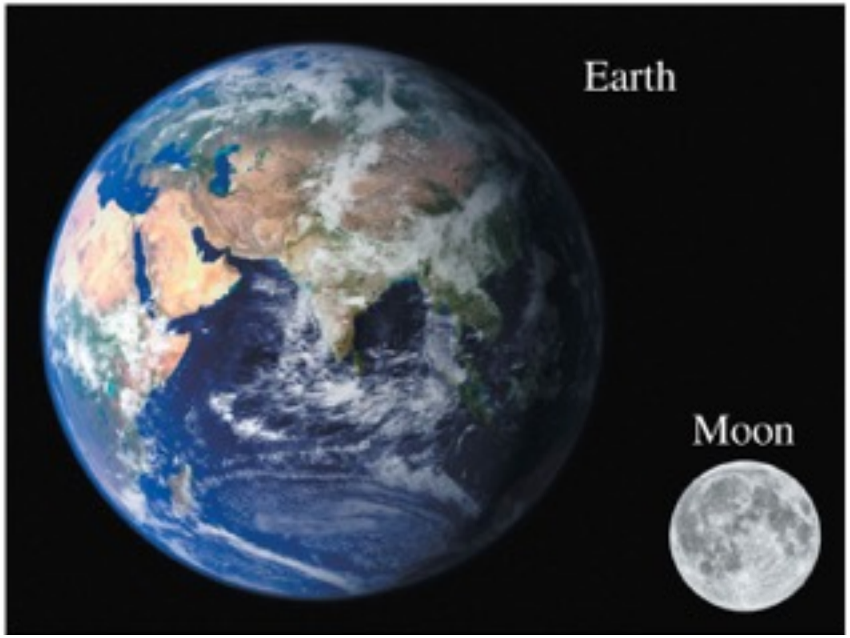
Lecture 31

PHYC 161 Fall 2016

Currents and planetary magnetism

- The earth's magnetic field is caused by currents circulating within its molten, conducting interior.
- These currents are stirred by our planet's relatively rapid spin (one rotation per 24 hours).
- The moon's internal currents are much weaker; it is much smaller than the earth, has a predominantly solid interior, and spins slowly (one rotation per 27.3 days).
- Hence the moon's magnetic field is only about 10^{-4} as strong as that of the earth.

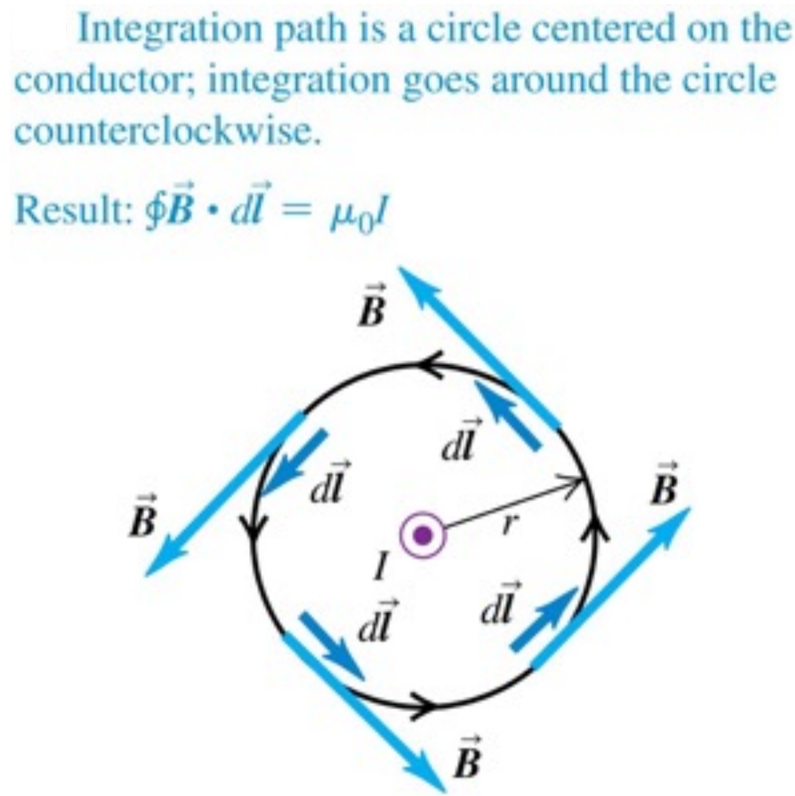


Magnetic fields of current-carrying wires

- Computer cables, or cables for audio-video equipment, create little or no magnetic field.
- This is because within each cable, closely spaced wires carry current in both directions along the length of the cable.
- The magnetic fields from these opposing currents cancel each other.

Ampere's law (special case)

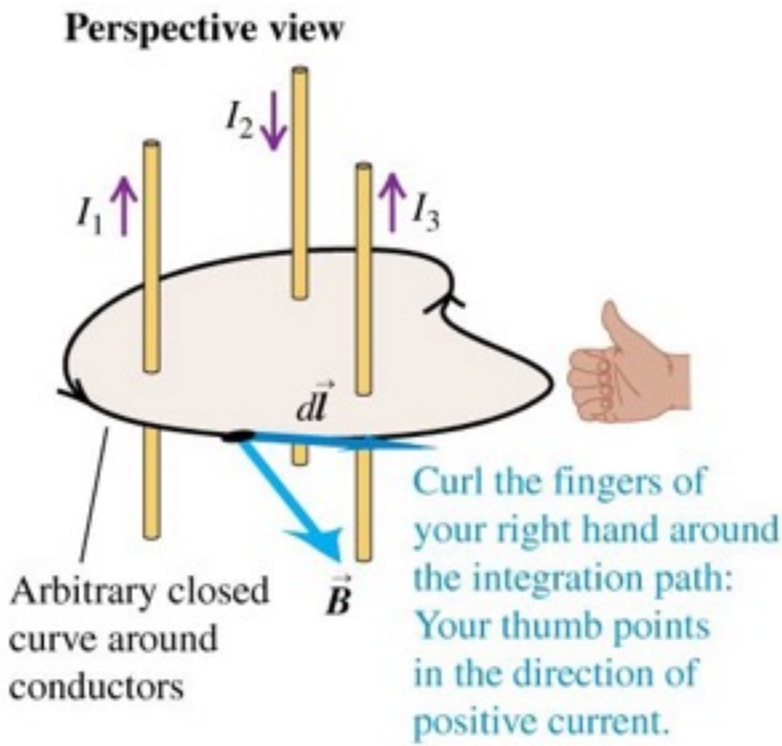
- **Ampere's law** relates electric current to the line integral around a closed path.
- Shown is the special case of a circular closed path centered on a long, straight conductor carrying current I out of the page.
- In this case the integral is simple:



$$\oint \vec{B} \cdot d\vec{l} = \oint B_{\parallel}\, dl = B \oint dl = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

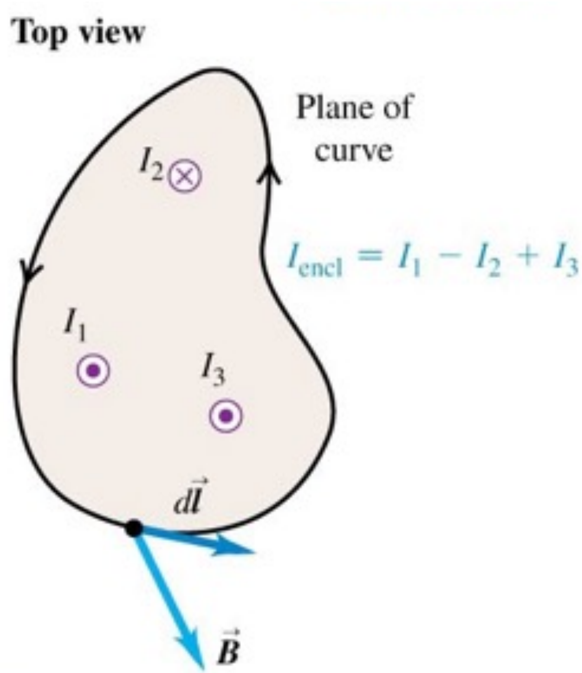
Ampere's law (general statement)

- Suppose several long, straight conductors pass through the surface bounded by the integration path.
- Thus the line integral of the total magnetic field is proportional to the *algebraic sum* of the currents.

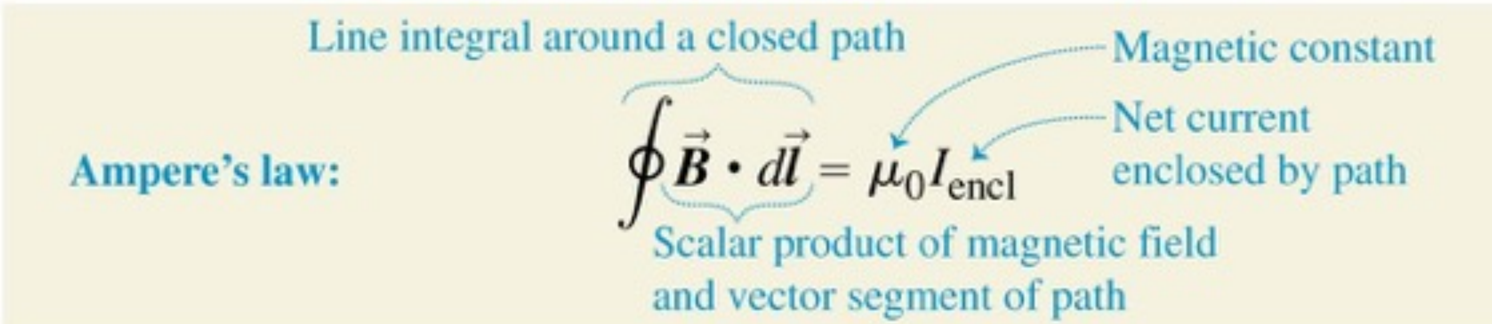


Ampere's law (general statement)

- For the general statement of Ampere's law, we can replace I with I_{encl}, the algebraic sum of the currents enclosed or linked by the integration path, with the sum evaluated by using the right-hand sign rule.



Ampere's law (general statement)

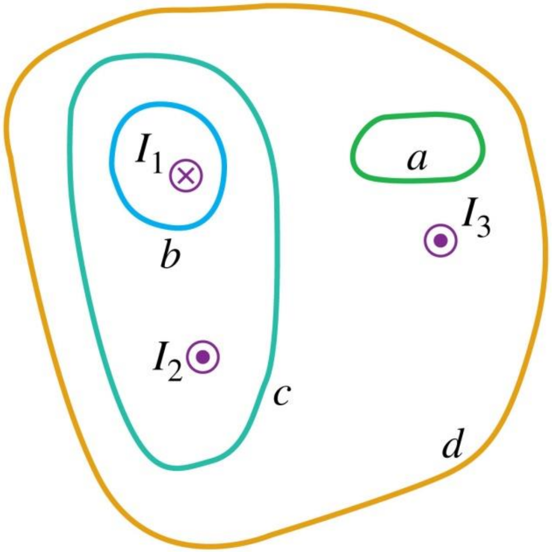


- This equation is valid for conductors and paths of any shape.
- If the integral around the closed path is zero, it *does not* necessarily mean that the magnetic field is everywhere along the path, only that the total current through an area bounded by the path is zero.

CPS 31-2

The figure shows, in cross section, three conductors that carry currents perpendicular to the plane of the figure.

If the currents I_1, I_2, and I_3 all have the same magnitude, for which path(s) is the line integral of the magnetic field equal to zero?



A. path *a* only

B. paths *a* and *c*

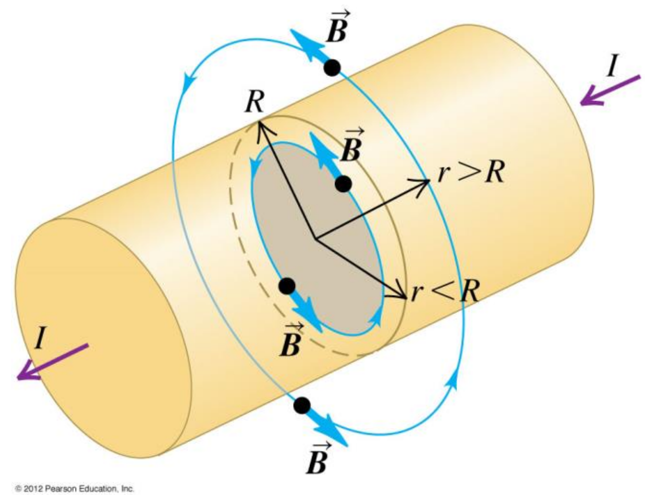
C. paths *b* and *d*

D. paths *a*, *b*, *c*, and *d*

E. The answer depends on whether the integral goes clockwise or counterclockwise around the path.

Applying Ampere's Law

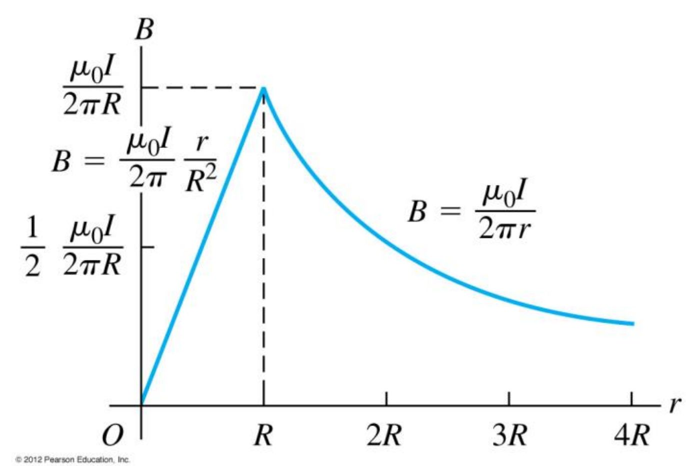
- We can look at the magnetic field inside a conducting wire with some (cylindrically symmetric) current density:



$$\int_{\text{Closed Path}} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} \Rightarrow$$

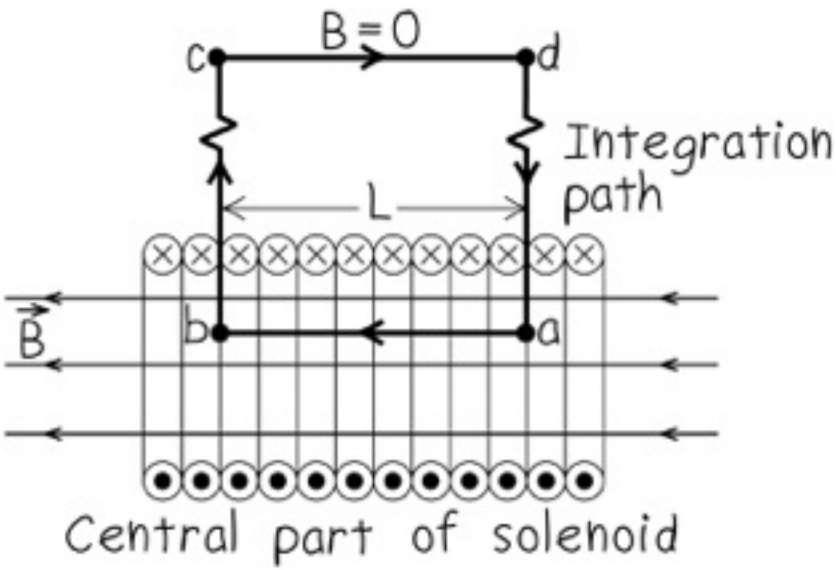
$$\int_{\text{Circle}} \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 I_{\text{enc}} = \begin{cases} \mu_0 J\left(\pi r^2\right) & \text{inside wire} \\ \mu_0 I & \text{outside wire} \end{cases}$$

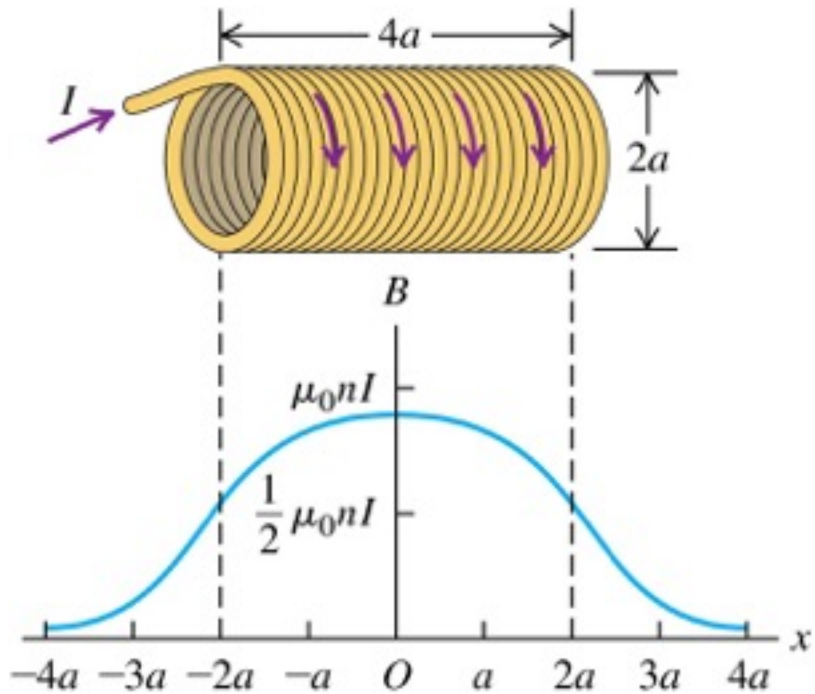
$$B = \begin{cases} \dfrac{\mu_0 J r}{2} & \text{inside wire} \\ \dfrac{\mu_0 I}{2\pi r} & \text{outside wire} \end{cases}$$



Field of a solenoid

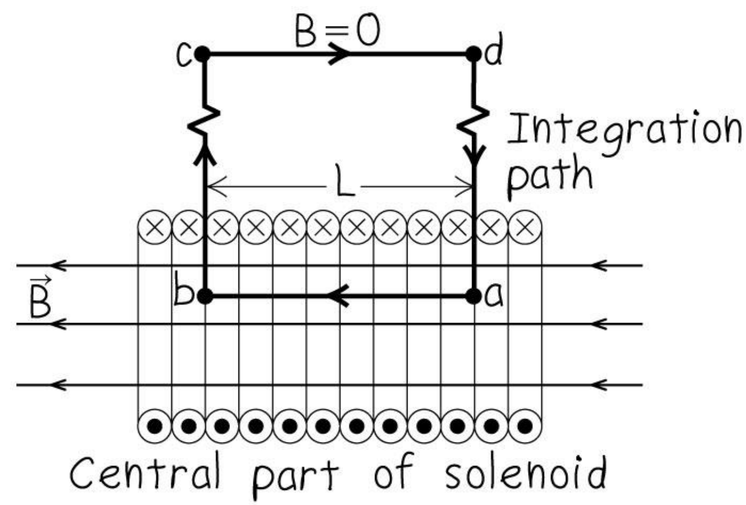
- A *solenoid* consists of a helical winding of wire on a cylinder.
- Follow Example 28.9 using the figures below.





Applying Ampere's Law

- If we have a current, I, and N turns per unit length, L, then:



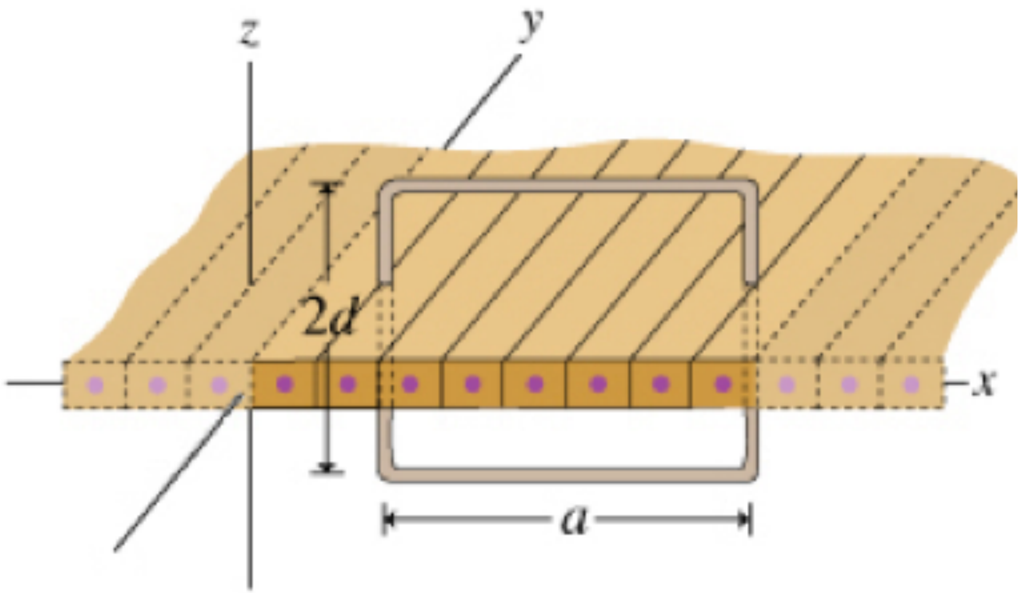
$$\int_{\text{Closed Path}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}} \Rightarrow$$

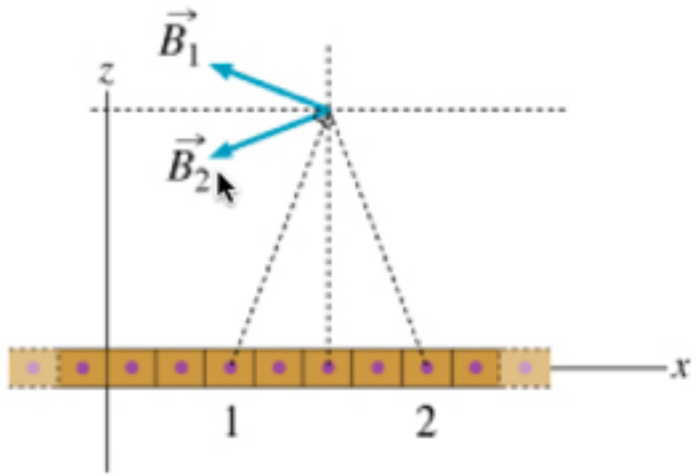
$$\int_{a\to b} \vec{B}\cdot d\vec{l} + \int_{b\to c} \vec{B}\not{\cdot}\, d\vec{l} + \int_{c\to d} \not{\vec{B}}\cdot d\vec{l} + \int_{d\to a} \vec{B}\not{\cdot}\, d\vec{l} = \mu_0 NI \Rightarrow$$

$$BL = \mu_0 NI \Rightarrow$$

$$B = \mu_0 \frac{N}{L} I$$

HW - Apply Amperes Law to a current sheet





Infinite sheet => B is constant
outside the sheet

Faraday's law of induction

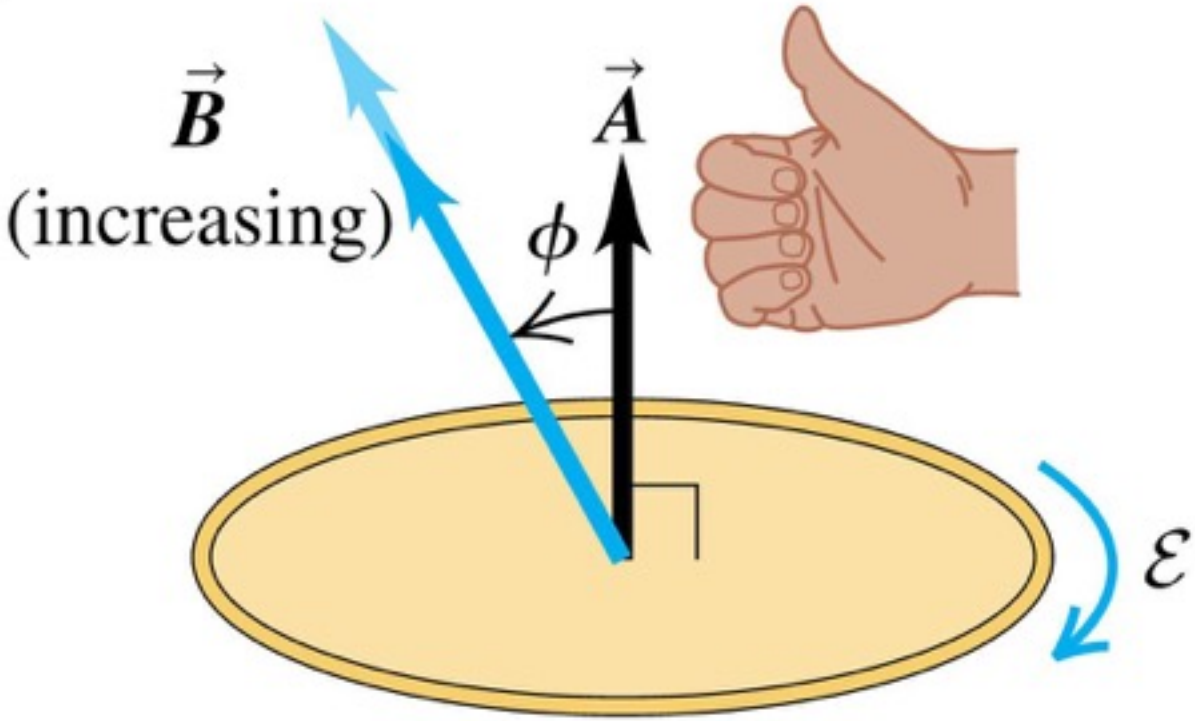
- When the magnetic flux through a single closed loop changes with time, there is an induced emf that can drive a current around the loop:

Faraday's law: The induced emf in a closed loop ... ... equals the negative of the time rate of change of magnetic flux through the loop.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

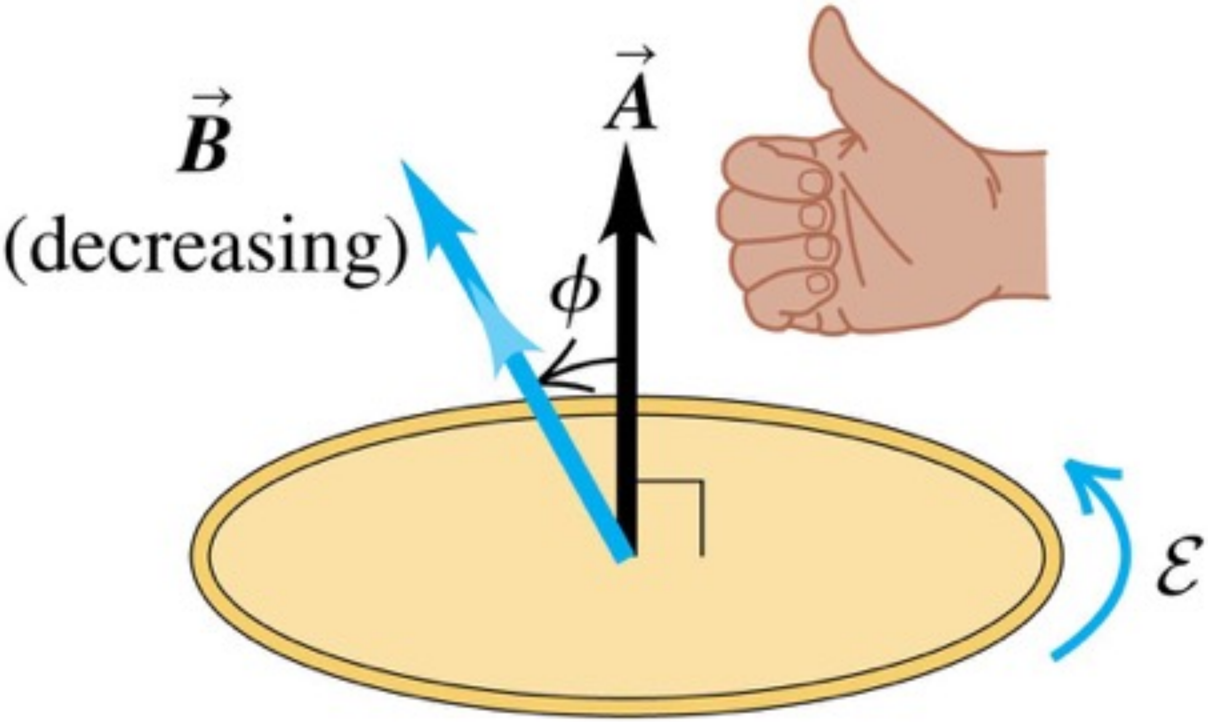
- Recall that the unit of magnetic flux is the weber (Wb).
- $1\ \text{T} \cdot \text{m}^2 = 1\ \text{Wb}$, so $1\ \text{V} = 1\ \text{Wb/s}$.

Determining the direction of the induced emf: Slide 1 of 4



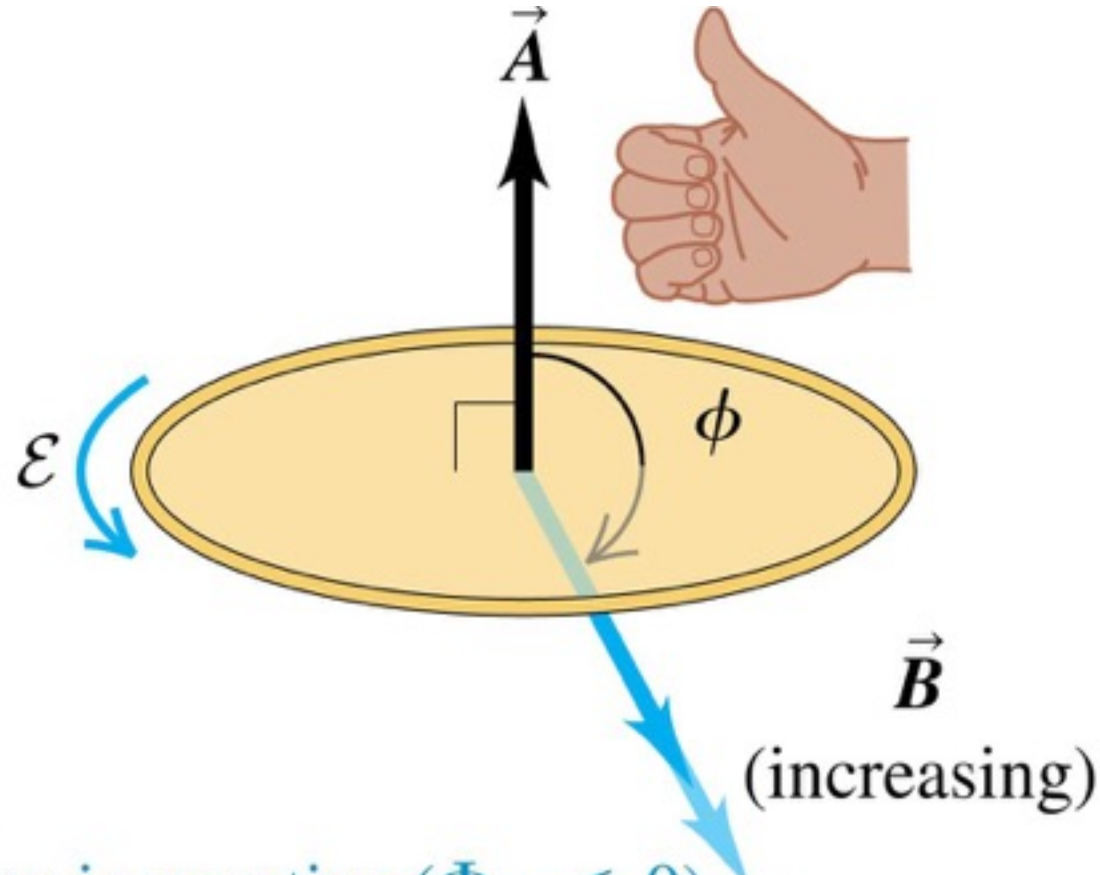
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming more positive ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

Determining the direction of the induced emf: Slide 2 of 4

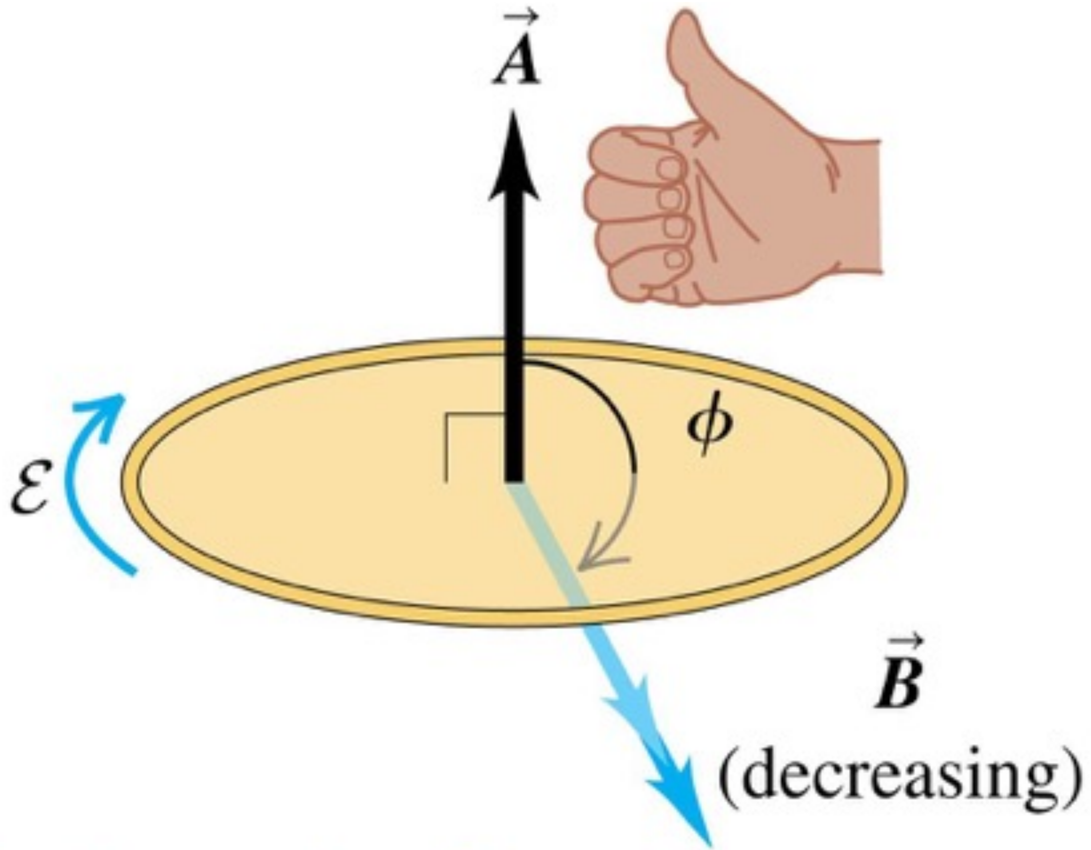


- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming less positive ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

Determining the direction of the induced emf: Slide 3 of 4



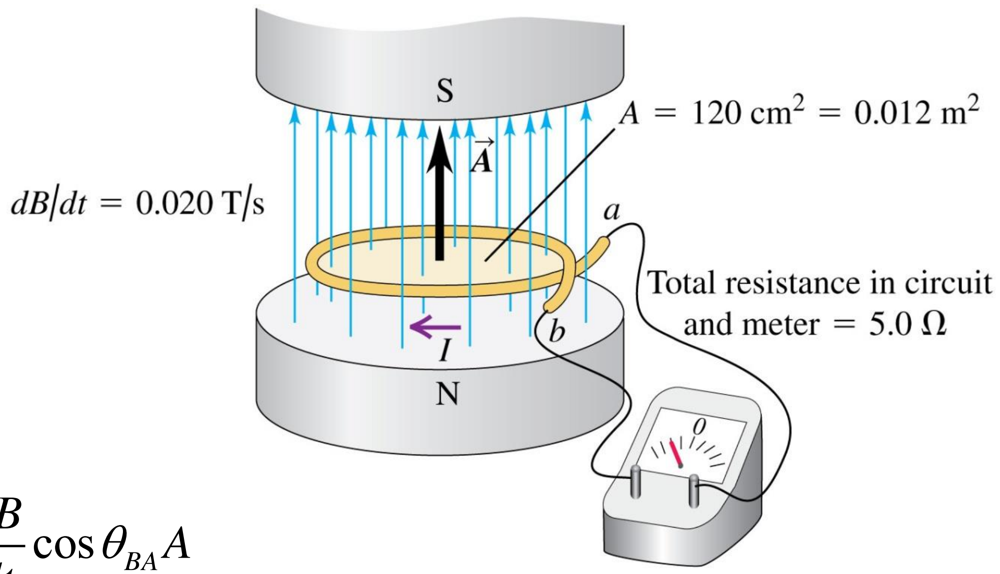
- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming more negative ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).



- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming less negative ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

Example

- Let's put some numbers in to see how this might work:



$$\mathcal{E} = -\frac{d}{dt}\left[N \int_{\text{Surface}} \vec{B}\cdot d\vec{A}\right] = -N\frac{dB}{dt}\cos\theta_{BA}A$$

$$= -(1)(0.020T/s)(1)(0.012m^2) = 2.4\times10^{-4}\frac{Tm^2}{s}$$

$$I = \frac{\mathcal{E}}{R} = \frac{2.4\times10^{-4}\frac{Tm^2}{s}}{5.0\Omega} = 4.8\times10^{-4}\frac{Tm^2}{\Omega s}$$

Unit Check!!!

- Let's put some numbers in to see how this might work:

$$\mathcal{E} = -\frac{d}{dt}\left[N\int_{\text{Surface}} \vec{B}\cdot d\vec{A}\right] = -N\frac{dB}{dt}\cos\theta_{BA}A$$

$$= -(1)(0.020\,T/s)(1)(0.012m^2) = 2.4\times10^{-4}\frac{Tm^2}{s}$$

$$I = \frac{\mathcal{E}}{R} = \frac{2.4\times10^{-4}\frac{Tm^2}{s}}{5.0\Omega} = 4.8\times10^{-4}\frac{Tm^2}{\Omega s}$$

$$d\vec{F} = Id\vec{l}\times\vec{B} \Rightarrow$$

$$N = AmT \Rightarrow$$

$$T = \frac{N}{Am}$$

$$V = IR \Rightarrow$$

$$\frac{Nm}{C} = A\Omega \Rightarrow$$

$$\Omega = \frac{Nm}{AC}$$

$$\Rightarrow \frac{Tm^2}{\Omega s} = \frac{\frac{N}{Am}m^2}{\frac{Nm}{AC}s} = \frac{C}{s} = A$$

Faraday's law for a coil

- A commercial alternator uses many loops of wire wound around a barrel-like structure called an armature.
- The resulting induced emf is far larger than would be possible with a single loop of wire.

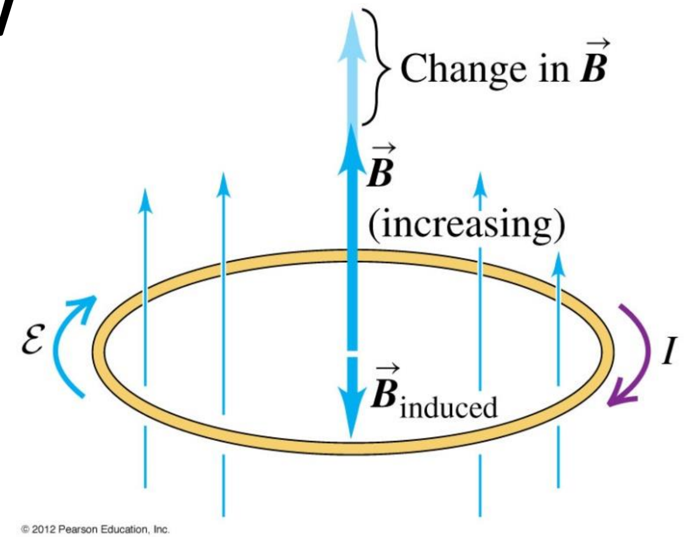
- If a coil has N identical turns and if the flux varies at the same rate through each turn, total emf is:

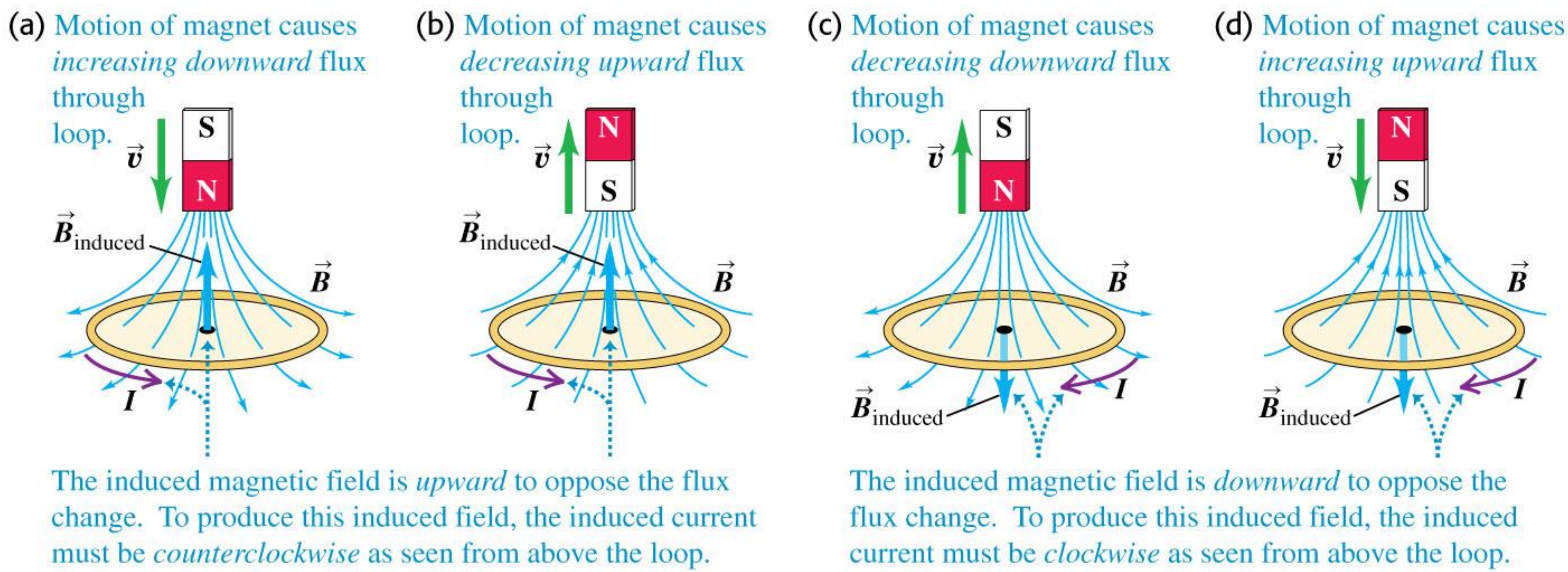
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

Lenz's Law

- To get the direction of the induced EMF (and thus, the current in a circuit), remember:

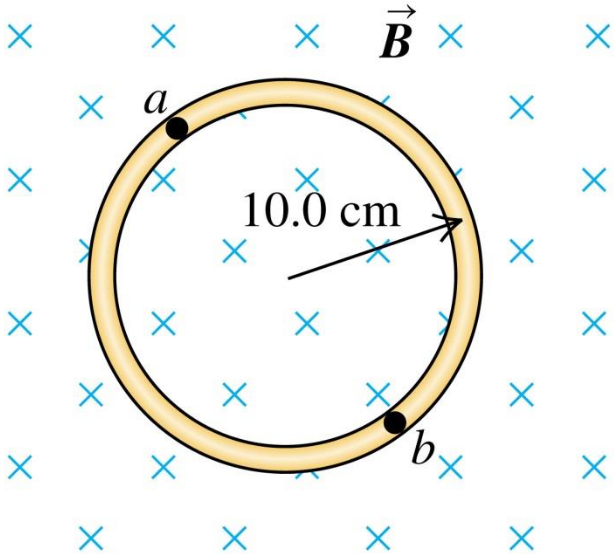
$$\mathcal{E} = -\frac{d}{dt}\Phi_B$$





CPS 32-1

A circular loop of wire is in a region of spatially uniform magnetic field. The magnetic field is directed into the plane of the figure. If the magnetic field magnitude is *decreasing*,



A. the induced emf is clockwise.

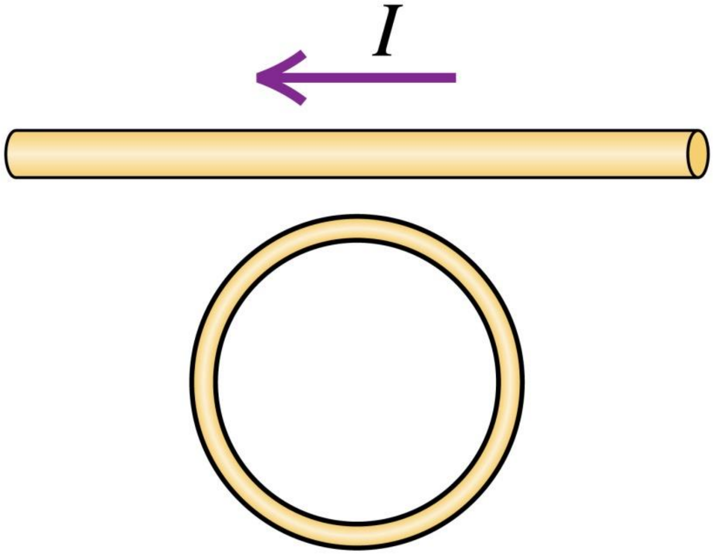
B. the induced emf is counterclockwise.

C. the induced emf is zero.

D. The answer depends on the strength of the field.

CPS 32-2

A circular loop of wire is placed next to a long straight wire. The current *I* in the long straight wire is increasing. What current does this induce in the circular loop?



A. a clockwise current

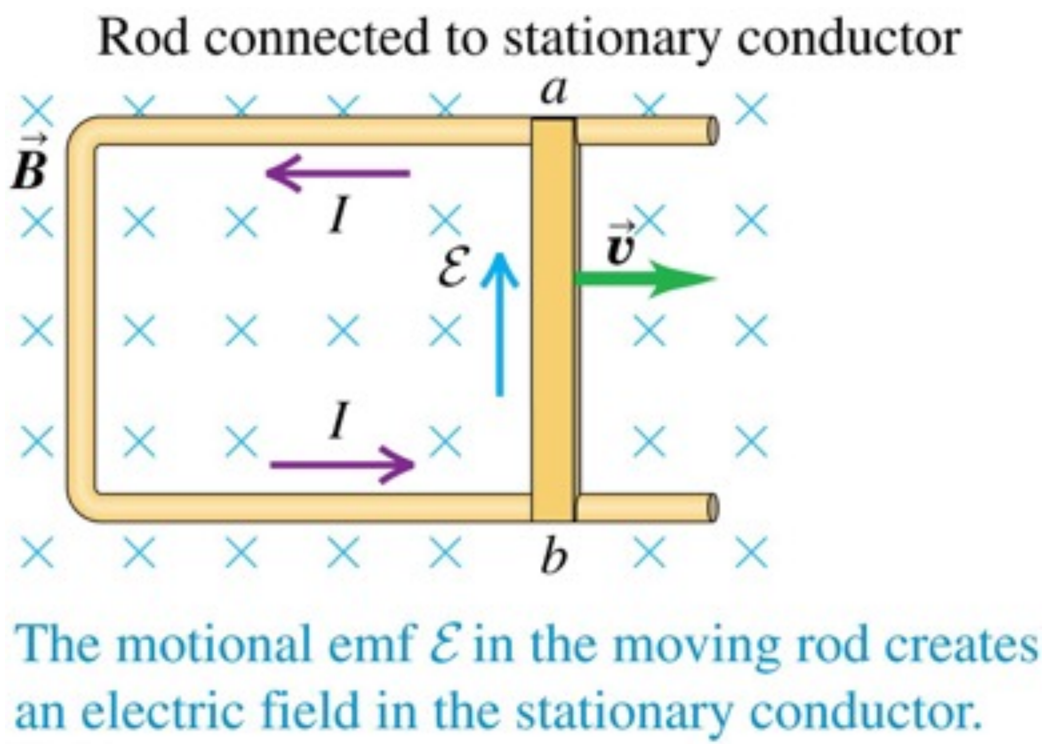
B. a counterclockwise current

C. zero current

D. not enough information given to decide

Motional electromotive force

- When a conducting rod moves perpendicular to a uniform magnetic field, there is a **motional emf** induced.



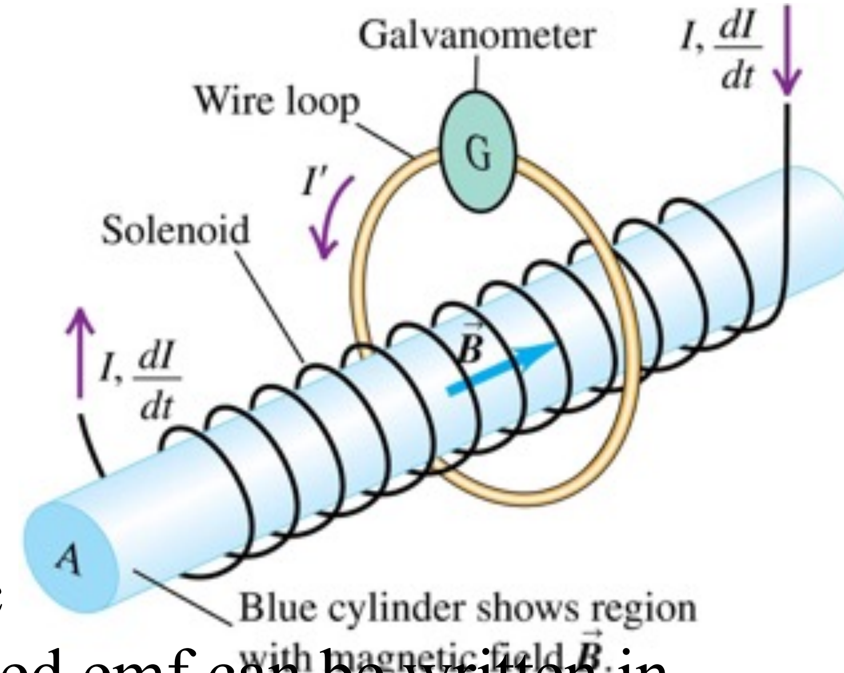
Motional emf, conductor length and velocity perpendicular to uniform $\vec{B}$

$$\mathcal{E} = vBL$$

- v: Conductor speed
- L: Conductor length
- B: Magnitude of uniform magnetic field

Induced electric fields

- A long, thin solenoid is encircled by a circular conducting loop.
- Electric field in the loop is what must drive the current.
- When the solenoid current I changes with time, the magnetic flux also changes, and the induced emf can be written in terms of **induced electric field**:



Faraday's law for a stationary integration path:

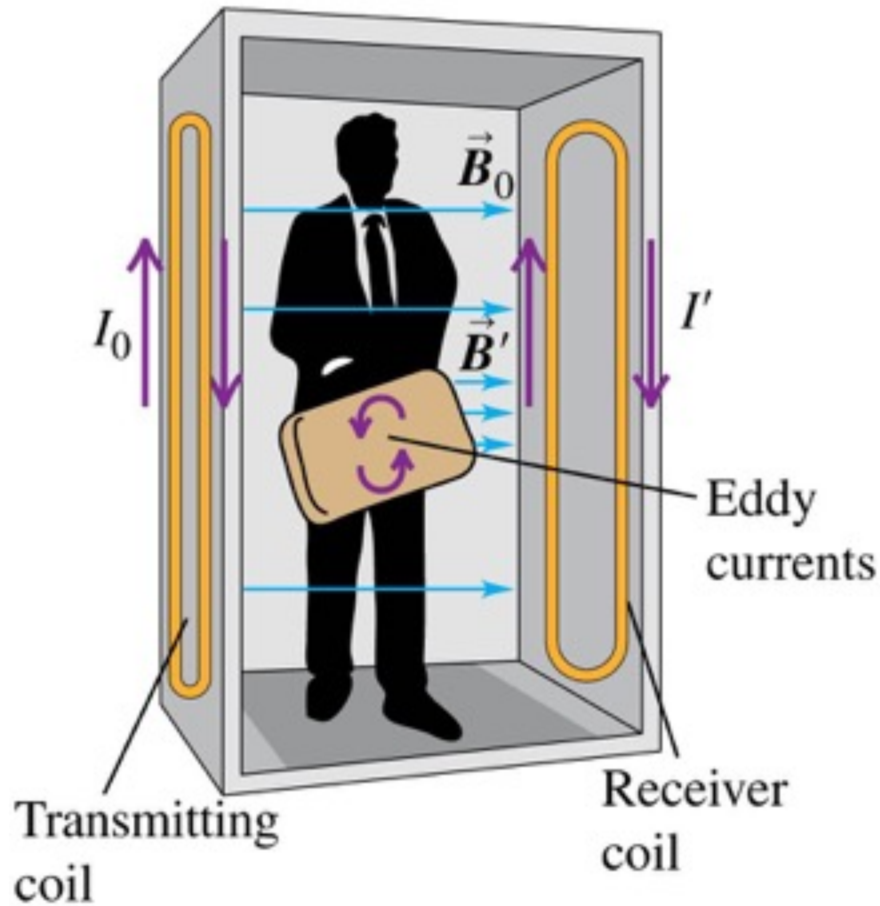
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Line integral of electric field around path ($\oint \vec{E} \cdot d\vec{l}$)

Negative of the time rate of change of magnetic flux through path ($-\frac{d\Phi_B}{dt}$)

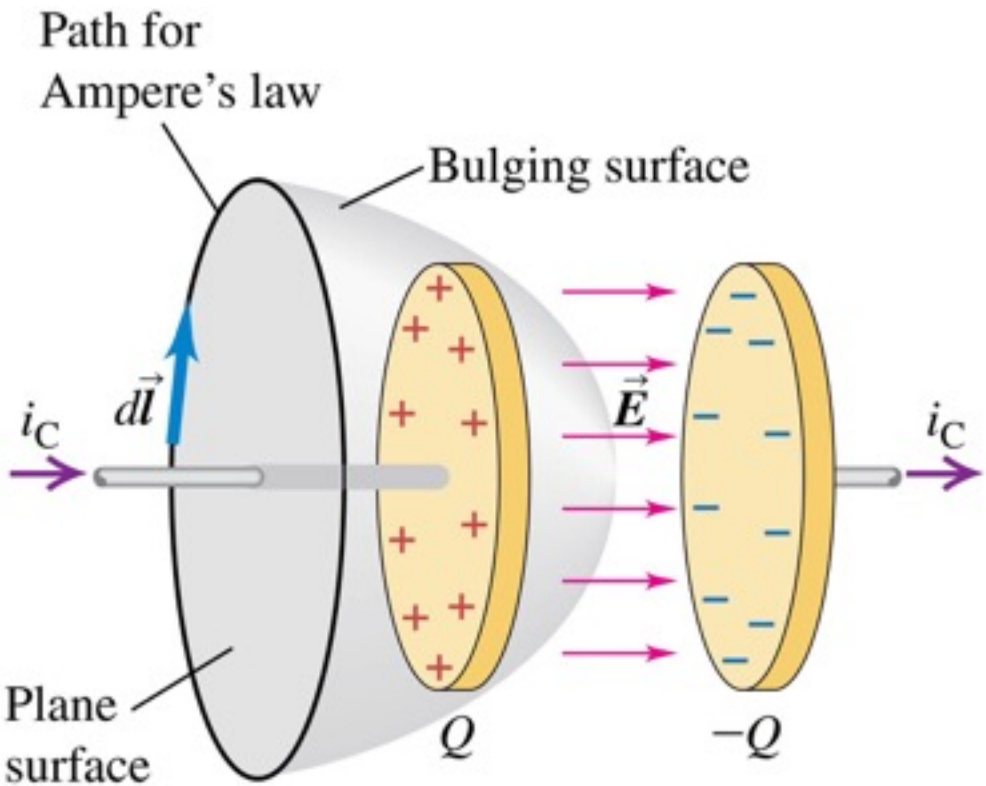
Eddy currents

- When a piece of metal moves through a magnetic field or is located in a changing magnetic field, **eddy currents** of electric current are induced.
- The metal detectors used at airport security checkpoints operate by detecting eddy currents induced in metallic objects.



Displacement current

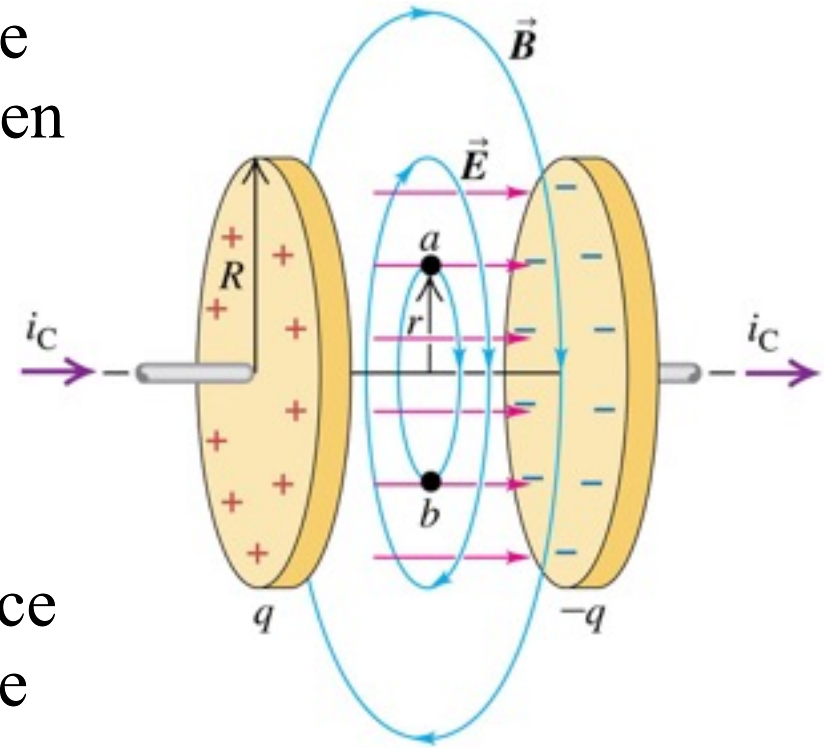
- Ampere's law is *incomplete*, as can be shown by considering the process of charging a capacitor, as shown.



- For the plane circular area bounded by the circle, I_{encl} is the current i_C in the left conductor.
- But the surface that bulges out to the right is bounded by the same circle, and the current through that surface is zero.
- This leads to a contradiction.

Displacement current

- When a capacitor is charging, the electric field is increasing between the plates.
- We can define a fictitious **displacement current** i_D in the region between the plates.
- This can be regarded as the source of the magnetic field between the plates.



Displacement current through an area → $$i_D = \epsilon \frac{d\Phi_E}{dt}$$ ← Time rate of change of electric flux through area

ϵ: Permittivity of material in area

Maxwell's equations of electromagnetism

- All the relationships between electric and magnetic fields and their sources are summarized by four equations, called **Maxwell's equations**.
- The first Maxwell equation is Gauss's law for electric fields from Chapter 22:

Gauss's law for $\vec{E}$:

Flux of electric field through a closed surface

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Q_{encl}: Charge enclosed by surface

ϵ_0: Electric constant

- The [illegible]ic fields from Chapter 27:

Gauss's law for $\vec{B}$:

Flux of magnetic field through any closed surface ...

$$\oint \vec{B} \cdot d\vec{A} = 0$$

... equals zero.

Maxwell's equations of electromagnetism

- The third Maxwell equation is this chapter's formulation of Faraday's law:

Faraday's law for a stationary integration path:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Line integral of electric field around path

Negative of the time rate of change of magnetic flux through path

- The fourth Maxwell equation is Ampere's law, including displacement current:

Ampere's law for a stationary integration path:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0\left(i_C + \epsilon_0 \frac{d\Phi_E}{dt}\right)_{\text{encl}}$$

Line integral of magnetic field around path

Electric constant

Time rate of change of electric flux through path

Magnetic constant

Conduction current through path

Displacement current through path

Maxwell's equations in empty space

- There is a remarkable symmetry in Maxwell's equations.
- In empty space where there is no charge, the first two equations are identical in form.
- The third equation says that a changing magnetic flux creates an electric field, and the fourth says that a changing electric flux creates a magnetic field.

In empty space there are no charges, so the fluxes of $\vec{E}$ and $\vec{B}$ through any closed surface are equal to zero.

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

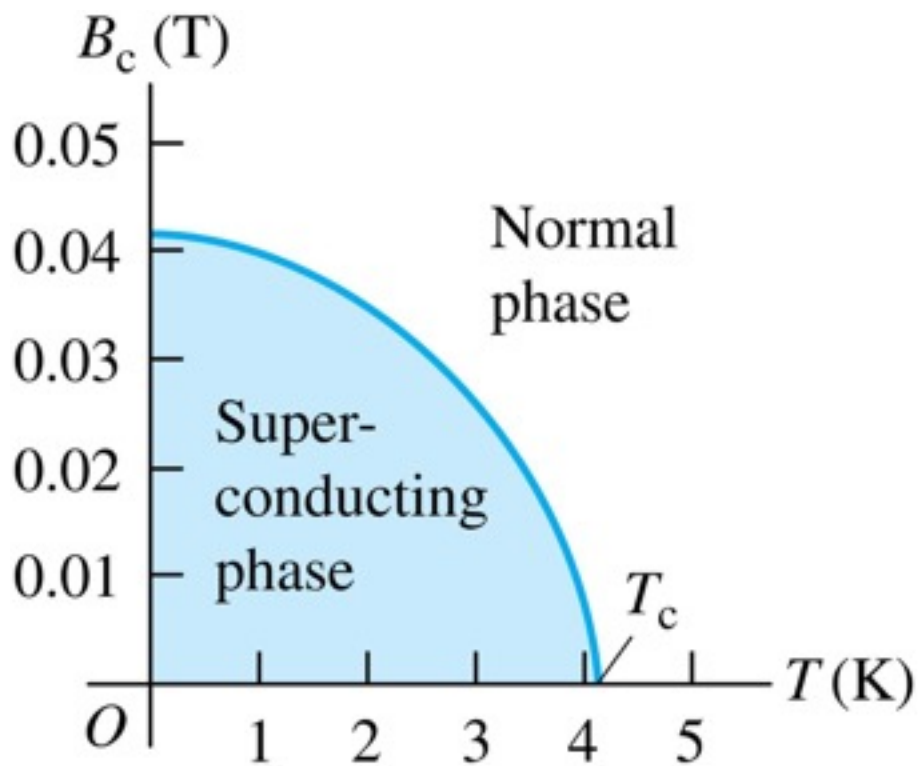
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In empty space there are no conduction currents, so the line integrals of $\vec{E}$ and $\vec{B}$ around any closed path are related to the rate of change of flux of the other field.

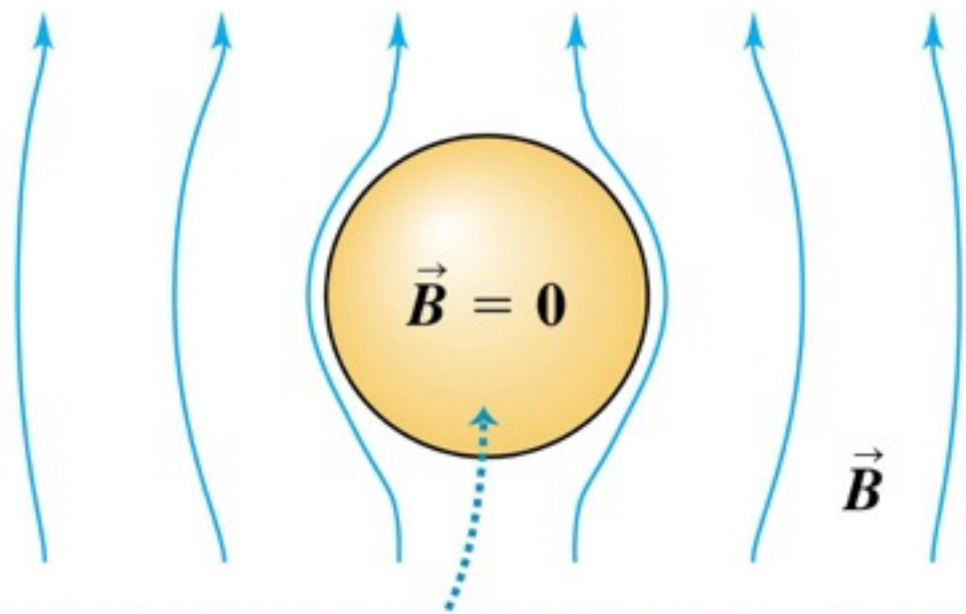
Superconductivity in a magnetic field

- When a superconductor is cooled below its critical temperature T_c, it loses all electrical resistance.
- For any superconducting material the critical temperature T_c changes when the material is placed in an externally produced magnetic field.
- Shown is this dependence for mercury.
- As the external field magnitude increases, the superconducting transition occurs at a lower and lower temperature.



The Meissner effect

- If we place a superconducting material in a uniform applied magnetic field, and then lower the temperature until the superconducting transition occurs, then all of the magnetic flux is expelled from the superconductor.
- The expulsion of magnetic flux is called the **Meissner effect**.



Magnetic flux is expelled from the material, and the field inside it is zero (Meissner effect).