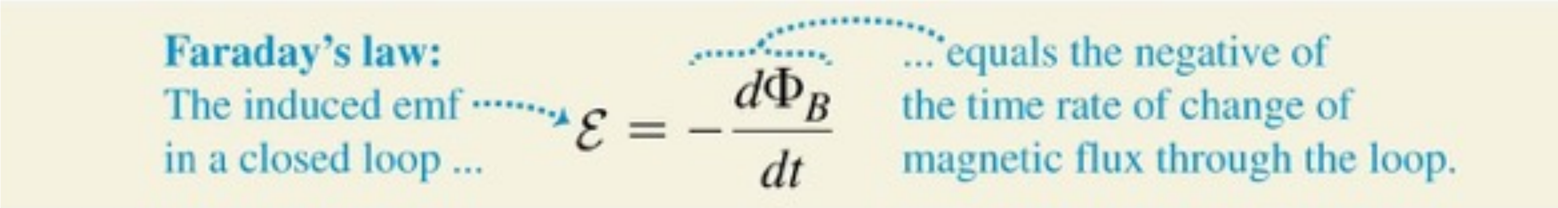
Lecture 33

PHYC 161 Fall 2016

Faraday's law of induction

- When the magnetic flux through a single closed loop changes with time, there is an induced emf that can drive a current around the loop:



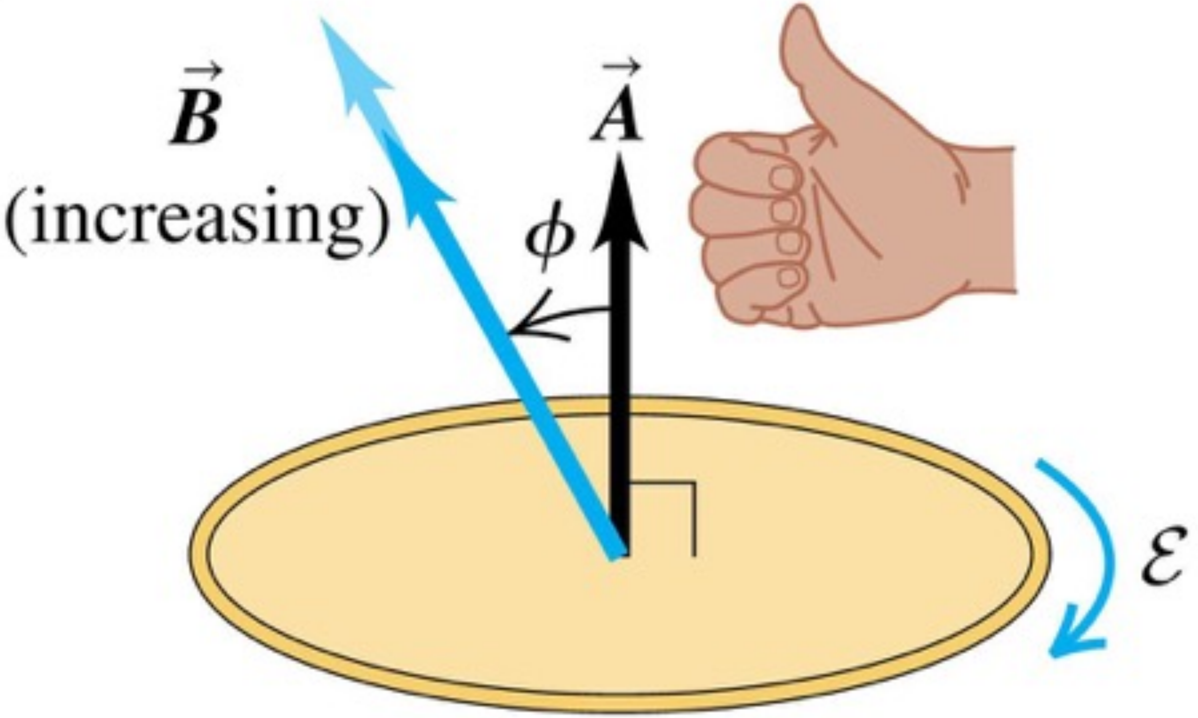
Faraday's law:
The induced emf in a closed loop ...

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

... equals the negative of the time rate of change of magnetic flux through the loop.

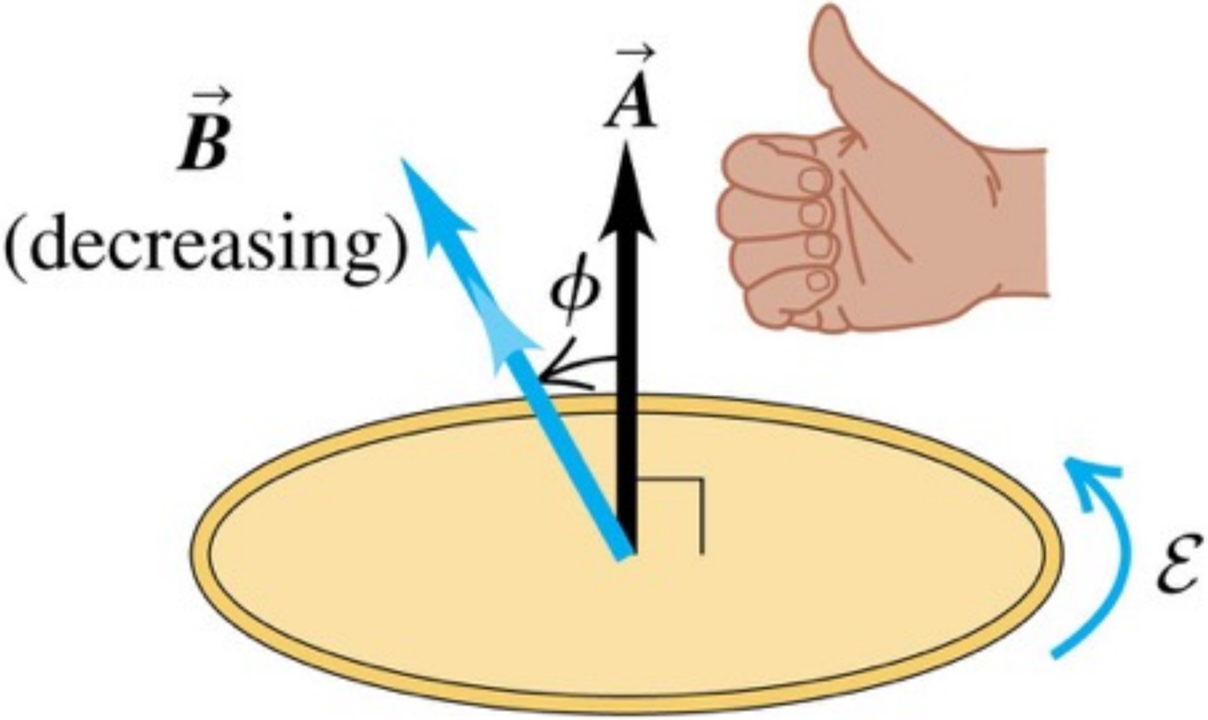
- Recall that the unit of magnetic flux is the weber (Wb).
- $1\ \mathrm{T} \cdot \mathrm{m}^2 = 1\ \mathrm{Wb}$, so $1\ \mathrm{V} = 1\ \mathrm{Wb/s}$.

Determining the direction of the induced emf: Slide 1 of 4



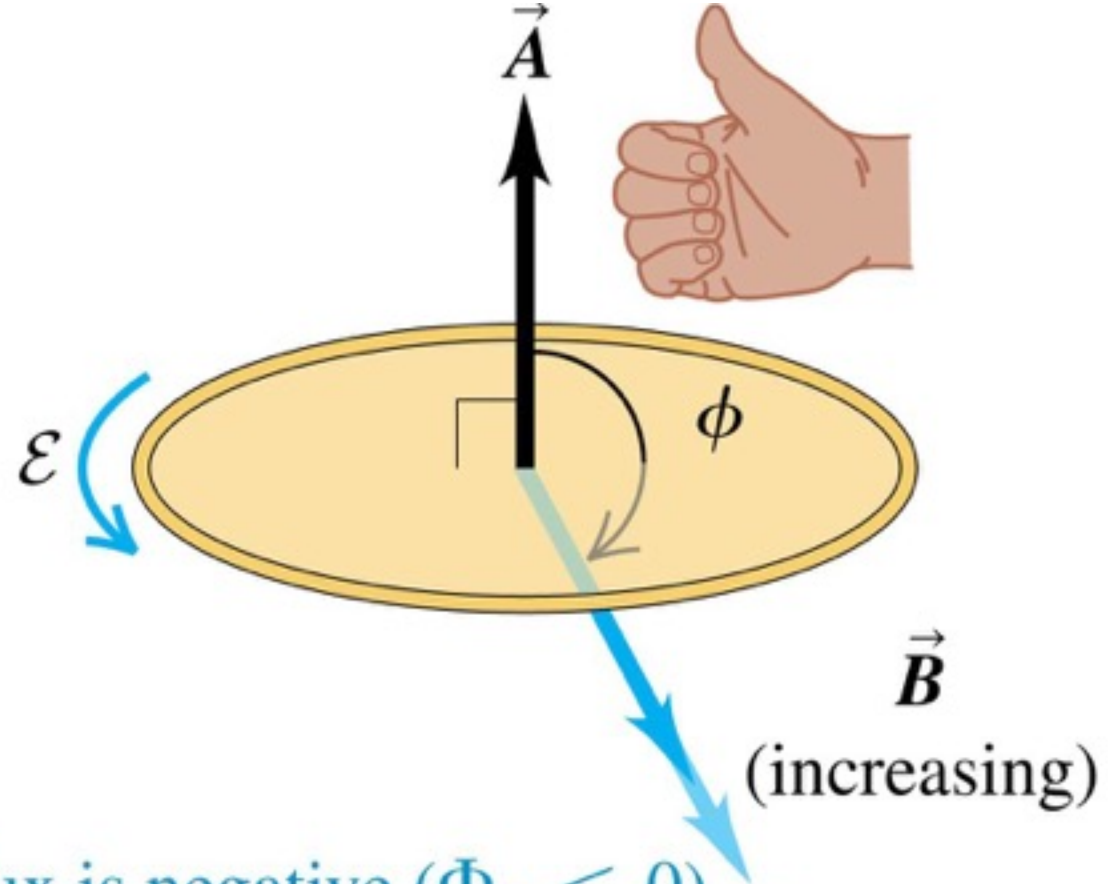
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming more positive ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

Determining the direction of the induced emf: Slide 2 of 4



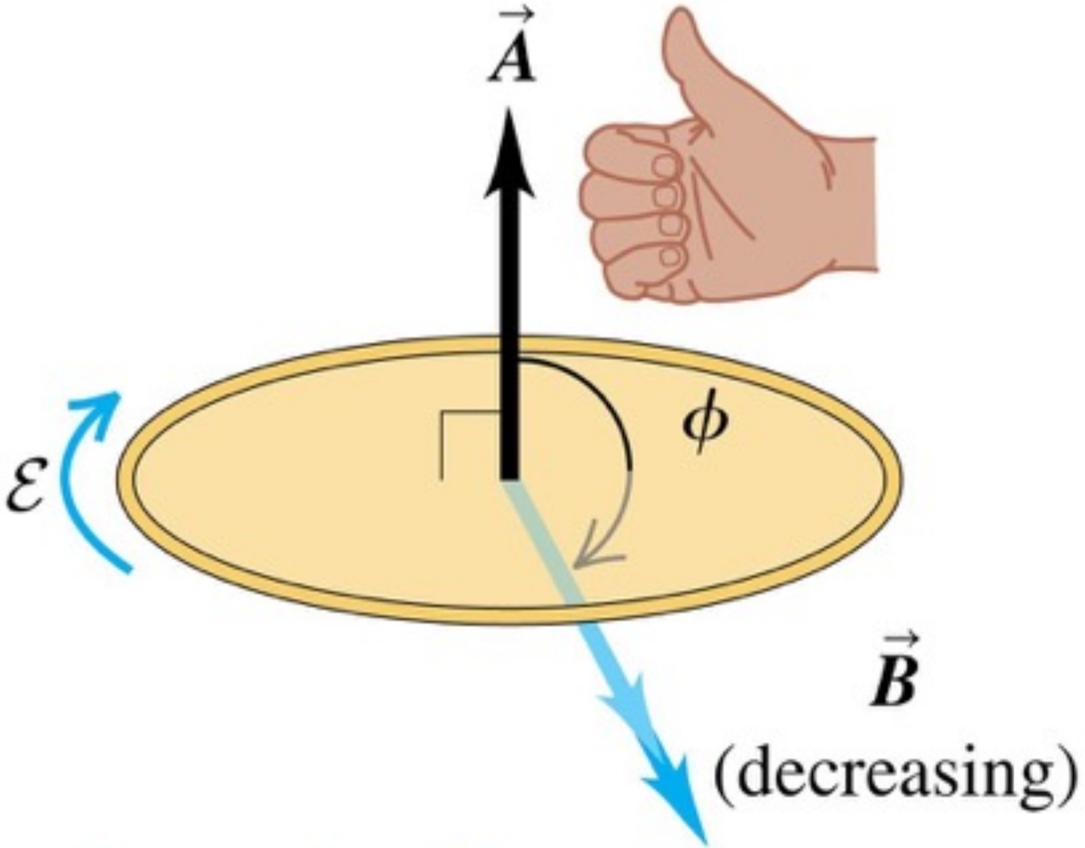
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming less positive ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

Determining the direction of the induced emf: Slide 3 of 4



- Flux is negative ($\Phi_B < 0$) …
- … and becoming more negative ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

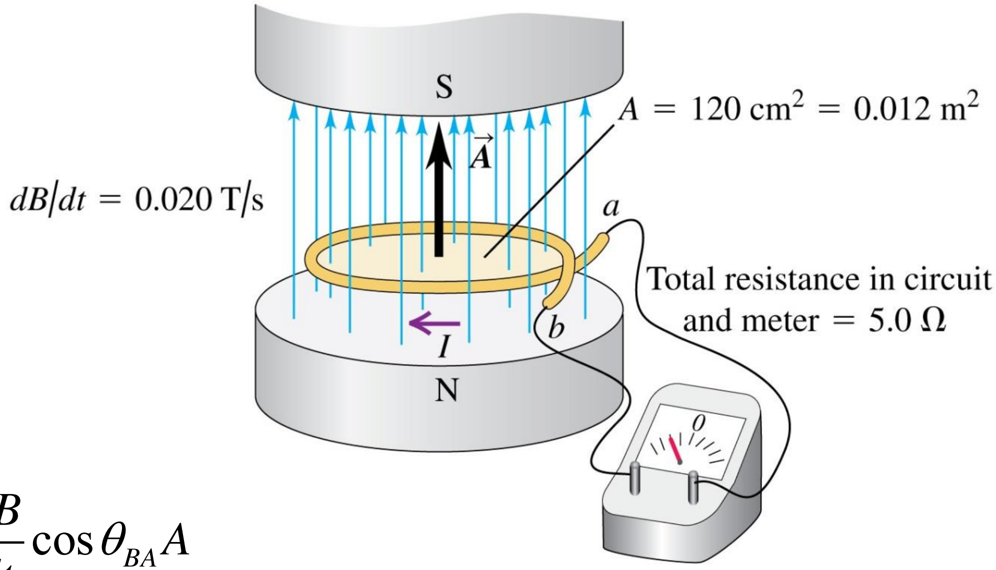
Determining the direction of the induced emf: Slide 4 of 4



- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming less negative ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

Example

- Let's put some numbers in to see how this might work:



$$\mathcal{E} = -\frac{d}{dt}\left[N\int_{\text{Surface}}\vec{B}\cdot d\vec{A}\right] = -N\frac{dB}{dt}\cos\theta_{BA}A$$

$$= -(1)(0.020T/s)(1)\left(0.012m^2\right) = 2.4\times10^{-4}\frac{Tm^2}{s}$$

$$I = \frac{\mathcal{E}}{R} = \frac{2.4\times10^{-4}\dfrac{Tm^2}{s}}{5.0\Omega} = 4.8\times10^{-4}\frac{Tm^2}{\Omega s}$$

Unit Check!!!

- Let's put some numbers in to see how this might work:

$$\mathcal{E} = -\frac{d}{dt}\left[N \int_{\text{Surface}} \vec{B} \cdot d\vec{A} \right] = -N \frac{dB}{dt} \cos\theta_{BA} A$$

$$= -(1)(0.020T/s)(1)(0.012m^2) = 2.4 \times 10^{-4} \frac{Tm^2}{s}$$

$$I = \frac{\mathcal{E}}{R} = \frac{2.4 \times 10^{-4} \frac{Tm^2}{s}}{5.0\Omega} = 4.8 \times 10^{-4} \frac{Tm^2}{\Omega s}$$

$$d\vec{F} = Id\vec{l} \times \vec{B} \Rightarrow$$

$$N = AmT \Rightarrow$$

$$T = \frac{N}{Am}$$

$$V = IR \Rightarrow$$

$$\frac{Nm}{C} = A\Omega \Rightarrow$$

$$\Omega = \frac{Nm}{AC}$$

$$\Rightarrow \frac{Tm^2}{\Omega s} = \frac{\frac{N}{Am} m^2}{\frac{Nm}{AC} s} = \frac{C}{s} = A$$

Faraday’s law for a coil

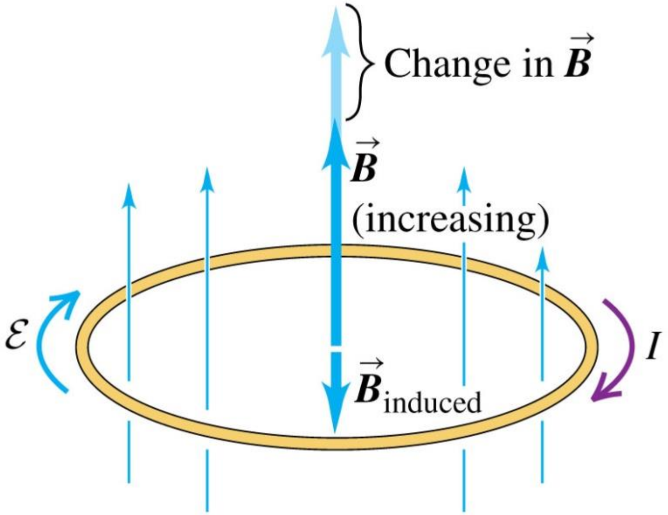
- A commercial alternator uses many loops of wire wound around a barrel-like structure called an armature.
- The resulting induced emf is far larger than would be possible with a single loop of wire.
- If a coil has N identical turns and if the flux varies at the same rate through each turn, total emf is:

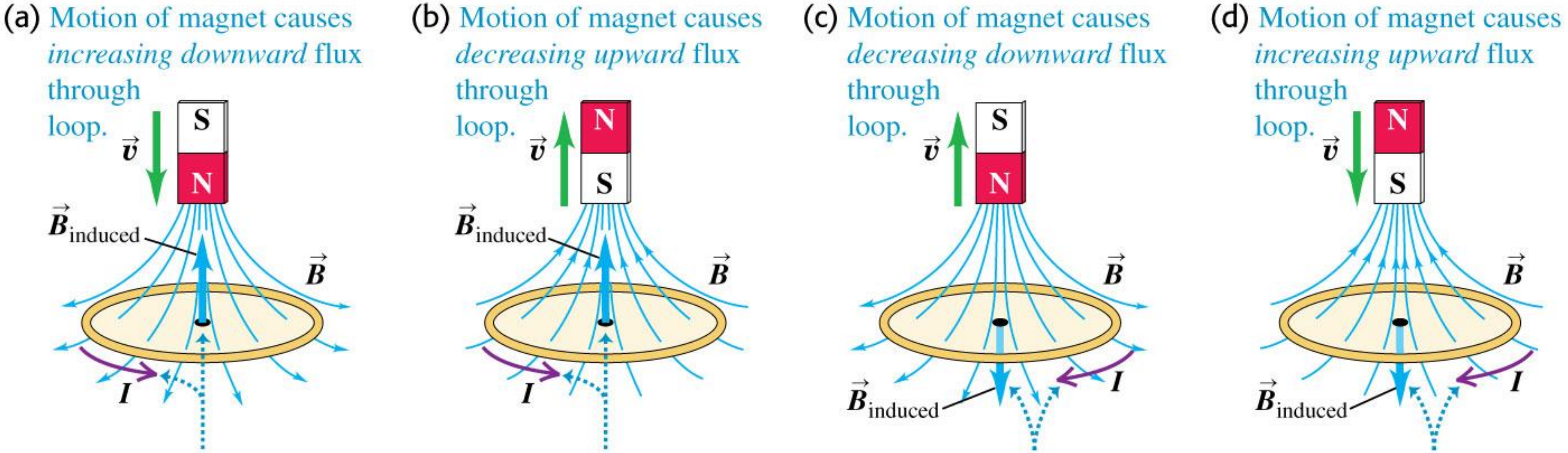
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

Lenz's Law

- To get the direction of the induced EMF (and thus, the current in a circuit), remember:

$$\mathcal{E} = \boxed{-\frac{d}{dt}}\Phi_B$$




(a) Motion of magnet causes *increasing downward* flux through loop.

(b) Motion of magnet causes *decreasing upward* flux through loop.

(c) Motion of magnet causes *decreasing downward* flux through loop.

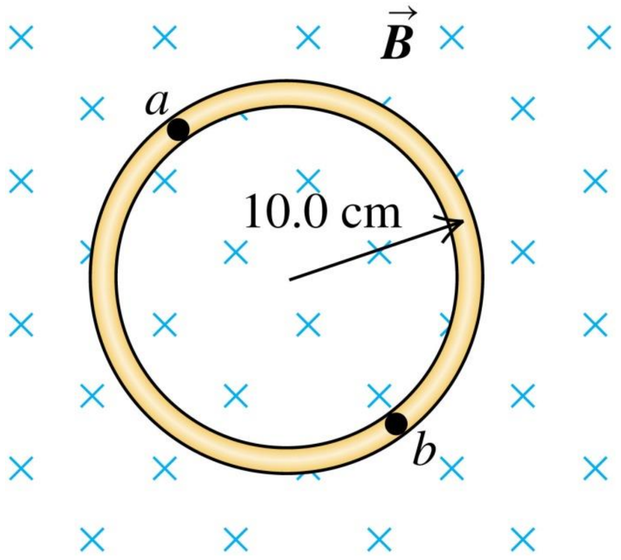
(d) Motion of magnet causes *increasing upward* flux through loop.

The induced magnetic field is *upward* to oppose the flux change. To produce this induced field, the induced current must be *counterclockwise* as seen from above the loop.

The induced magnetic field is *downward* to oppose the flux change. To produce this induced field, the induced current must be *clockwise* as seen from above the loop.

CPS 32-1

A circular loop of wire is in a region of spatially uniform magnetic field. The magnetic field is directed into the plane of the figure. If the magnetic field magnitude is *decreasing*,



A. the induced emf is clockwise.

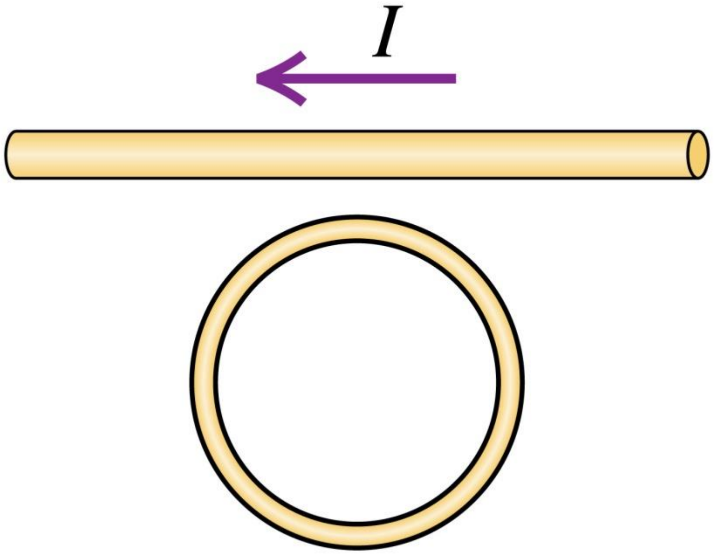
B. the induced emf is counterclockwise.

C. the induced emf is zero.

D. The answer depends on the strength of the field.

CPS 32-2

A circular loop of wire is placed next to a long straight wire. The current *I* in the long straight wire is increasing. What current does this induce in the circular loop?



A. a clockwise current

B. a counterclockwise current

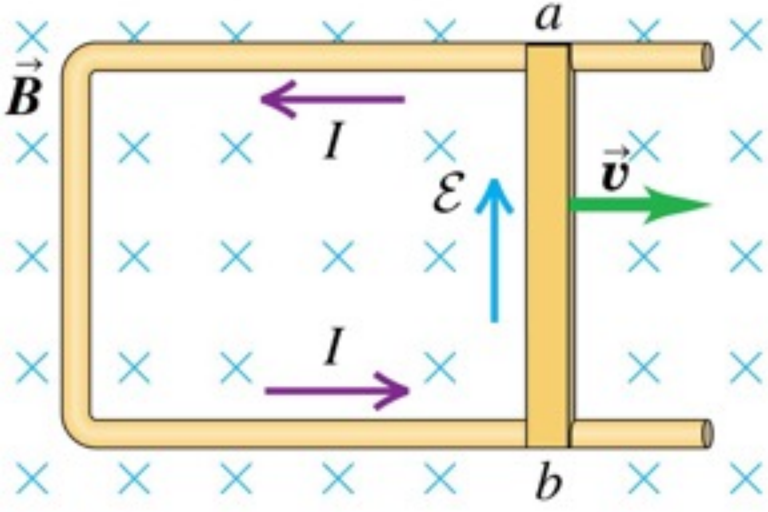
C. zero current

D. not enough information given to decide

Motional electromotive force

- When a conducting rod moves perpendicular to a uniform magnetic field, there is a **motional emf** induced.

Rod connected to stationary conductor



The motional emf $\mathcal{E}$ in the moving rod creates an electric field in the stationary conductor.

Motional emf, conductor length and velocity perpendicular to uniform $\vec{B}$ $\mathcal{E} = vBL$

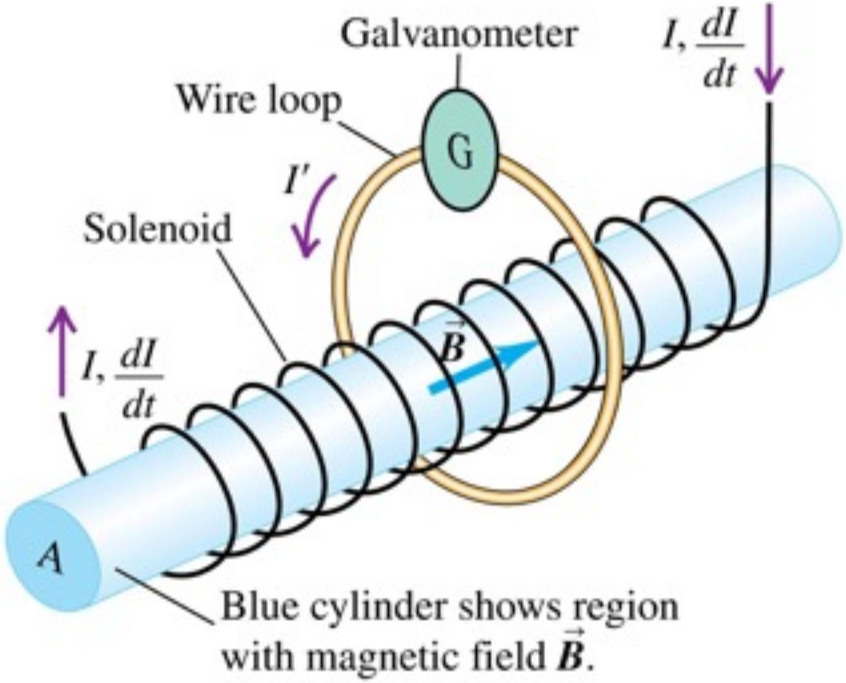
Conductor speed

Conductor length

Magnitude of uniform magnetic field

Induced electric fields

- A long, thin solenoid is encircled by a circular conducting loop.
- Electric field in the loop is what must drive the current.
- When the solenoid current *I* changes with time, the magnetic flux also changes, and the induced emf can be written in terms of **induced electric field**:

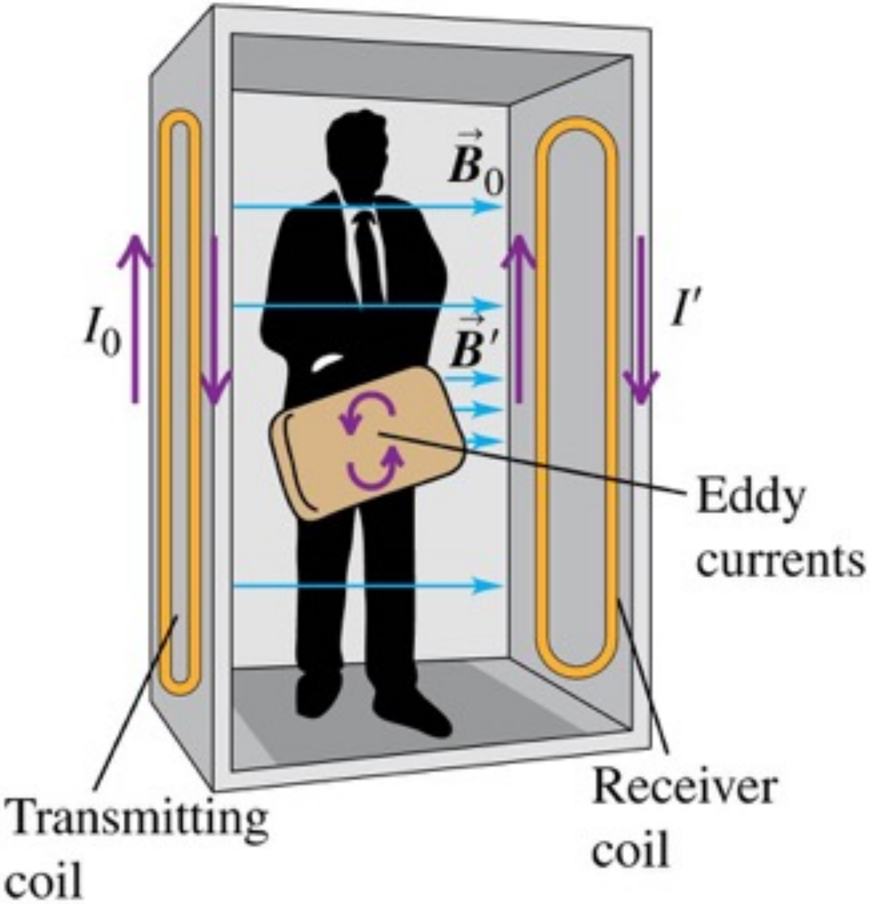


Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E} \cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

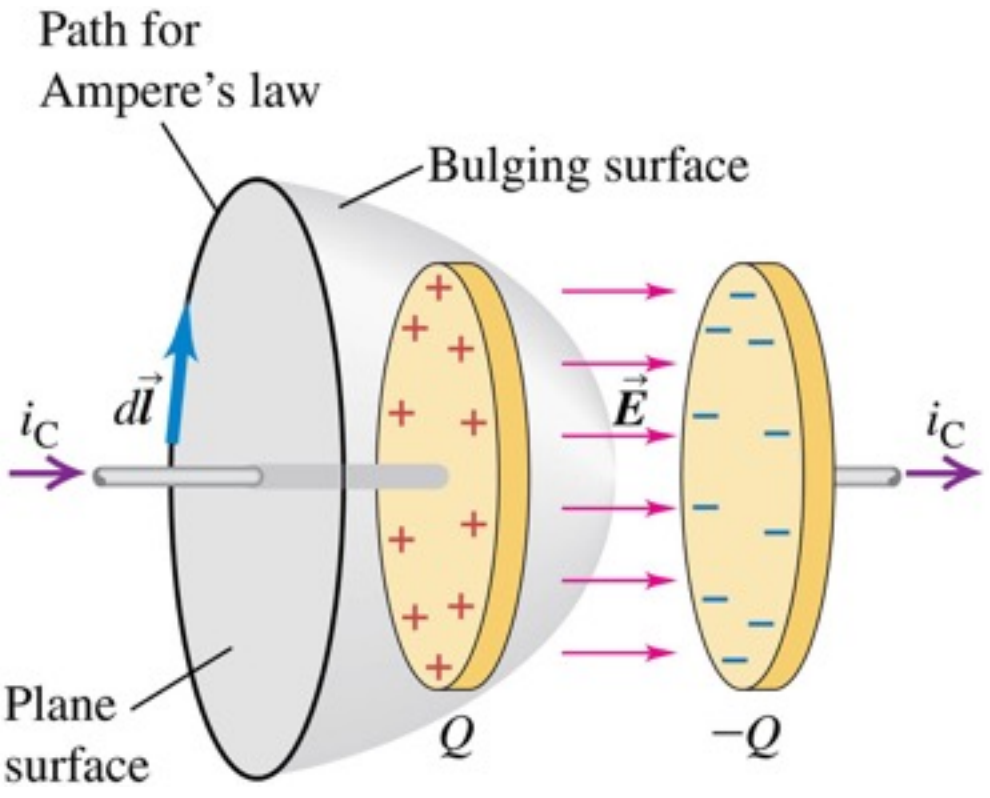
Eddy currents

- When a piece of metal moves through a magnetic field or is located in a changing magnetic field, **eddy currents** of electric current are induced.
- The metal detectors used at airport security checkpoints operate by detecting eddy currents induced in metallic objects.



Displacement current

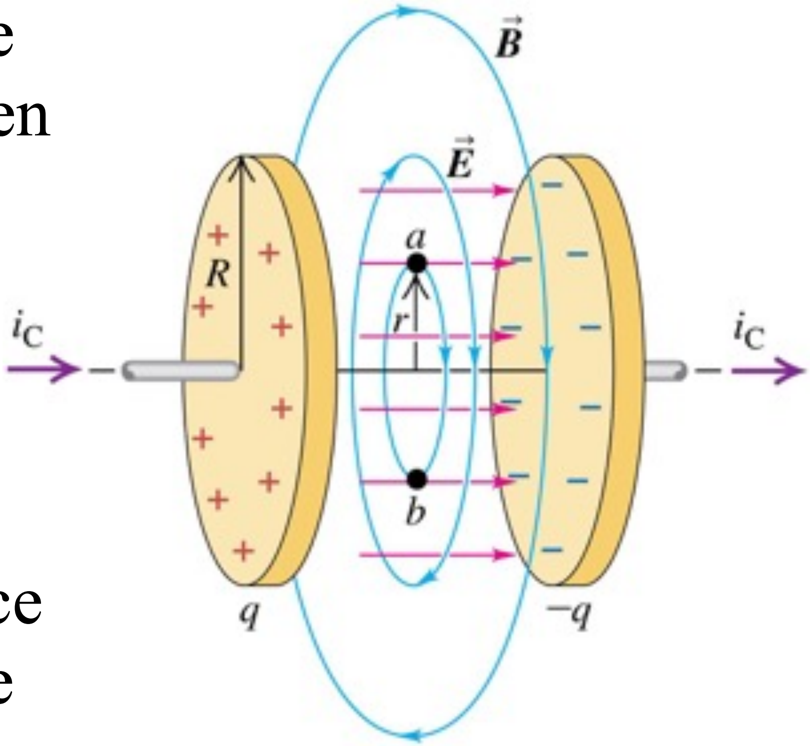
- Ampere's law is *incomplete*, as can be shown by considering the process of charging a capacitor, as shown.



- For the plane circular area bounded by the circle, I_{encl} is the current i_C in the left conductor.
- But the surface that bulges out to the right is bounded by the same circle, and the current through that surface is zero.
- This leads to a contradiction.

Displacement current

- When a capacitor is charging, the electric field is increasing between the plates.
- We can define a fictitious **displacement current** i_D in the region between the plates.
- This can be regarded as the source of the magnetic field between the plates.



Displacement current through an area

$$i_D = \epsilon \frac{d\Phi_E}{dt}$$

Time rate of change of electric flux through area

Permittivity of material in area

Maxwell's equations of electromagnetism

- All the relationships between electric and magnetic fields and their sources are summarized by four equations, called **Maxwell's equations**.
- The first Maxwell equation is Gauss's law for electric fields from Chapter 22:

Gauss's law for $\vec{E}$:

$$\overbrace{\oint \vec{E}\cdot d\vec{A}}^{\text{Flux of electric field through a closed surface}} = \frac{Q_{\text{encl}} \leftarrow \text{Charge enclosed by surface}}{\epsilon_0 \leftarrow \text{Electric constant}}$$

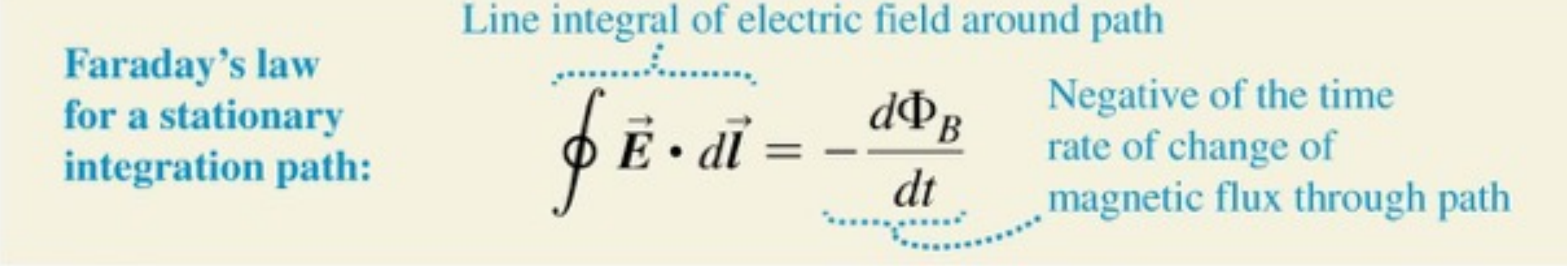
- The ic fields from Chapter 27:

Gauss's law for $\vec{B}$:

$$\overbrace{\oint \vec{B}\cdot d\vec{A}}^{\text{Flux of magnetic field through any closed surface ...}} = 0 \leftarrow \text{... equals zero.}$$

Maxwell's equations of electromagnetism

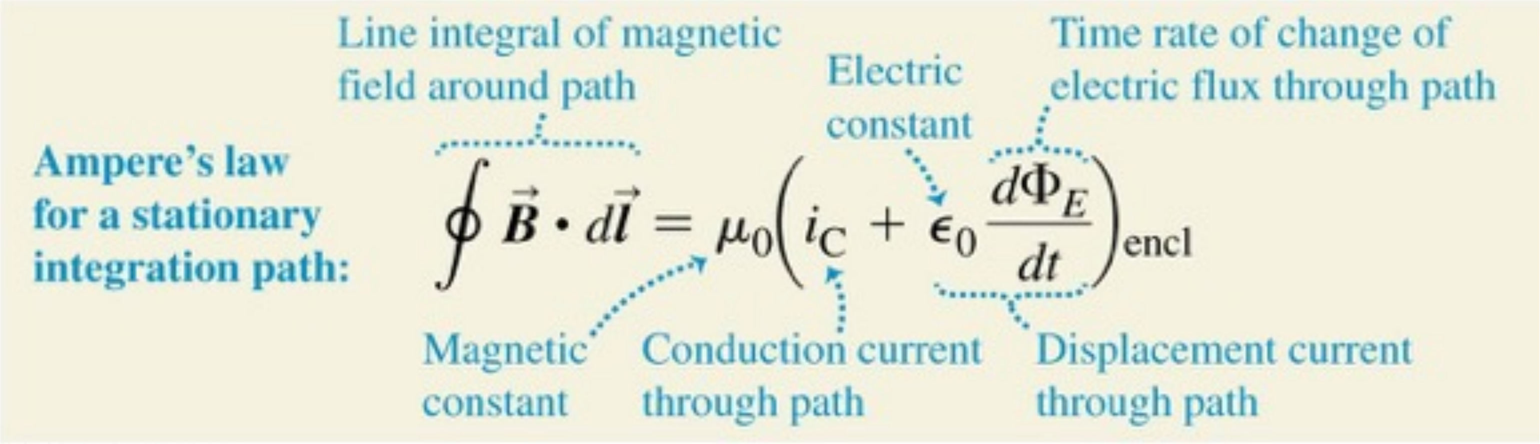
- The third Maxwell equation is this chapter's formulation of Faraday's law:



Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E} \cdot d\vec{l}}^{\text{Line integral of electric field around path}} = \underbrace{-\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

- The fourth Maxwell equation is Ampere's law, including displacement current:

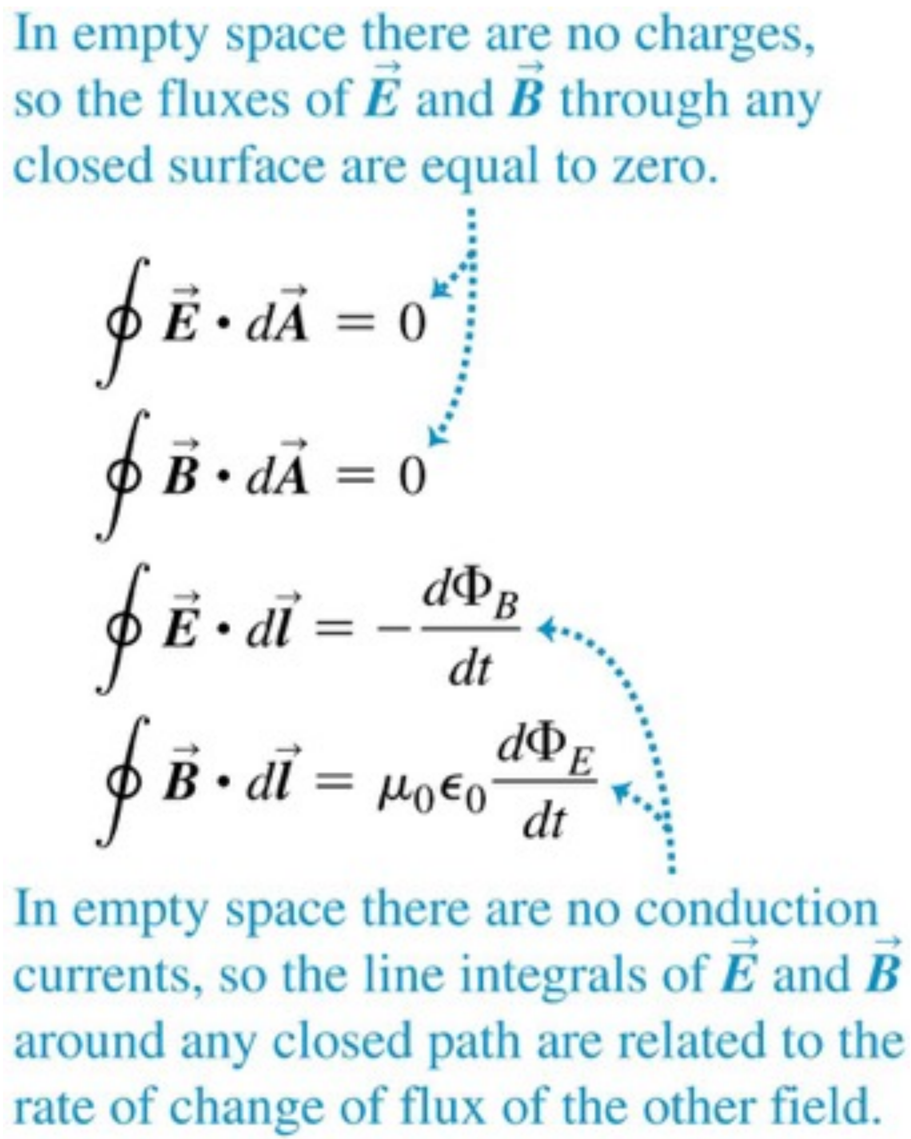


Ampere's law for a stationary integration path:

$$\overbrace{\oint \vec{B} \cdot d\vec{l}}^{\text{Line integral of magnetic field around path}} = \underbrace{\mu_0}_{\text{Magnetic constant}}\left(\underbrace{i_C}_{\text{Conduction current through path}} + \underbrace{\overset{\text{Electric constant}}{\epsilon_0}\overbrace{\frac{d\Phi_E}{dt}}^{\text{Time rate of change of electric flux through path}}}_{\text{Displacement current through path}}\right)_{\text{encl}}$$

Maxwell's equations in empty space

- There is a remarkable symmetry in Maxwell's equations.
- In empty space where there is no charge, the first two equations are identical in form.
- The third equation says that a changing magnetic flux creates an electric field, and the fourth says that a changing electric flux creates a magnetic field.



In empty space there are no charges, so the fluxes of $\vec{E}$ and $\vec{B}$ through any closed surface are equal to zero.

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

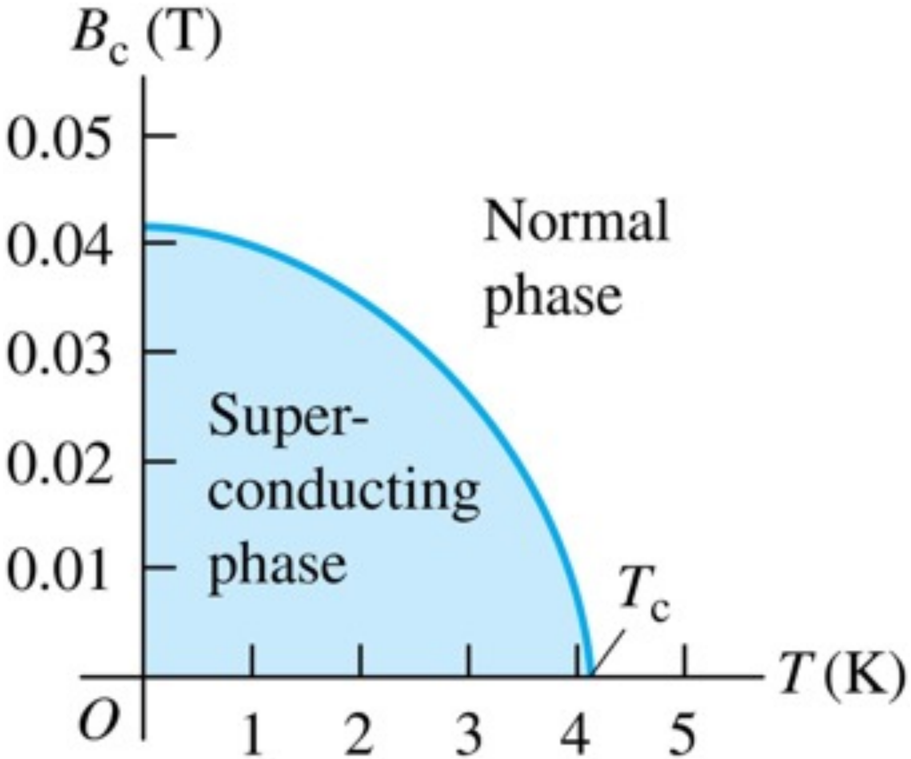
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In empty space there are no conduction currents, so the line integrals of $\vec{E}$ and $\vec{B}$ around any closed path are related to the rate of change of flux of the other field.

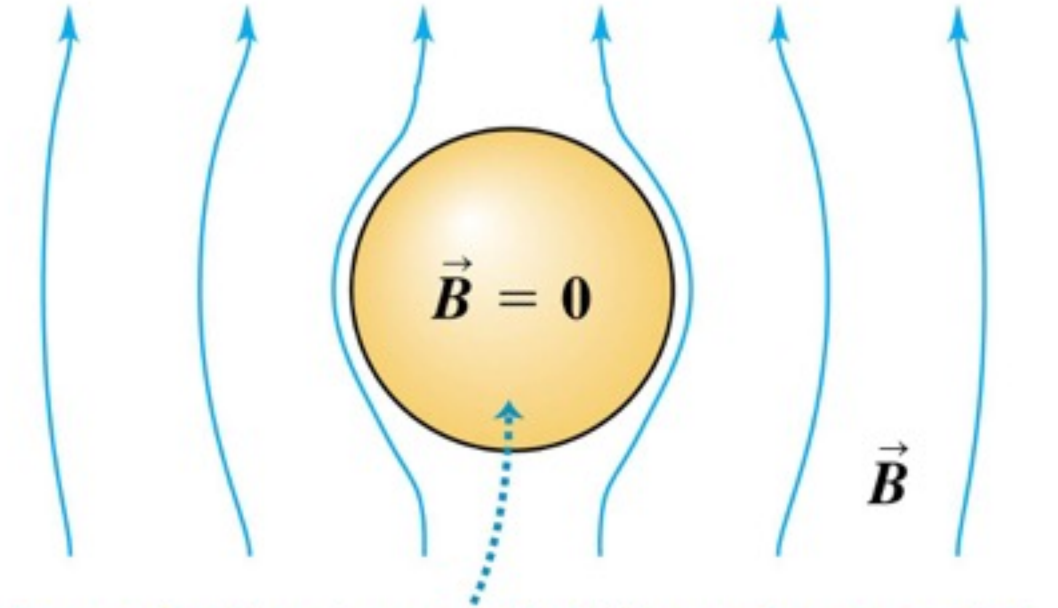
Superconductivity in a magnetic field

- When a superconductor is cooled below its critical temperature T_c, it loses all electrical resistance.
- For any superconducting material the critical temperature T_c changes when the material is placed in an externally produced magnetic field.
- Shown is this dependence for mercury.
- As the external field magnitude increases, the superconducting transition occurs at a lower and lower temperature.



The Meissner effect

- If we place a superconducting material in a uniform applied magnetic field, and then lower the temperature until the superconducting transition occurs, then all of the magnetic flux is expelled from the superconductor.
- The expulsion of magnetic flux is called the **Meissner effect**.



Magnetic flux is expelled from the material, and the field inside it is zero (Meissner effect).