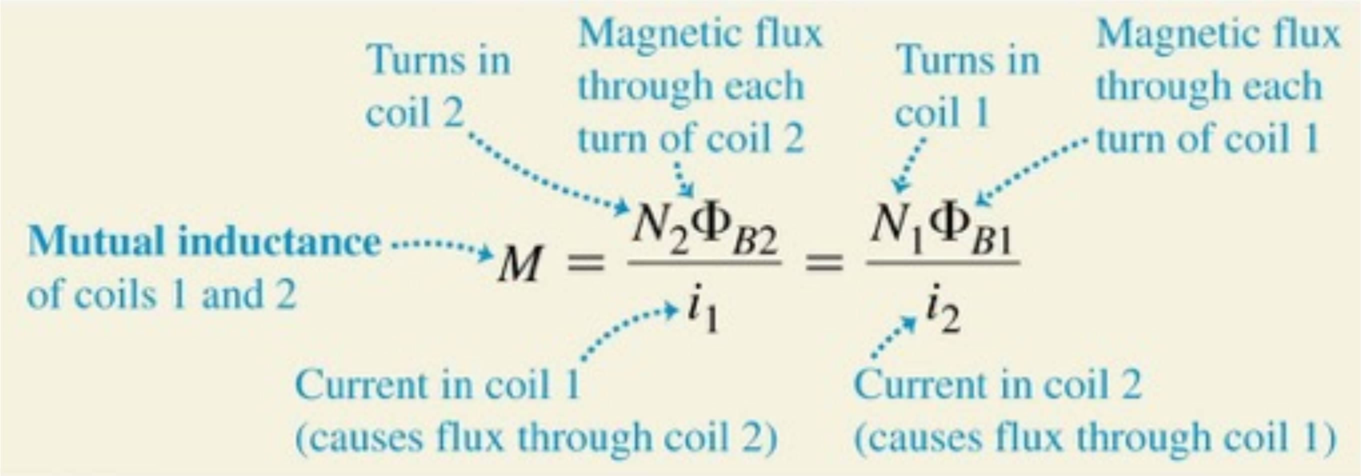
- Consider two neighboring coils of wire, as shown.
- If the current in coil 1 changes, this induces an emf in coil 2, and vice versa.
- The proportionality constant for this pair of coils is called the **mutual inductance**, *M*.

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B2}}{dt}$$



Mutual inductance

- The mutual inductance M is:



$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

- The SI unit of mutual inductance is called the henry (1 H), in honor of the American physicist Joseph Henry.

$$1\ \text{H} = 1\ \text{Wb/A} = 1\ \text{V} \cdot \text{s/A} = 1\ \Omega \cdot \text{s} = 1\ \text{J/A}^2$$