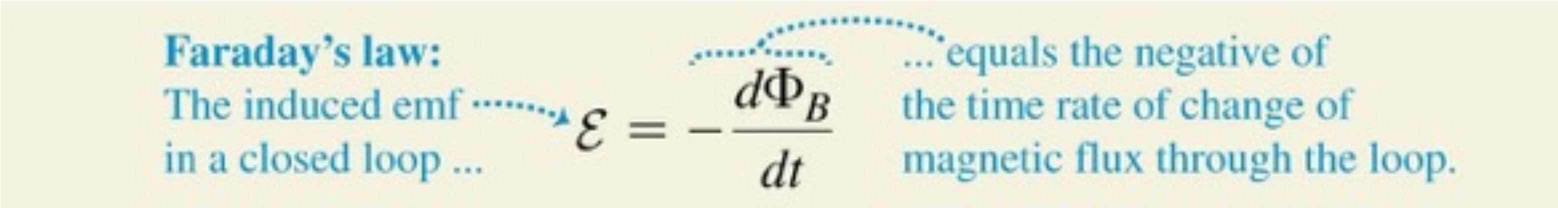
Lecture 34

PHYC 161 Fall 2016

Faraday's law of induction

- When the magnetic flux through a single closed loop changes with time, there is an induced emf that can drive a current around the loop:



Faraday's law:
The induced emf in a closed loop ...

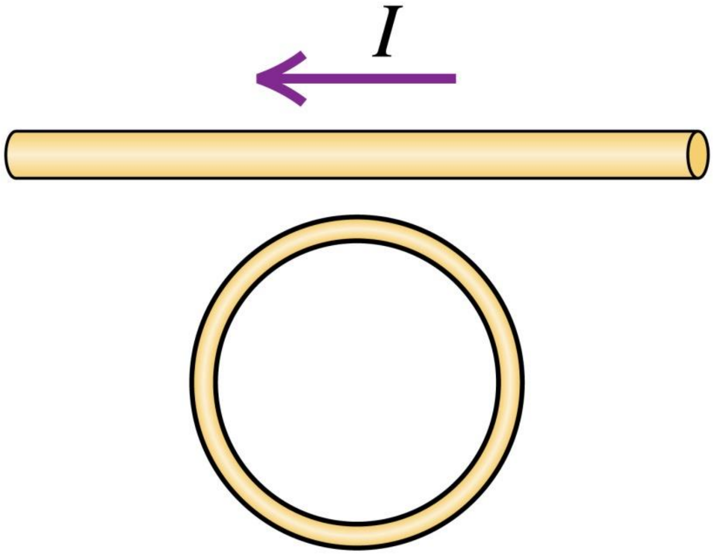
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

... equals the negative of the time rate of change of magnetic flux through the loop.

- Recall that the unit of magnetic flux is the weber (Wb).
- 1 T · m^2 = 1 Wb, so 1 V = 1 Wb/s.

CPS 32-2

A circular loop of wire is placed next to a long straight wire. The current *I* in the long straight wire is increasing. What current does this induce in the circular loop?



A. a clockwise current

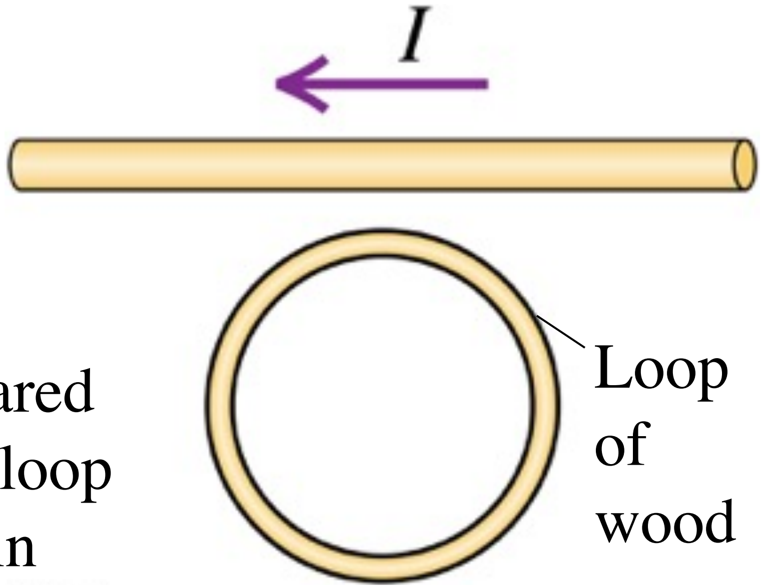
B. a counterclockwise current

C. zero current

D. not enough information given to decide

Q29.4

A circular loop of *wood* is placed next to a long, straight wire. The resistivity of wood is about 10^{20} times greater than that of copper. The current I in the long, straight wire is increasing. Compared to the emf that would be induced if the loop were made of copper, the emf induced in the loop of wood is



A. about 10^{-20} as great.

B. about 10^{-10} as great.

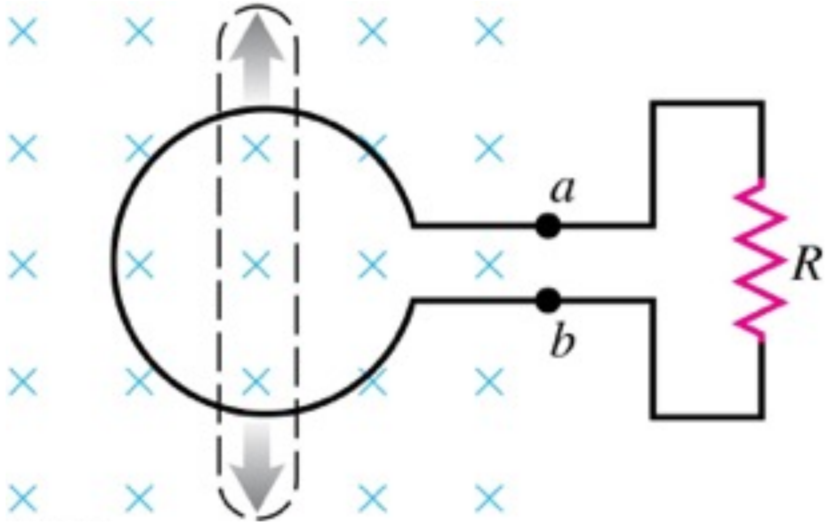
C. about 10^{-5} as great.

D. the same.

E. greater.

Q29.5

A flexible loop of wire lies in a uniform magnetic field of magnitude B directed into the plane of the picture. The loop is pulled as shown, reducing its area. The induced current flows



A. downward through resistor R and is proportional to B.

B. upward through resistor R and is proportional to B.

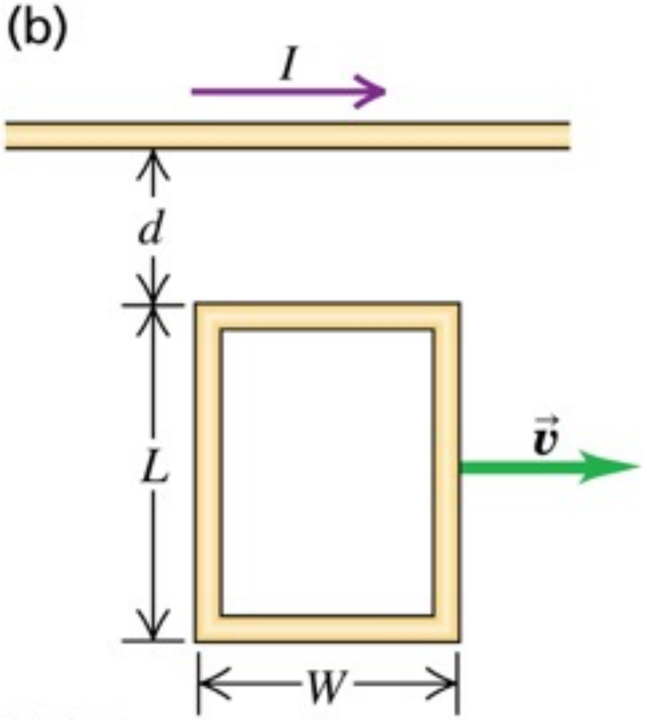
C. downward through resistor R and is proportional to B^2.

D. upward through resistor R and is proportional to B^2.

E. None of the above is true.

Q29.6

The rectangular loop of wire is being moved to the right at constant velocity. A constant current I flows in the long, straight wire in the direction shown. The current induced in the loop is



A. clockwise and proportional to I.

B. counterclockwise and proportional to I.

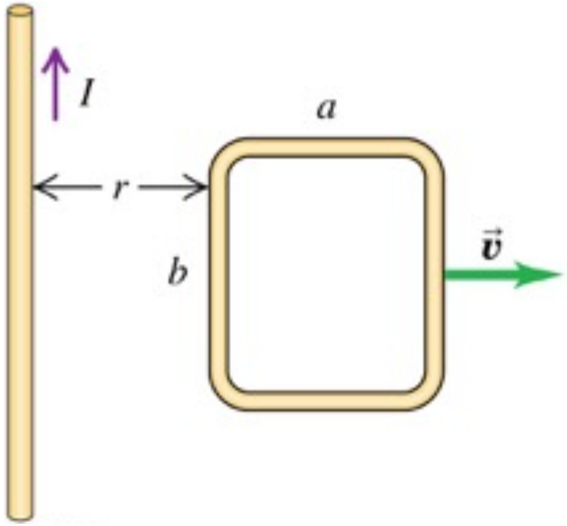
C. clockwise and proportional to I^2.

D. counterclockwise and proportional to I^2.

E. zero.

Q29.7

The loop of wire is being moved to the right at constant velocity. A constant current I flows in the long, straight wire in the direction shown. The current induced in the loop is



A. clockwise and proportional to I.

B. counterclockwise and proportional to I.

C. clockwise and proportional to I^2.

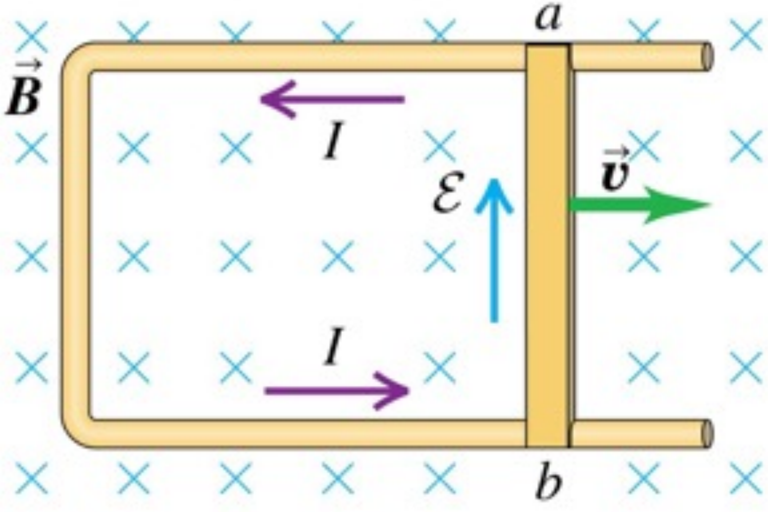
D. counterclockwise and proportional to I^2.

E. zero.

Motional electromotive force

- When a conducting rod moves perpendicular to a uniform magnetic field, there is a **motional emf** induced.

Rod connected to stationary conductor



The motional emf $\mathcal{E}$ in the moving rod creates an electric field in the stationary conductor.

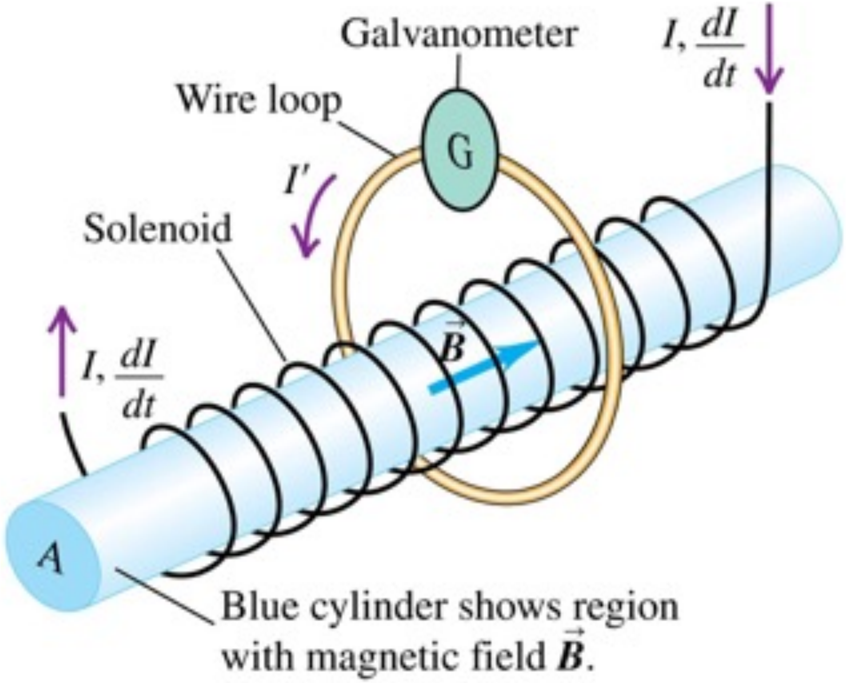
Motional emf, conductor length and velocity perpendicular to uniform $\vec{B}$

$$\mathcal{E} = vBL$$

v: Conductor speed
L: Conductor length
B: Magnitude of uniform magnetic field

Induced electric fields

- A long, thin solenoid is encircled by a circular conducting loop.
- Electric field in the loop is what must drive the current.
- When the solenoid current *I* changes with time, the magnetic flux also changes, and the induced emf can be written in terms of **induced electric field**:



Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E}\cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

Two equivalent statements:

Faraday's law:
The induced emf in a closed loop ...

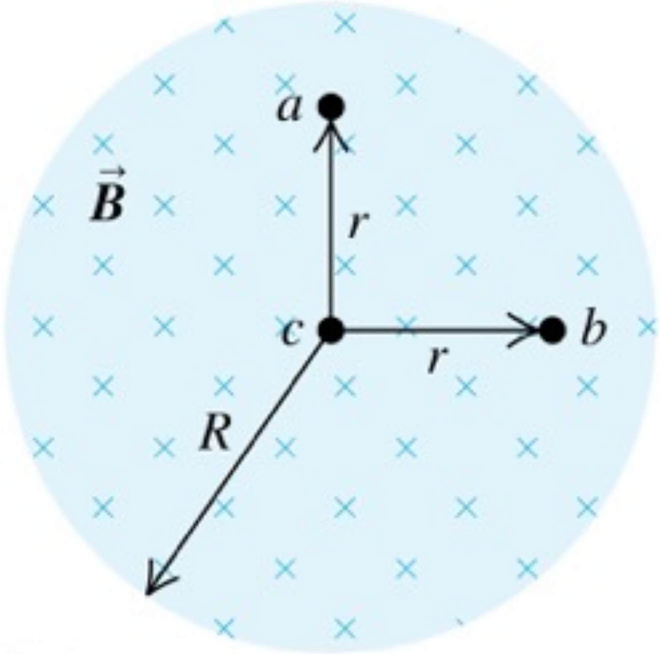
$$\mathcal{E} = \overbrace{-\frac{d\Phi_B}{dt}}^{\text{... equals the negative of the time rate of change of magnetic flux through the loop.}}$$

Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E} \cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

Q29.9

The drawing shows the uniform magnetic field inside a long, straight solenoid. The field is directed into the plane of the drawing and is increasing. What is the direction of the *electric* force on a positive point charge placed at point *a*?



A. to the left

B. to the right

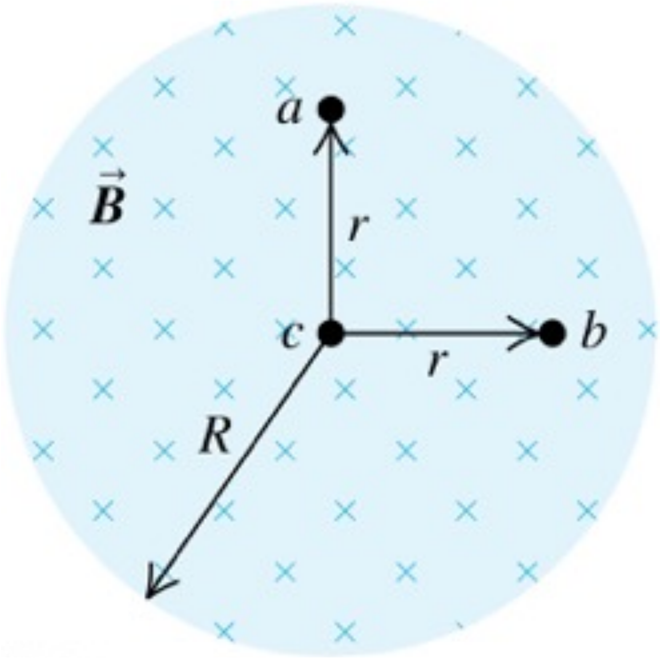
C. straight up

D. straight down

E. Misleading question—the electric force at this point is zero.

Q29.10

The drawing shows the uniform magnetic field inside a long, straight solenoid. The field is directed into the plane of the drawing and is increasing. What is the direction of the *electric* force on a positive point charge placed at point *b*?



A. to the left

B. to the right

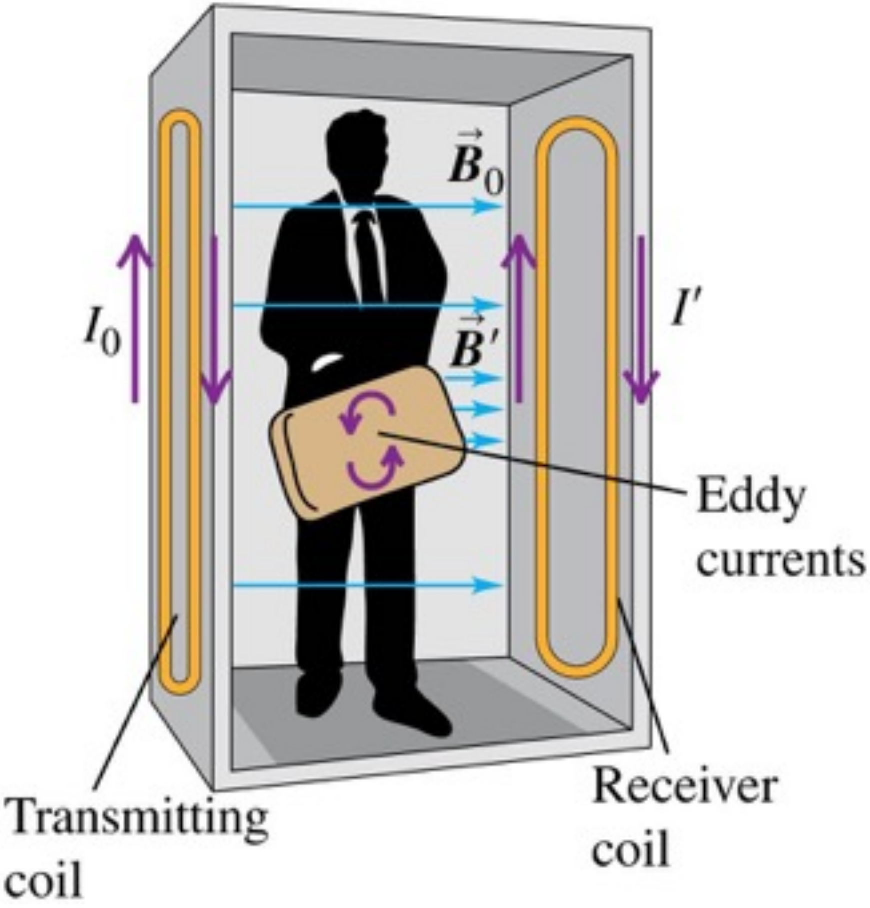
C. straight up

D. straight down

E. Misleading question—the electric force at this point is zero.

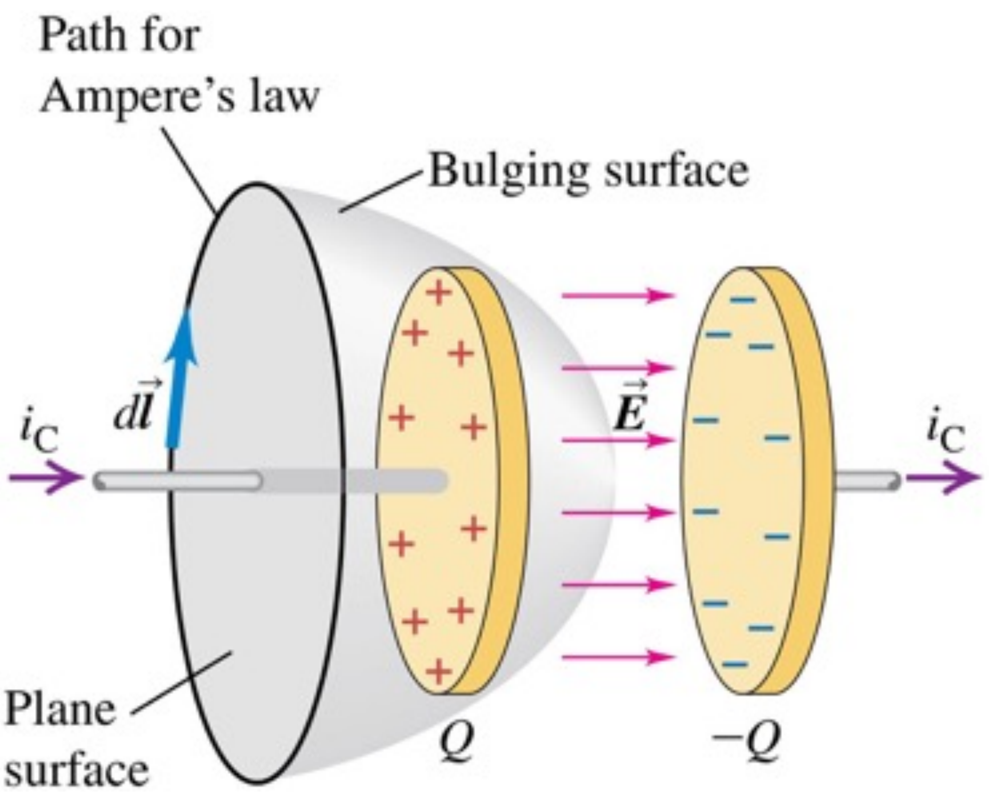
Eddy currents

- When a piece of metal moves through a magnetic field or is located in a changing magnetic field, **eddy currents** of electric current are induced.
- The metal detectors used at airport security checkpoints operate by detecting eddy currents induced in metallic objects.



Displacement current

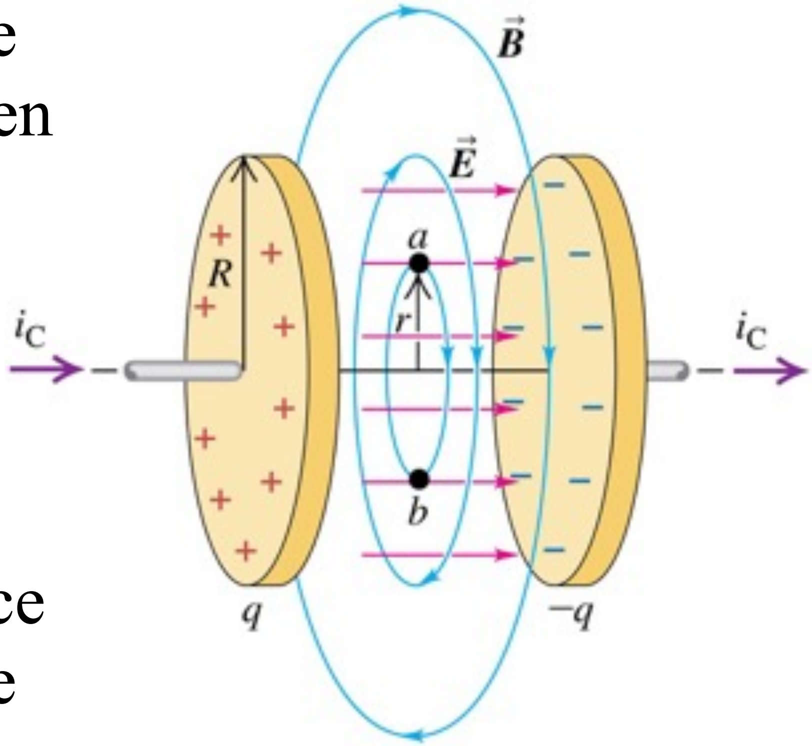
- Ampere's law is *incomplete*, as can be shown by considering the process of charging a capacitor, as shown.



- For the plane circular area bounded by the circle, I_{encl} is the current i_C in the left conductor.
- But the surface that bulges out to the right is bounded by the same circle, and the current through that surface is zero.
- This leads to a contradiction.

Displacement current

- When a capacitor is charging, the electric field is increasing between the plates.
- We can define a fictitious **displacement current** i_D in the region between the plates.
- This can be regarded as the source of the magnetic field between the plates.



Displacement current through an area

$$i_D = \epsilon \frac{d\Phi_E}{dt}$$

Time rate of change of electric flux through area

Permittivity of material in area

Maxwell's equations of electromagnetism

- All the relationships between electric and magnetic fields and their sources are summarized by four equations, called **Maxwell's equations**.
- The first Maxwell equation is Gauss's law for electric fields from Chapter 22:

Gauss's law for $\vec{E}$:

$$\overbrace{\oint \vec{E}\cdot d\vec{A}}^{\text{Flux of electric field through a closed surface}} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Charge enclosed by surface

Electric constant

- The second Maxwell equation is Gauss's law for magnetic fields from Chapter 27:

Gauss's law for $\vec{B}$:

$$\overbrace{\oint \vec{B}\cdot d\vec{A}}^{\text{Flux of magnetic field through any closed surface ...}} = 0$$

... equals zero.

Maxwell’s equations of electromagnetism

- The third Maxwell equation is this chapter’s formulation of Faraday’s law:

Faraday’s law for a stationary integration path:

$$\overbrace{\oint \vec{E}\cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

- The fourth Maxwell equation is Ampere’s law, including displacement current:

Ampere’s law for a stationary integration path:

$$\overbrace{\oint \vec{B}\cdot d\vec{l}}^{\text{Line integral of magnetic field around path}} = \underset{\text{Magnetic constant}}{\mu_0}\left(\underset{\text{Conduction current through path}}{i_C} + \underbrace{\overset{\text{Electric constant}}{\epsilon_0}\overbrace{\frac{d\Phi_E}{dt}}^{\text{Time rate of change of electric flux through path}}}_{\text{Displacement current through path}}\right)_{\text{encl}}$$

Maxwell's equations in empty space

- There is a remarkable symmetry in Maxwell's equations.
- In empty space where there is no charge, the first two equations are identical in form.
- The third equation says that a changing magnetic flux creates an electric field, and the fourth says that a changing electric flux creates a magnetic field.

In empty space there are no charges, so the fluxes of $\vec{E}$ and $\vec{B}$ through any closed surface are equal to zero.

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

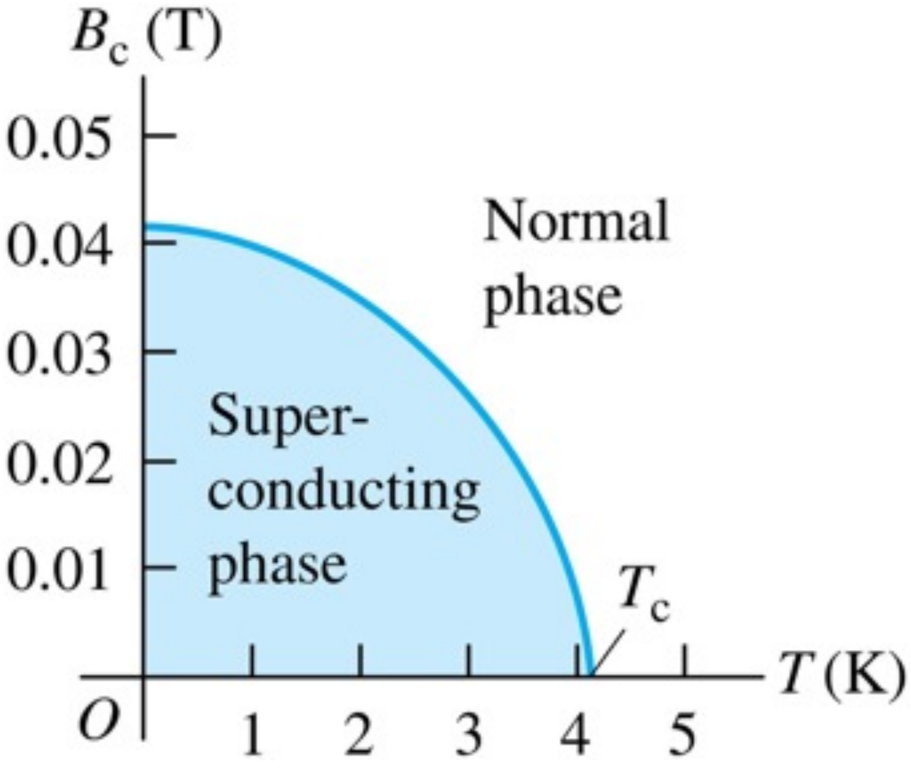
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In empty space there are no conduction currents, so the line integrals of $\vec{E}$ and $\vec{B}$ around any closed path are related to the rate of change of flux of the other field.

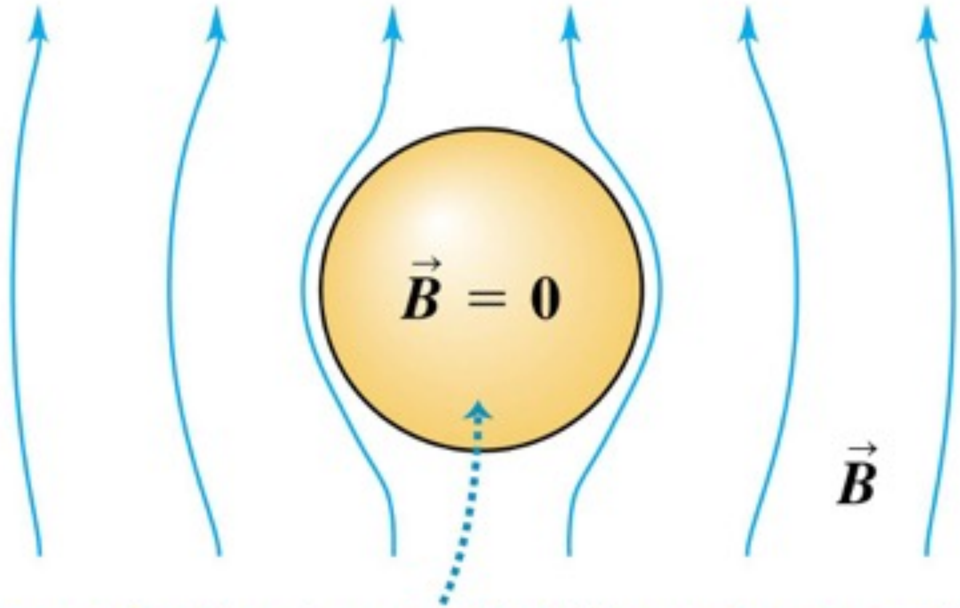
Superconductivity in a magnetic field

- When a superconductor is cooled below its critical temperature T_c, it loses all electrical resistance.
- For any superconducting material the critical temperature T_c changes when the material is placed in an externally produced magnetic field.
- Shown is this dependence for mercury.
- As the external field magnitude increases, the superconducting transition occurs at a lower and lower temperature.



The Meissner effect

- If we place a superconducting material in a uniform applied magnetic field, and then lower the temperature until the superconducting transition occurs, then all of the magnetic flux is expelled from the superconductor.
- The expulsion of magnetic flux is called the **Meissner effect**.



Magnetic flux is expelled from the material, and the field inside it is zero (Meissner effect).