Lecture 28

PHYC 161 Fall 2016

CPS 27-1

At which point is the magnitude of the magnetic field the largest?

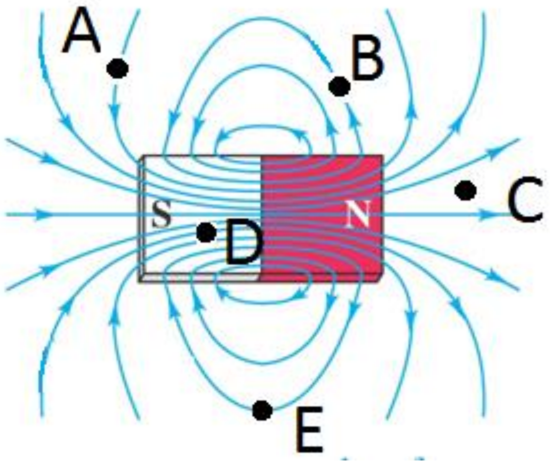
A.

B.

C.

D

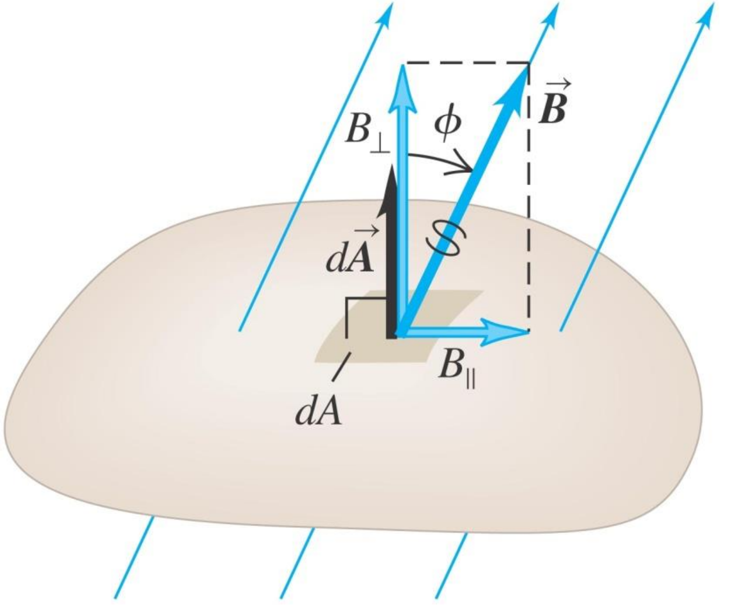
E.



Magnetic Flux

- Yes, back to flux, which means back to surface integrals.
- We can define the magnetic flux in the same way that we defined the electric flux:

$$\Phi_B = \int_{Surface} \vec{B} \cdot d\vec{A}$$



Units of magnetic field and magnetic flux

- The SI unit of **magnetic field** B is called the tesla (1 T), in honor of Nikola Tesla:

$$1 \text{ tesla} = 1 \text{ T} = 1 \text{ N/A} \cdot \text{m}$$

- Another unit of B, the gauss ($1 \text{ G} = 10^{-4} \text{ T}$), is also in common use.
- The magnetic field of the earth is on the order of 10^{-4} T or 1 G.
- The SI unit of **magnetic flux** Φ_B is called the weber (1 Wb), in honor of Wilhelm Weber:

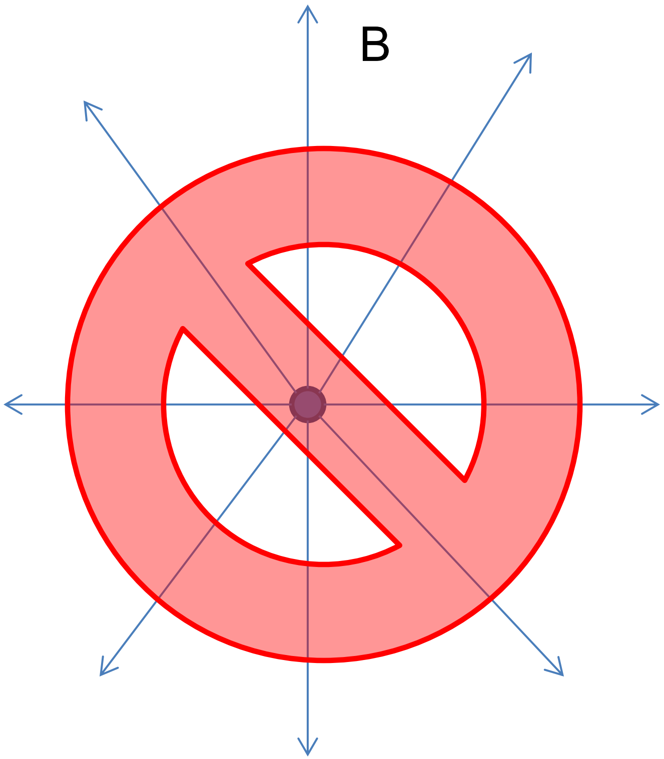
$$1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$$

Gauss's Law for Magnetic Fields

- Then, given what we understand about Gauss's Law for the electric field, we can deduce that:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- In other words, there is no magnetic charge (magnetic monopoles).

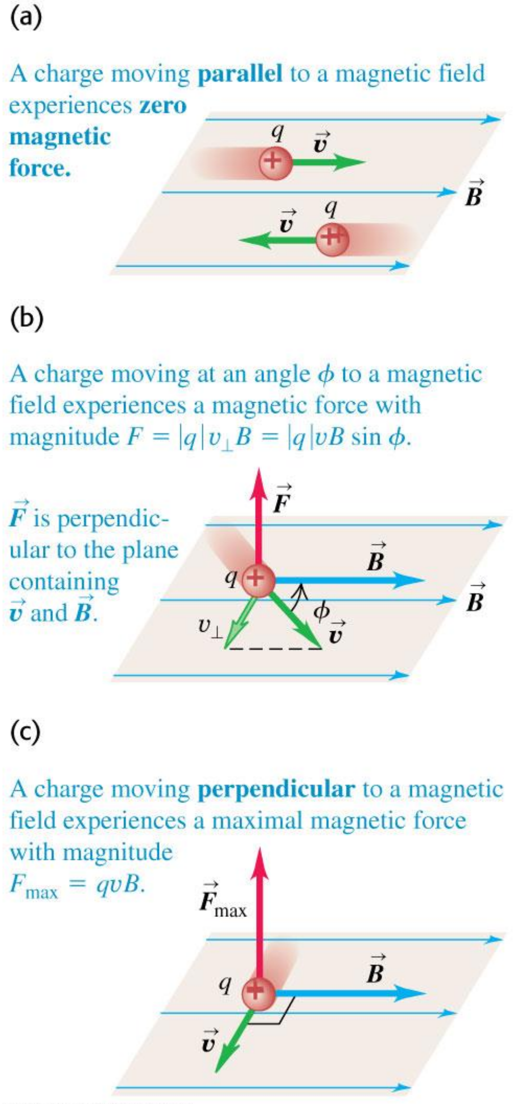


Magnetic Force

- Magnetic fields have an affect on *moving* charges.

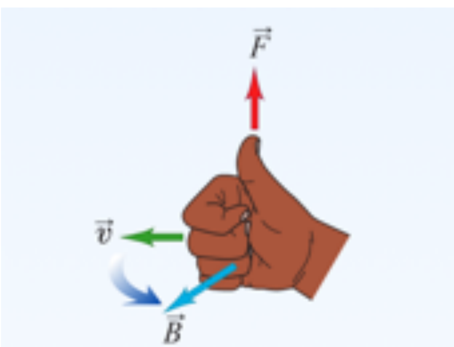
$$\vec{F} = q\vec{v} \times \vec{B}$$

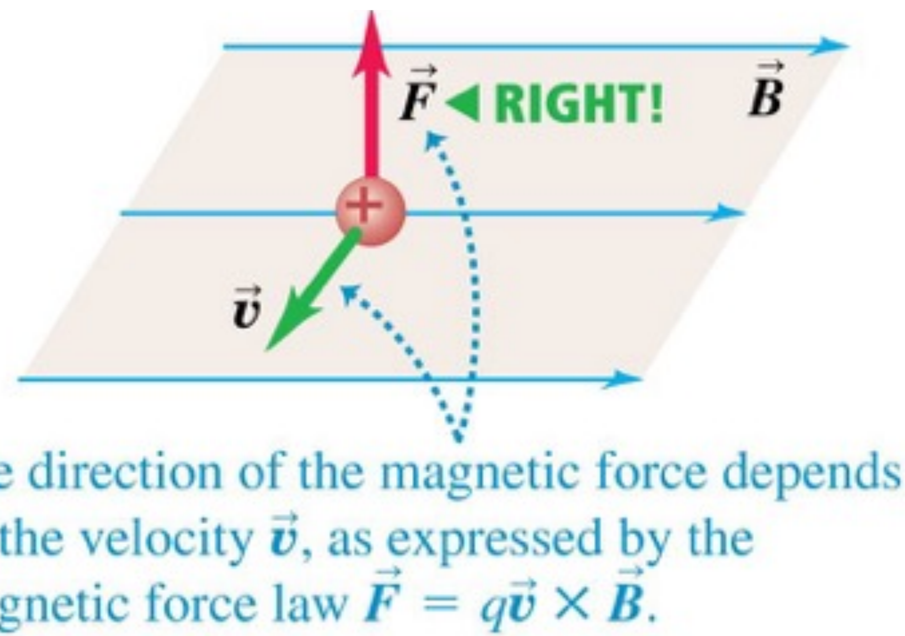
- Since it has been a while since we dealt with cross products, let's take a few minutes to review what this means.
- There is no force on a charge moving in the same direction as the magnetic field.
- The force on a moving charge in a magnetic field is perpendicular to both the field direction and the direction of motion.
- The direction of the force on a negative charge is opposite to that on a positive charge moving in the same direction.
- Since the force is perpendicular to the direction of motion, no work is done by the magnetic force.



Magnetic field lines are *not* lines of force

- It is important to remember that magnetic field lines are *not* lines of magnetic force.
- The force on a charged particle is not along the direction of a field line.

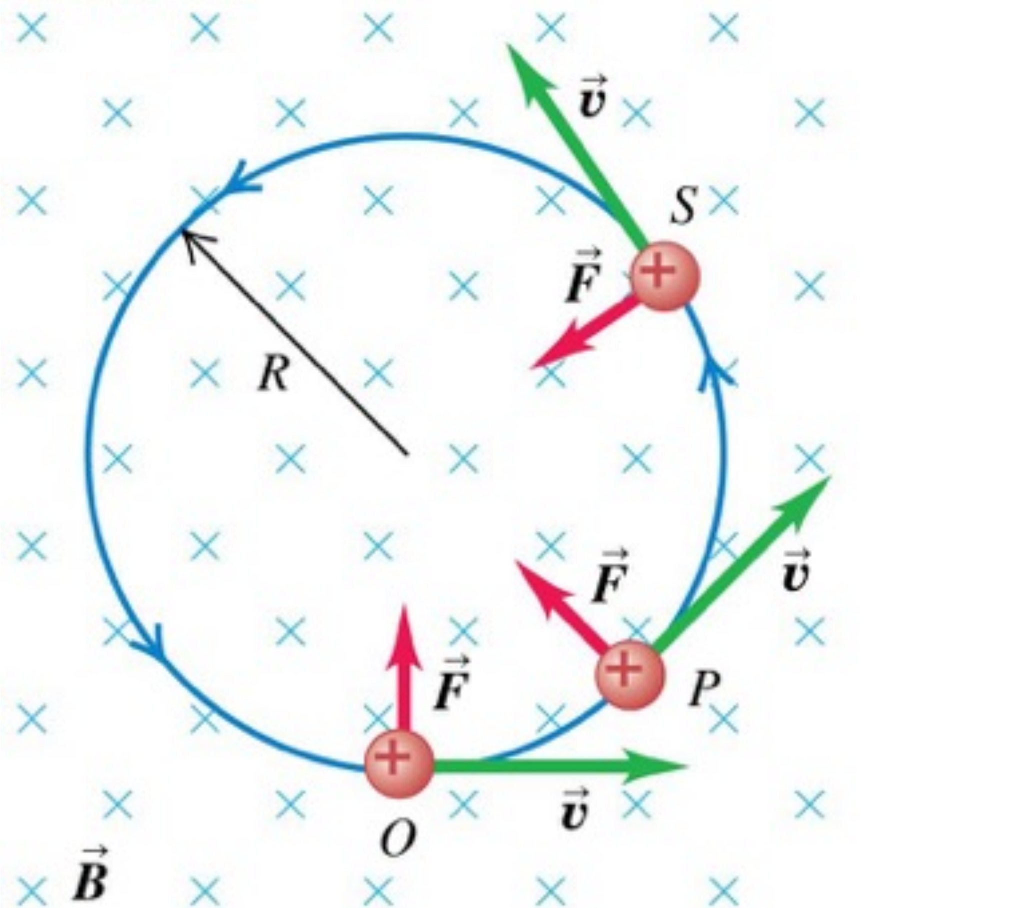




Motion of charged particles in a magnetic field

- When a charged particle moves in a magnetic field, it is acted on by the magnetic force.
- The force is always perpendicular to the velocity, so it cannot change the speed of the particle.

A charge moving at right angles to a uniform $\boldsymbol{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.



Consequences and Applications

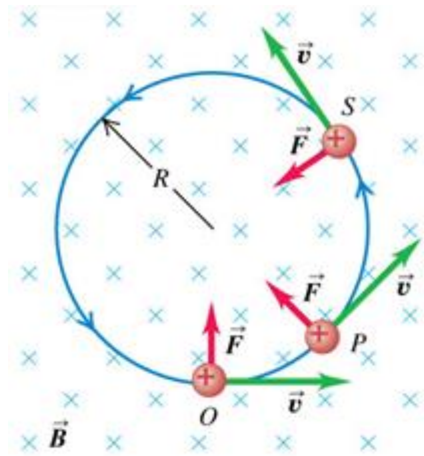
- A beam of charged particles will move in a circle at constant speed when they are sent into it perpendicular to a magnetic field.

$$\vec{F} = q\vec{v}\times\vec{B} = m\vec{a} \Rightarrow$$

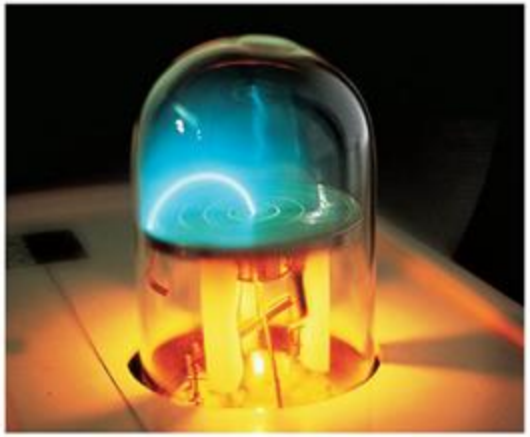
$$|\vec{a}| = \frac{|q\vec{v}\vec{B}|}{m} = \frac{v^2}{R} \Rightarrow$$

$$R = \frac{mv}{qB}$$

$$\left[\frac{kg\cdot\frac{m}{s}}{C\cdot T}\right] = \left[\frac{kg\cdot\frac{m}{s}}{C\cdot\frac{N}{A\cdot m}}\right] = \left[\frac{kg\cdot\frac{m}{s}}{C\cdot\frac{\frac{kg\cdot m}{s^2}}{\frac{C}{s}\cdot m}}\right] = [m]$$



(b) An electron beam (seen as a white arc) curving in a magnetic field



HW - Cyclotron

Charge Moving in a Cyclotron Orbit

Description: General problem, which goes through charged-particle motion perpendicular to a magnetic field; reviews cyclotron frequency derivation. Goes through kinematics and frequency invariance. rhr_B convention used.

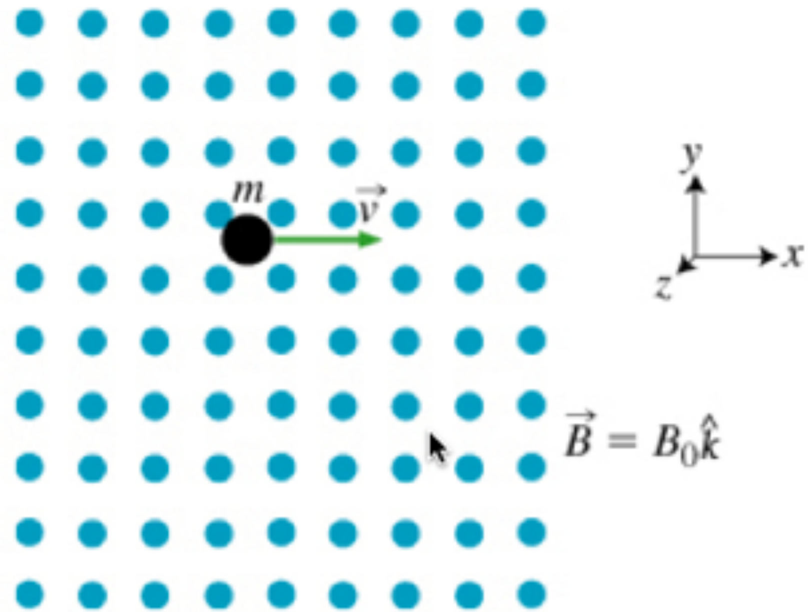
Learning Goal:

To understand why charged particles move in circles perpendicular to a magnetic field and why the frequency is an invariant.

A particle of charge q and mass m moves in a region of space where there is a uniform magnetic field $\vec{B} = B_0\hat{k}$ (i.e., a magnetic field of magnitude B_0 in the +z direction). In this problem, neglect any forces on the particle other than the magnetic

Which way is $\vec{F}$?

a) + x-hat
b) - x-hat
c) + y hat
d) - y hat
e) - k hat



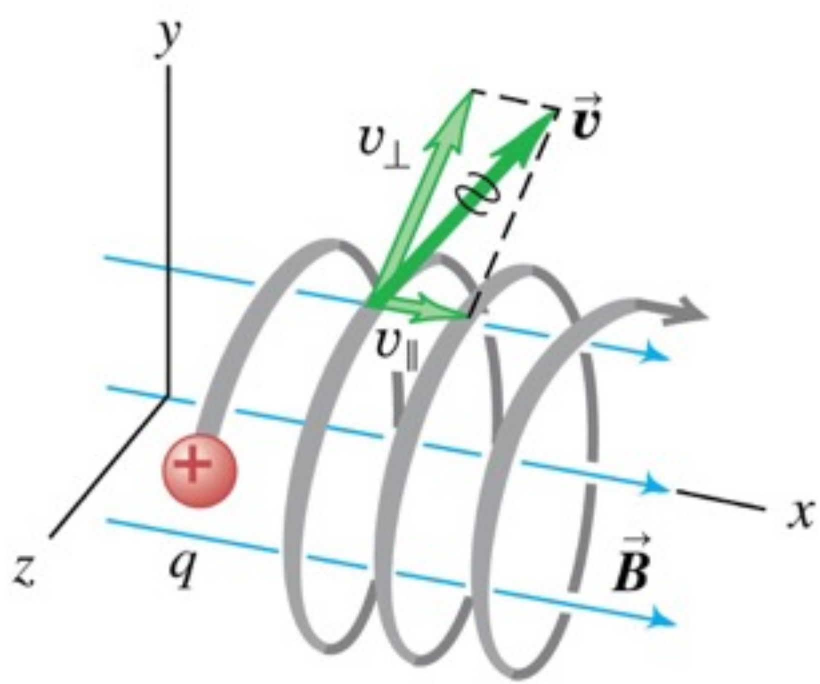
Part A

At a given moment the particle is moving in the +x direction (and the magnetic field is always in the +z direction). If q is positive, what is the direction of the force on the particle due to the magnetic field?

Helical motion

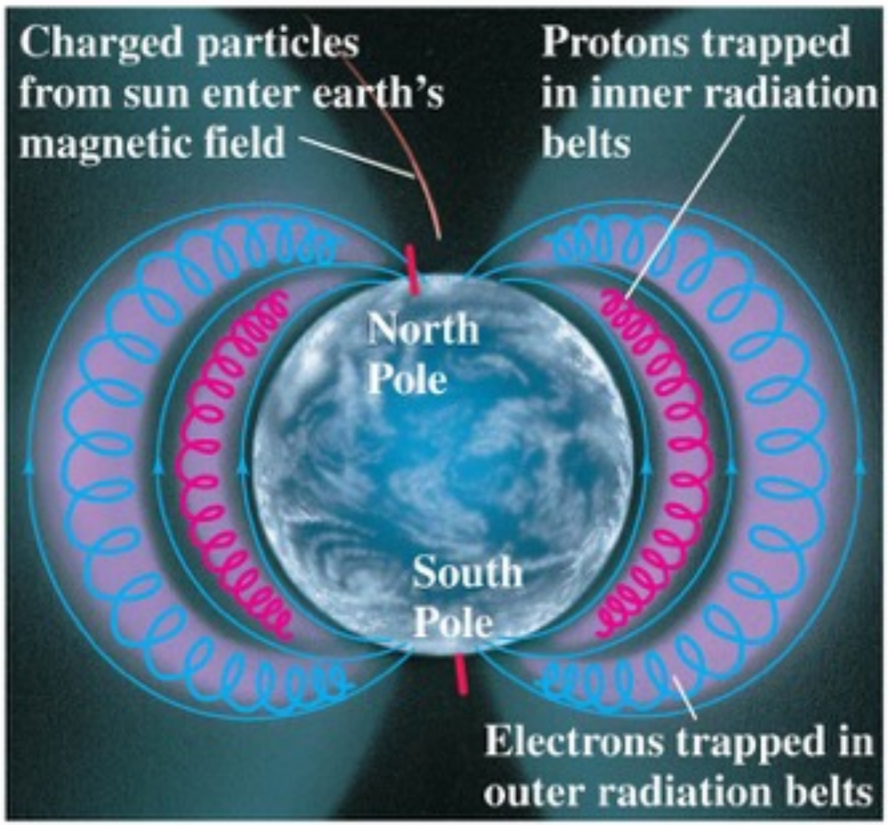
- If the particle has velocity components parallel to and perpendicular to the field, its path is a *helix*.
- The speed and kinetic energy of the particle remain constant.

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a helical path.



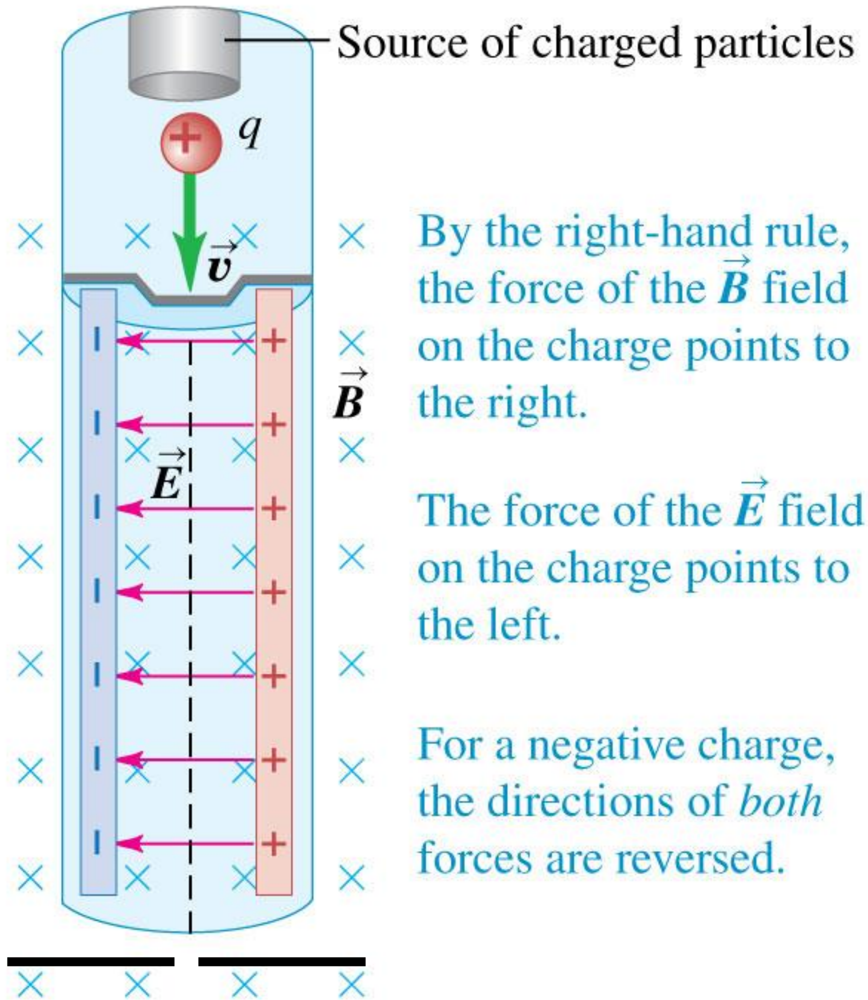
The Van Allen radiation belts

- Near the poles, charged particles from these belts can enter the atmosphere, producing the aurora borealis ("northern lights") and aurora australis ("southern lights").

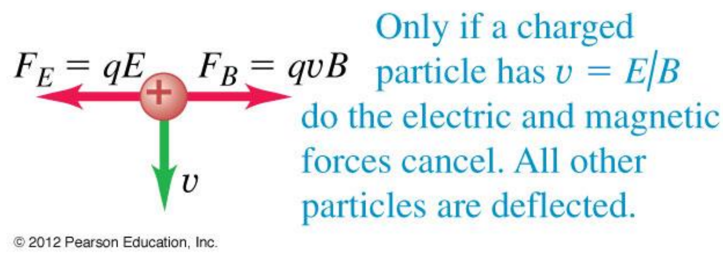


When both electric and magnetic fields present

(a) Schematic diagram of velocity selector



(b) Free-body diagram for a positive particle



$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{F}_E = q\vec{E}$$

HW Velocity Filter

- Only charged particles of a certain velocity pass through

Electromagnetic Velocity Filter

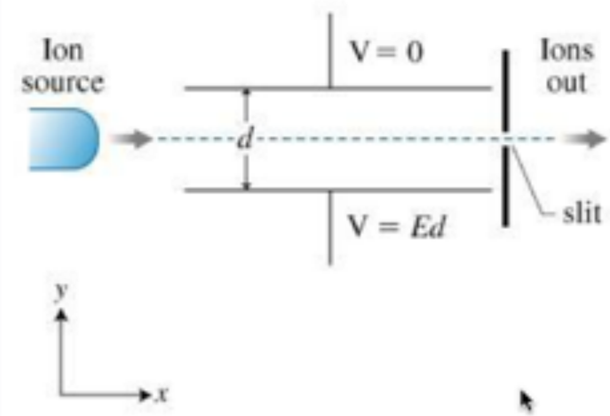
Description: Find the velocity of a charged particle that is undeflected in crossed electric and magnetic fields. Look at relation between mass, charge, and acceleration as charged particle traverses the fields.

When a particle with charge q moves across a magnetic field of magnitude B, it experiences a force to the side. If the proper electric field $\vec{E}$ is simultaneously applied, the electric force on the charge will be in such a direction as to cancel the magnetic force with the result that the particle will travel in a straight line. The balancing condition provides a relationship involving the velocity $\vec{v}$ of the particle. In this problem you will figure out how to arrange the fields to create this balance and then determine this relationship.

Part A

Consider the arrangement of ion source and electric field plates shown in the figure. The ion source sends particles with velocity $\vec{v}$ along the positive x axis. They encounter electric field plates spaced a distance d apart that generate a uniform electric field of magnitude E in the $+y$ direction. To cancel the resulting electric force with a magnetic force, a magnetic field (not shown) must be added in which direction? Using the right-hand rule, you can see that the positive z axis is directed out of the screen.

Choose the direction of $\hat{B}$.



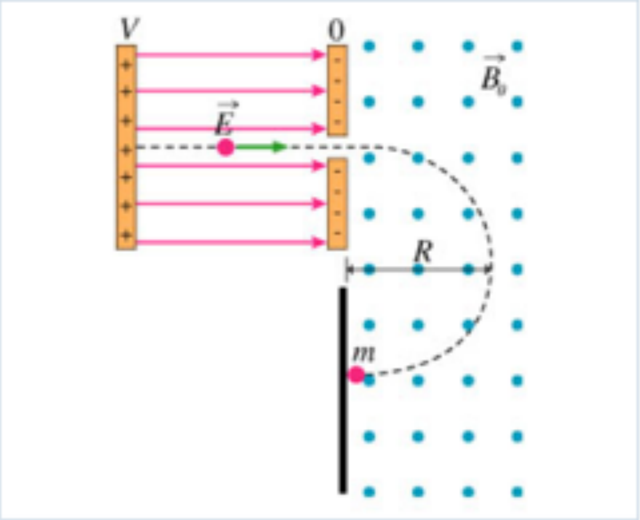
HW - Mass Spectrometer

Mass Spectrometer

Description: Potential V accelerates charged particle to speed u, then measure radius in magnetic field to find m/q.

J. J. Thomson is best known for his discoveries about the nature of cathode rays. Another important contribution of his was the invention, together with one of his students, of the mass spectrometer. The ratio of mass m to (positive) charge q of an ion may be accurately determined in a mass spectrometer. In essence, the spectrometer consists of two regions: one that accelerates the ion through a potential difference V and a second that measures its radius of curvature in a perpendicular magnetic field as shown in .

The ion begins at potential V and is accelerated toward zero potential. When the particle exits the region with the electric field it will have obtained a speed u.



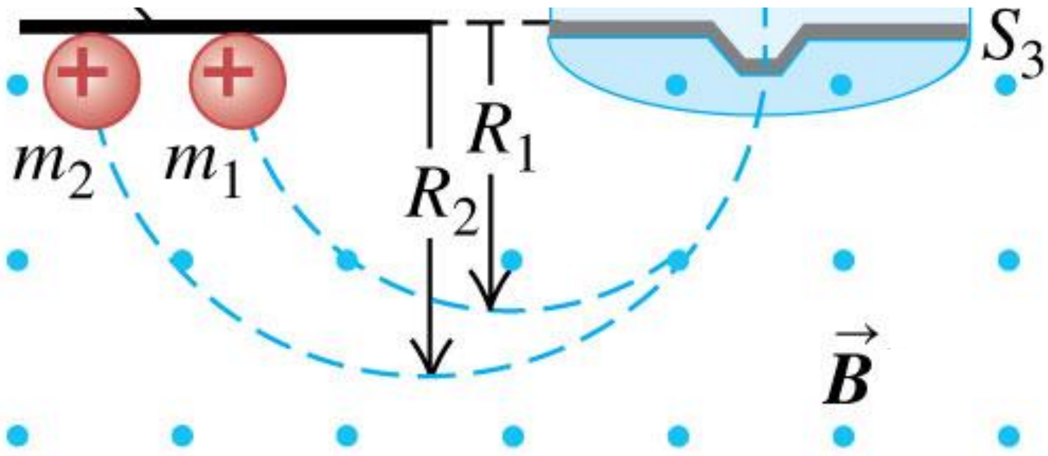
Part A

With what speed u does the ion exit the acceleration region?

Find the speed in terms of m, q, V, and any constants.

Mass-spec

- R=mv/qB



Magnetic field separates particles by mass; the greater a particle’s mass, the larger is the radius of its path.

Magnetic Force on a Current Element

- In electronics, we rarely deal with “beams” of charged particles, but rather deal with current in a wire.
- But current is just moving charged particles.

$$\vec{F}_B = q\vec{v}_d \times \vec{B}$$

- The total force on the wire segment of length *dl* is just the sum of the forces on all the moving charges:

$$d\vec{F} = n(V)q\vec{v}_d \times \vec{B}$$

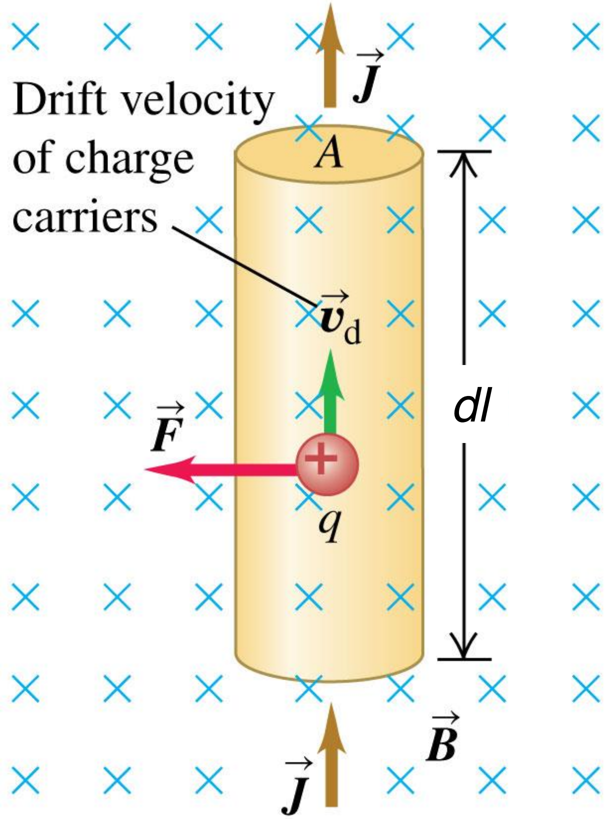
$$= n(Adl)q\vec{v}_d \times \vec{B}$$

- But nqv_d is just the current density, and the current density times the area is just the current:

$$d\vec{F} = n(Adl)q\vec{v}_d \times \vec{B}$$

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

- Where we have designated the direction of dl to be in the same direction as the current.

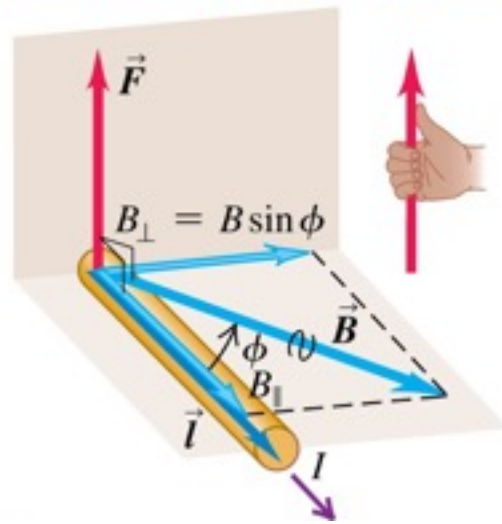


The magnetic force on a current-carrying conductor

- The force is always perpendicular to both the conductor and the field, with the direction determined by the same right-hand rule we used for a moving positive charge.

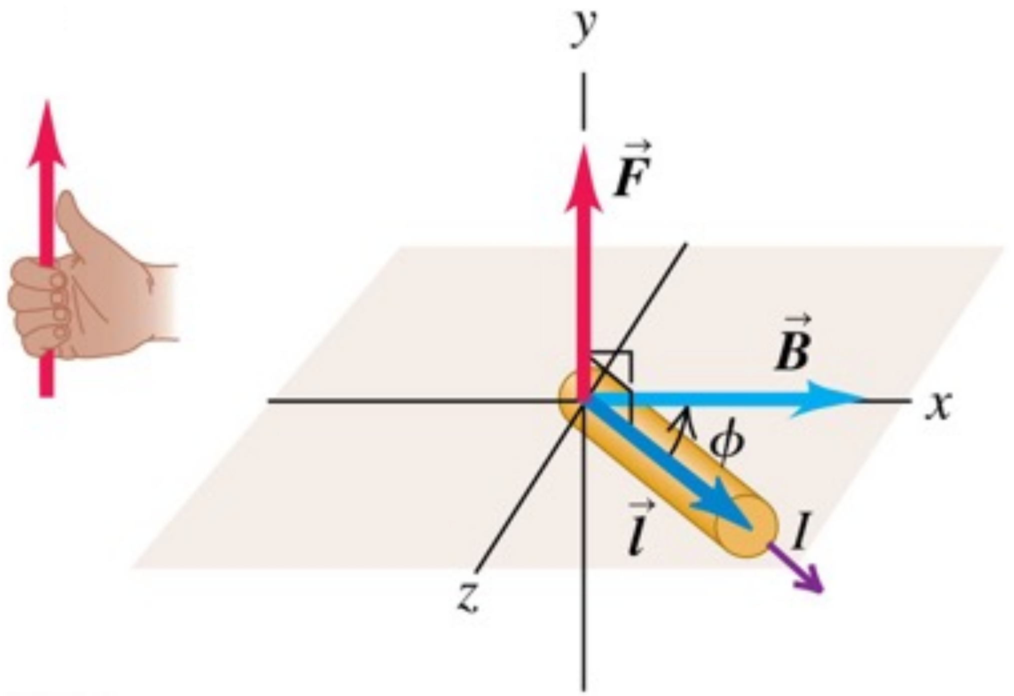
Force $\vec{F}$ on a straight wire carrying a positive current and oriented at an angle ϕ to a magnetic field $\vec{B}$:

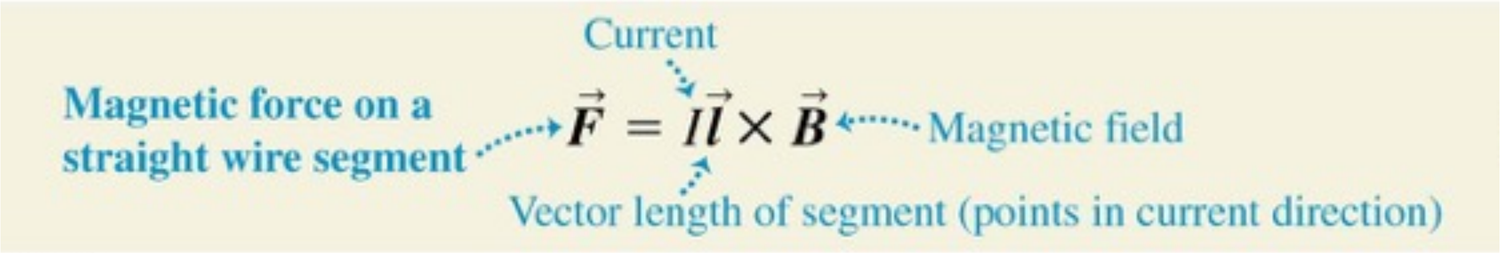
- Magnitude is $F = IlB_{\perp} = IlB\sin\phi$.
- Direction of $\vec{F}$ is given by the right-hand rule.



The magnetic force on a current-carrying conductor

- The magnetic force on a segment of a straight wire can be represented as a vector product.





Magnetic Force on a Current-Carrying Wire

- If the force isn't constant (either direction or magnitude), we must use our calculus tools:

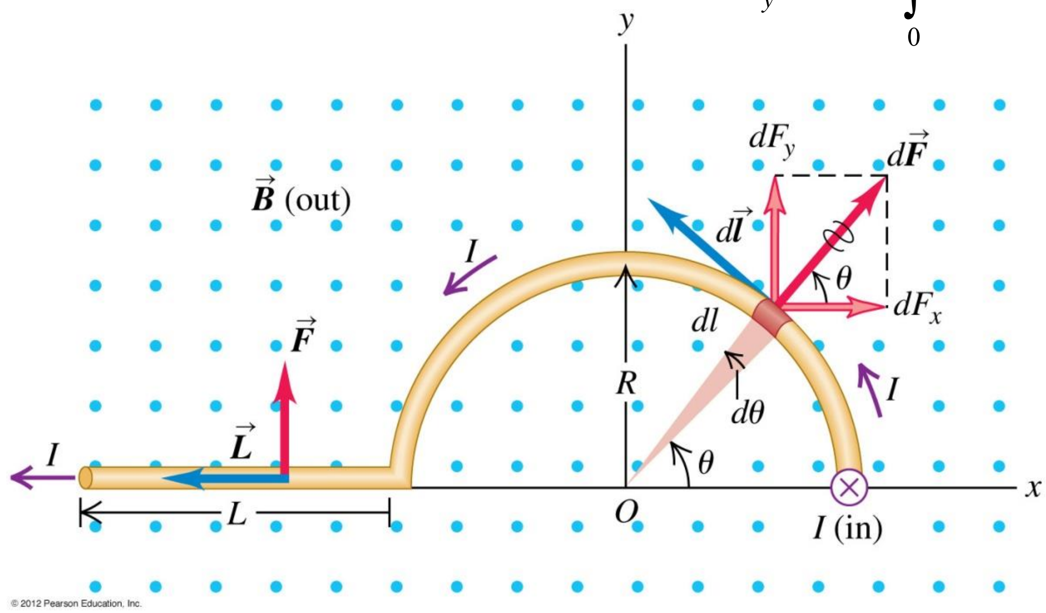
$$d\vec{F} = n\left(Adl\right)q\vec{v}_d \times \vec{B}$$

$$d\vec{F} = Id\vec{l} \times \vec{B} = I\left(Rd\theta\right)B\hat{r}$$

$$d\vec{F} = I\left(Rd\theta\right)B\left[\cos\theta\hat{i} + \sin\theta\hat{j}\right]$$

$$dF_y = IRB\sin\theta d\theta, \quad dF_x = IRB\cos\theta d\theta$$

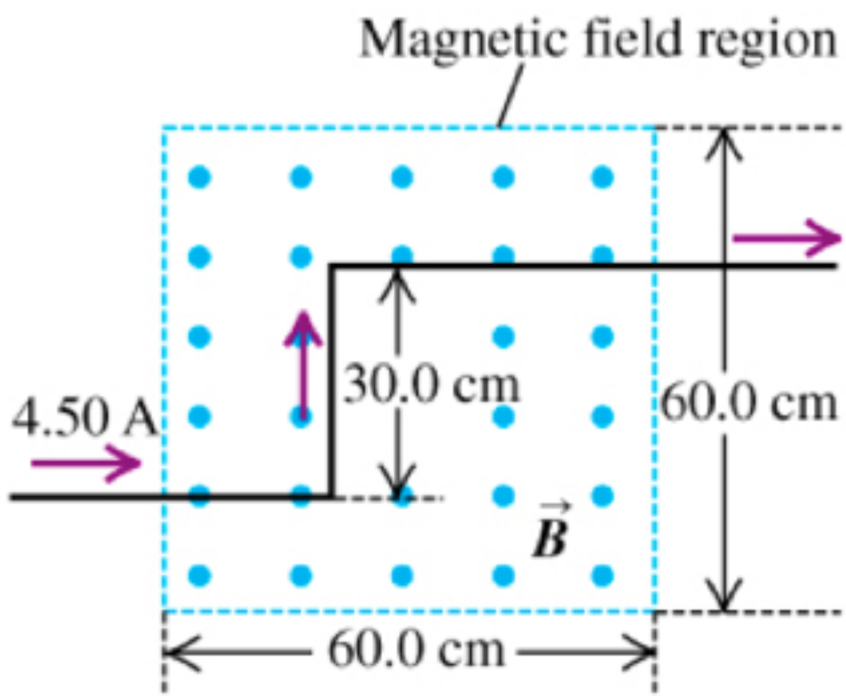
$$F_y = IRB\int_0^{\pi}\sin\theta d\theta$$



Exercise 27.35

A long wire carrying 4.50 A of current makes two 90 ° bends, as shown in the figure (Figure 1) . The bent part of the wire passes through a uniform 0.248 T magnetic field directed as shown in the figure and confined to a limited region of space.

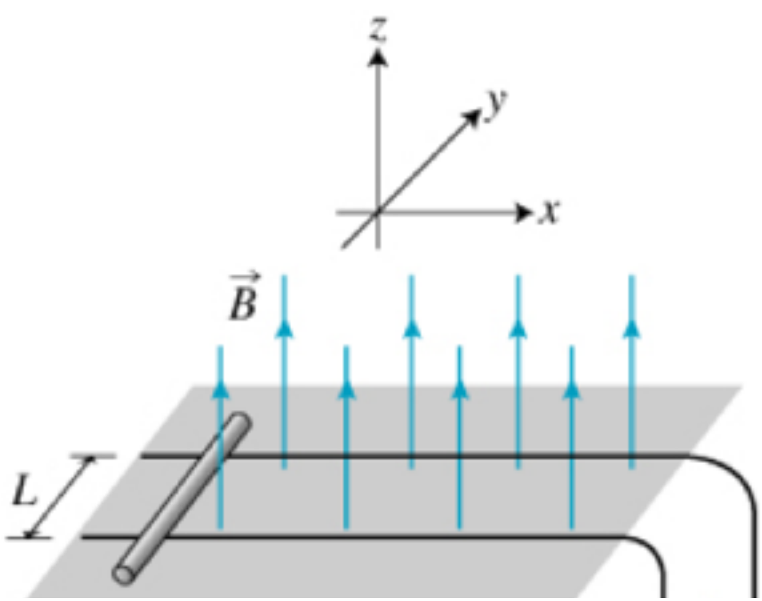
Figure 1 of 1



Rail Gun

A Rail Gun uses electromagnetic forces to accelerate a projectile to very high velocities. The basic mechanism of acceleration is relatively simple and can be illustrated in the following example. A metal rod of mass m and electrical resistance R rests on parallel

Figure 1 of 1



Which way should the battery go?

A

B

Magnetic Force and Torque on a Current Loop

- Let's look at the Net force and net torque on a current loop:

$$d\vec{F} = Id\vec{l} \times \vec{B} \Rightarrow$$

$$F = IaB \quad (\text{top and bottom})$$

$$F = IbB \quad (\text{sides})$$

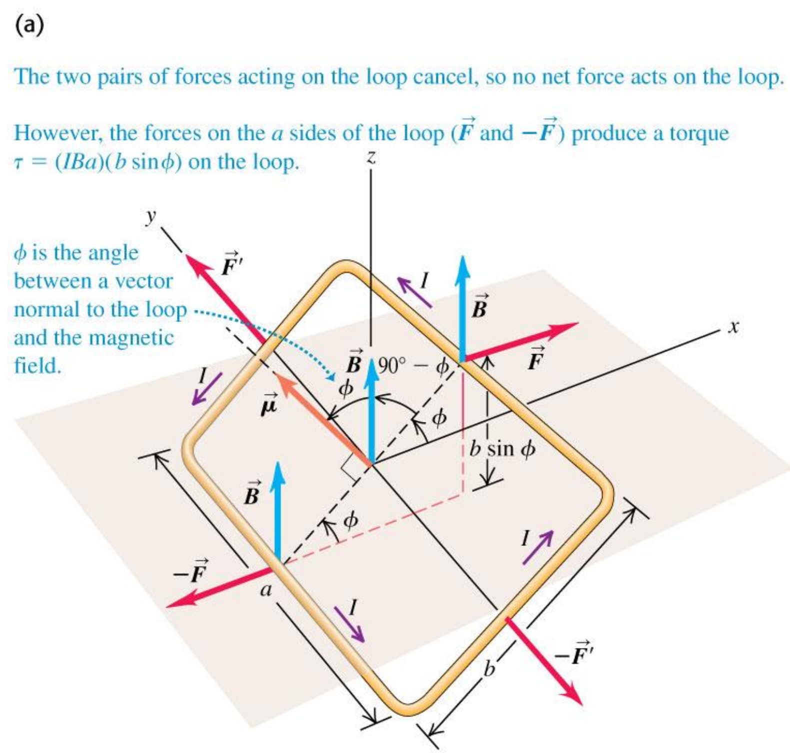
- But, the forces on opposite sides are opposing, so:

$$F_{Net} = 0$$

- Take the axis of rotation to be the y-axis, then the torque is:

$$|\vec{\tau}| = IaB\left(\frac{b}{2}\sin\phi\right) + IaB\left(\frac{b}{2}\sin\phi\right)$$

$$|\vec{\tau}| = IabB\sin\phi = IAB\sin\phi$$



Magnetic Torque on a Current Loop

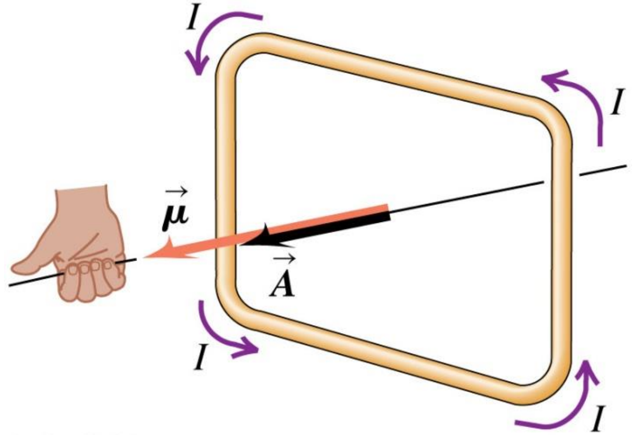
- We can rewrite this as:

$$|\vec{\tau}| = IAB\sin\phi = \mu B\sin\phi$$

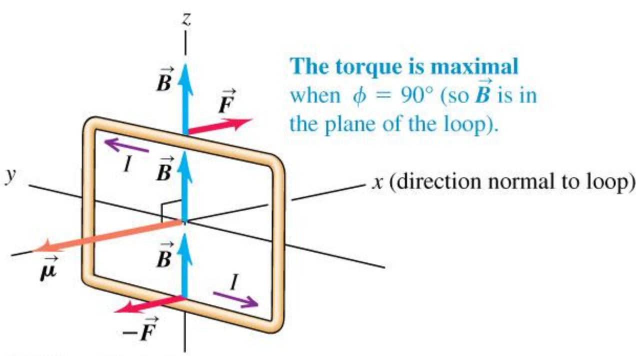
$$or$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

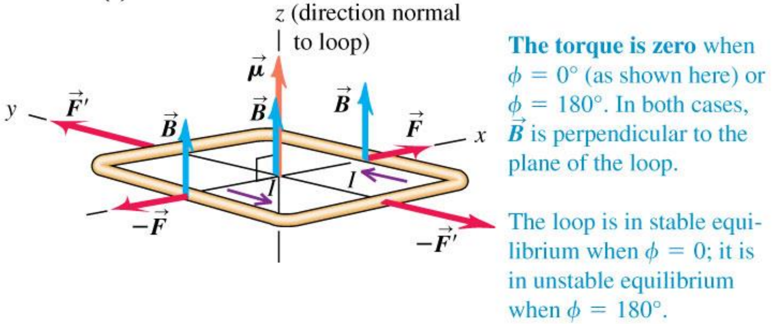
- Where the direction of the magnetic moment, μ, is given by the right-hand rule.



(b)

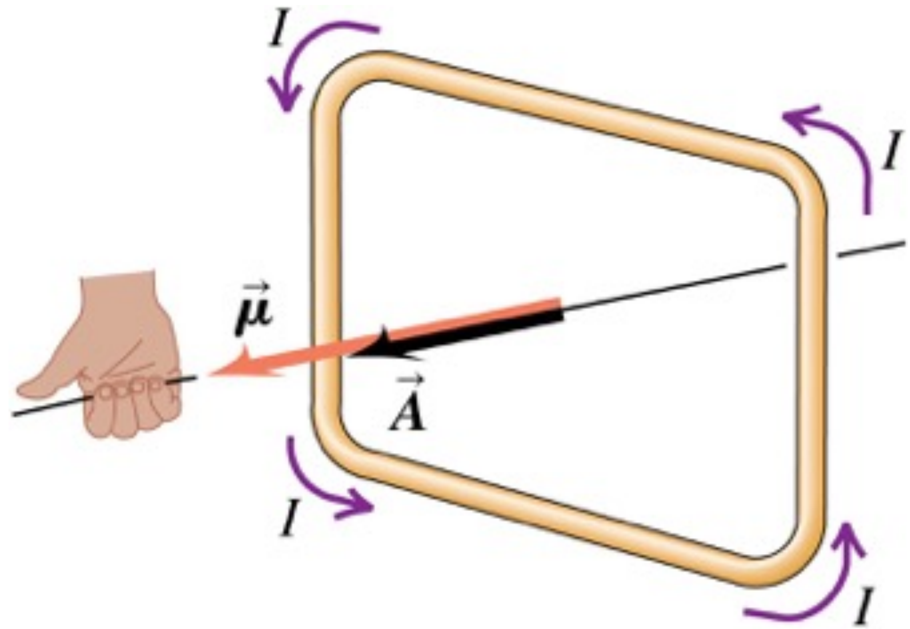


(c)



Force and torque on a current loop

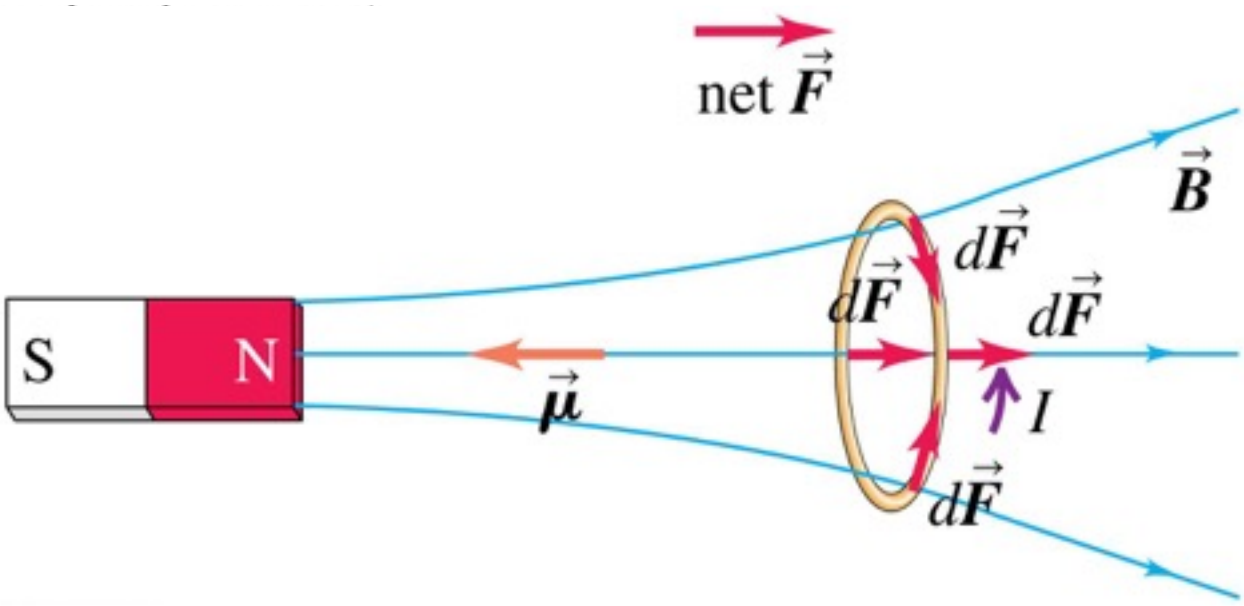
- The net force on a current loop in a *uniform* magnetic field is zero.
- We can define a magnetic moment $\boldsymbol{\mu}$ with magnitude IA, and direction as shown.
- The net torque on the loop is given by the vector product:



Vector magnetic torque on a current loop → $\vec{\tau} = \vec{\mu} \times \vec{B}$ (Magnetic dipole moment: $\vec{\mu}$; Magnetic field: $\vec{B}$)

Magnetic dipole in a nonuniform magnetic field

- A current loop with magnetic moment pointing to the left is in a magnetic field that decreases in magnitude to the right.
- When these forces are summed to find the net force on the loop, the radial components cancel so that the net force is to the right, away from the magnet.



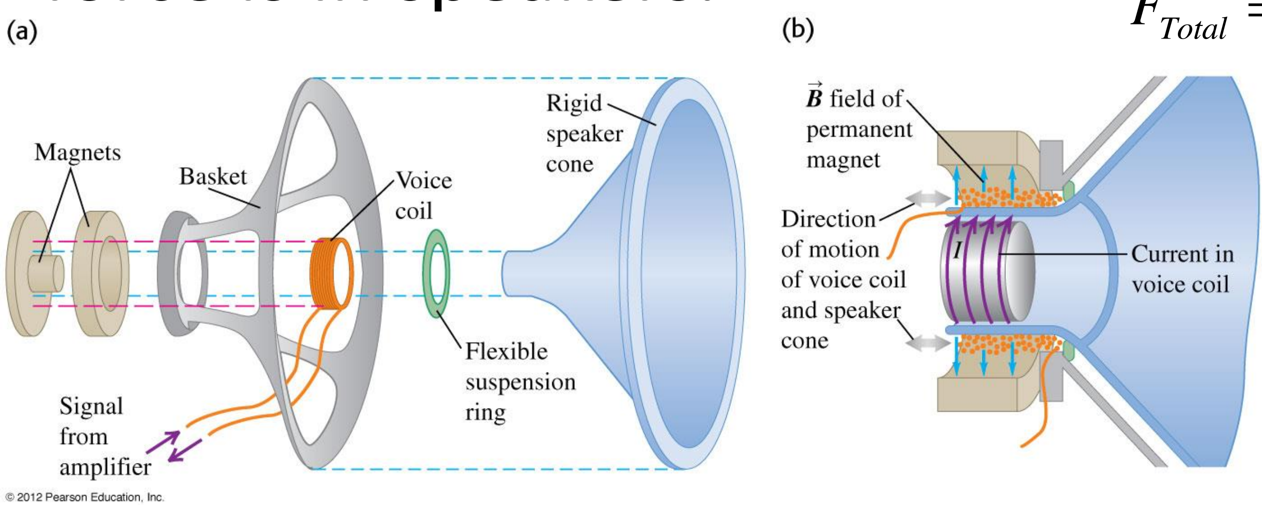
Magnetic Force on a Current

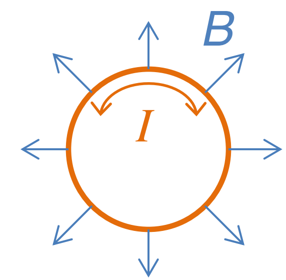
- One common use of this force is in speakers.

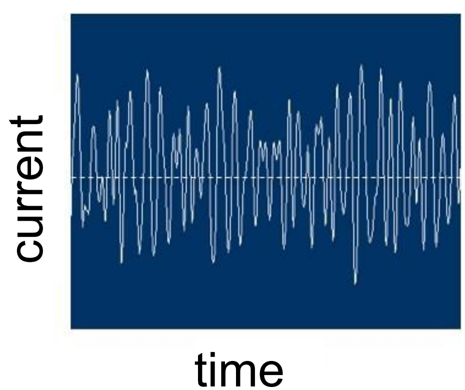
$$d\vec{F} = n\left(A\,dl\right)q\vec{v}_d \times \vec{B}$$

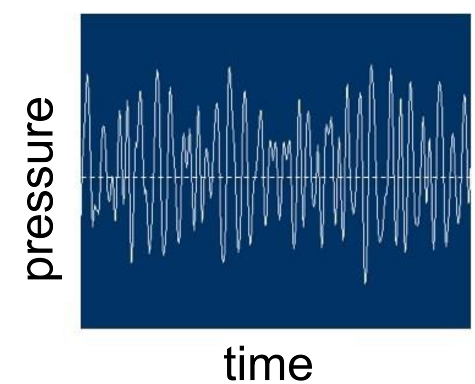
$$d\vec{F} = I\,d\vec{l} \times \vec{B} \Rightarrow$$

$$F_{Total} = IL_{Total}B$$









CPS 28-2

A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the *net magnetic force* on the loop is

A. perpendicular to the plane of the loop, in a direction given by a right-hand rule.

B. perpendicular to the plane of the loop, in a direction given by a left-hand rule.

C. in the same plane as the loop.

D. zero.

E. The answer depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.

CPS 28-3

A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the *net magnetic torque* on the loop

A. tends to orient the loop so that its plane is perpendicular to the direction of the magnetic field.

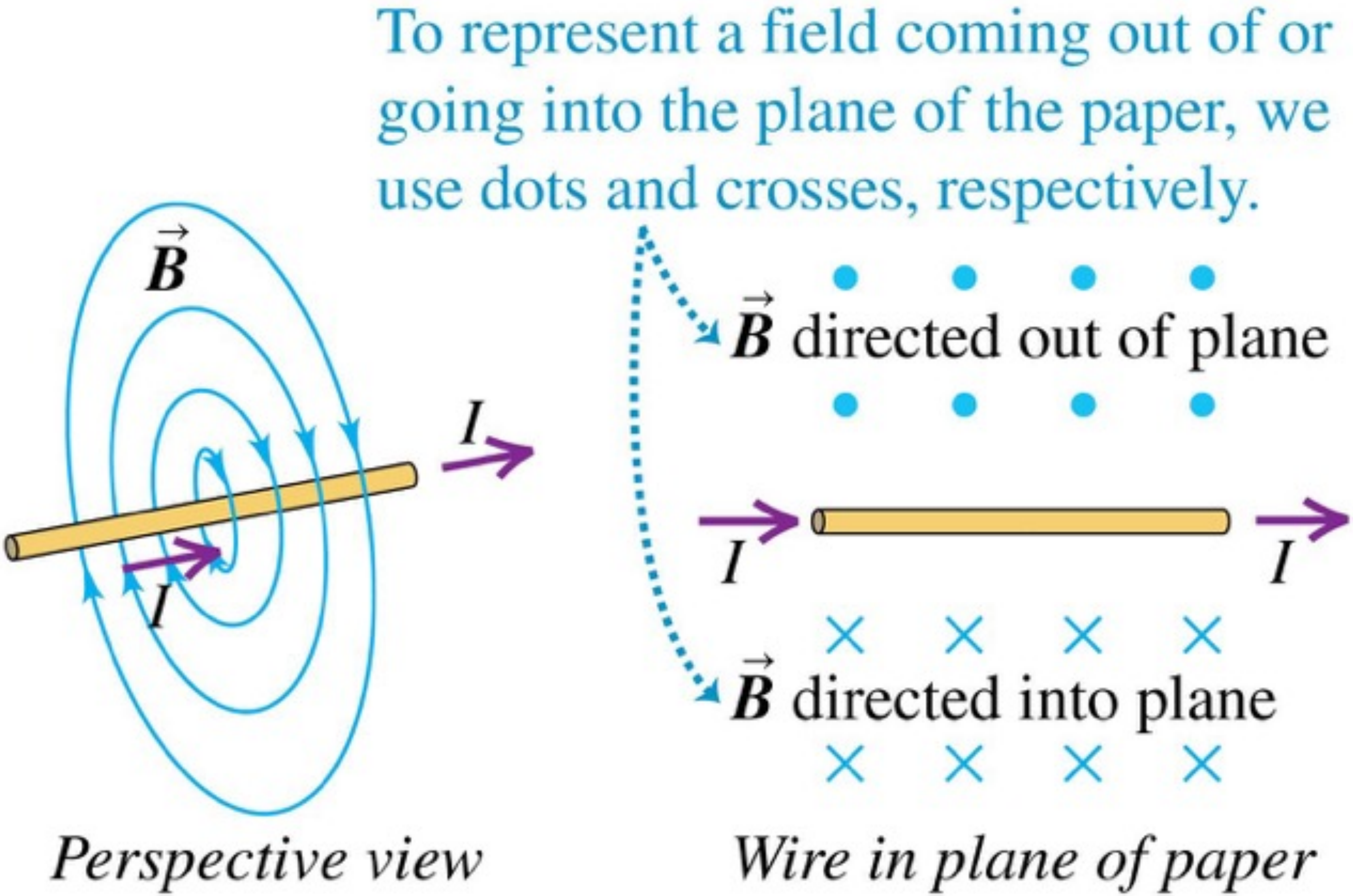
B. tends to orient the loop so that its plane is edge-on to the direction of the magnetic field.

C. tends to make the loop rotate around its axis.

D. is zero.

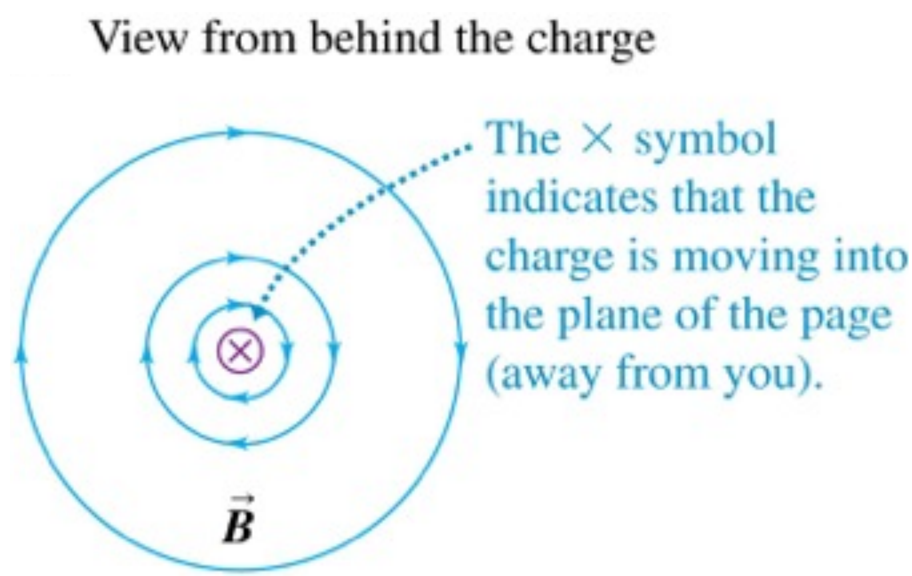
E. The answer depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.

Magnetic field of a straight current-carrying wire



The magnetic field of a moving charge

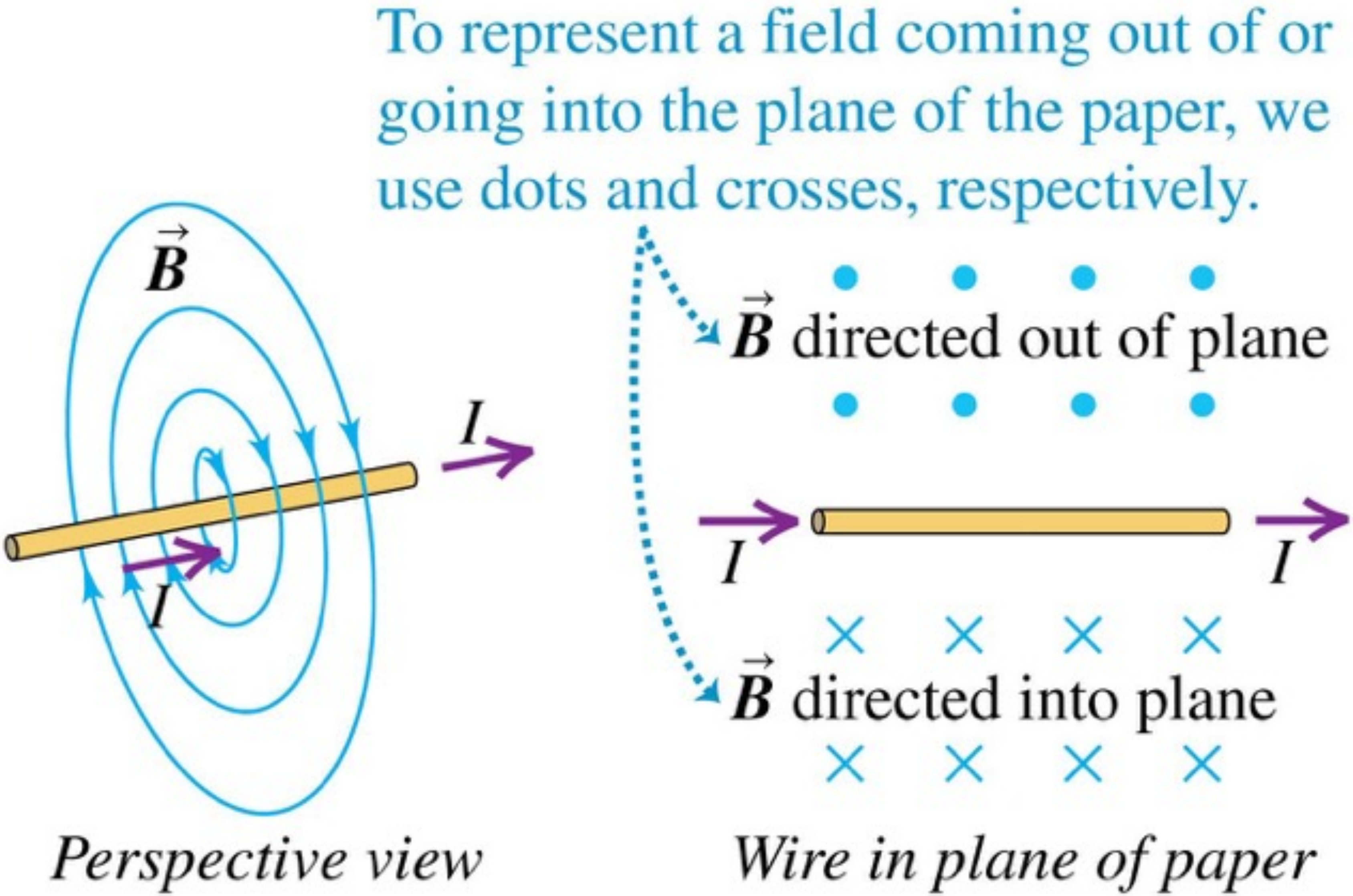
- A moving charge generates a magnetic field that depends on the velocity of the charge, and the distance from the charge.



$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

- $\vec{B}$: Magnetic field due to a point charge with constant velocity
- μ_0: Magnetic constant
- q: Charge
- $\vec{v}$: Velocity
- $\hat{r}$: Unit vector from point charge toward where field is measured
- r: Distance from point charge to where field is measured

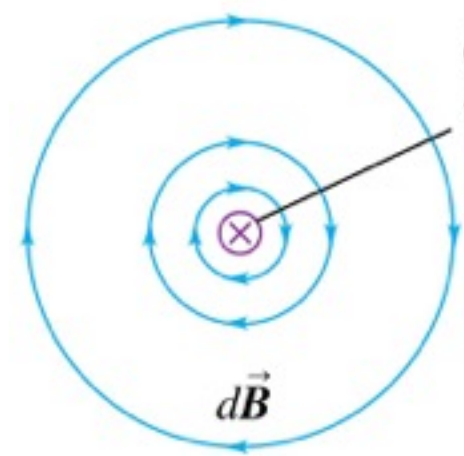
Magnetic field of a straight current-carrying wire



Magnetic field of a current element

- The total magnetic field of several moving charges is the vector sum of each field.
- The magnetic field caused by a short segment of a current-carrying conductor is found using the **law of Biot and Savart**:

View along the axis of the current element

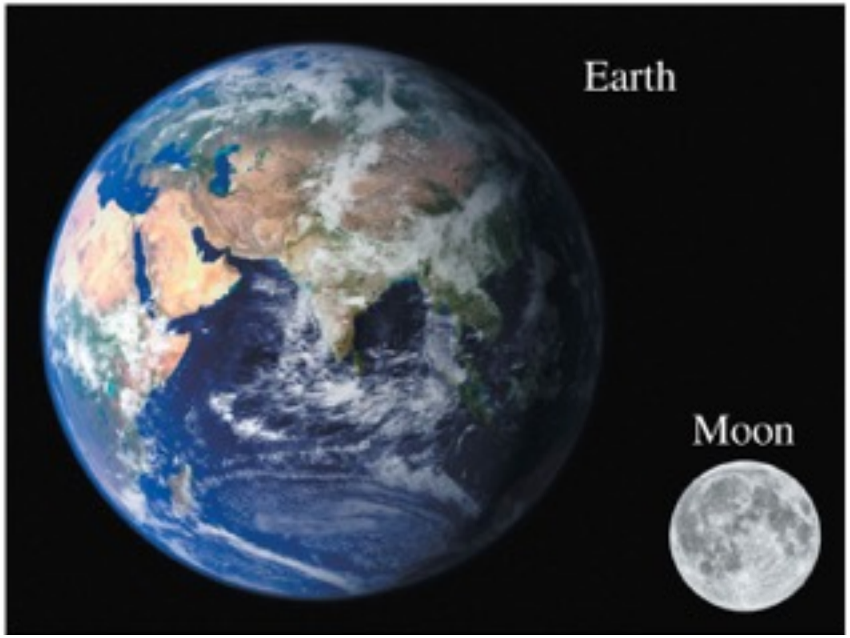


$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

- Magnetic field due to an infinitesimal current element: $d\vec{B}$
- Magnetic constant: μ_0
- Current: I
- Vector length of element (points in current direction): $d\vec{l}$
- Unit vector from element toward where field is measured: $\hat{r}$
- Distance from element to where field is measured: r

Currents and planetary magnetism

- The earth's magnetic field is caused by currents circulating within its molten, conducting interior.
- These currents are stirred by our planet's relatively rapid spin (one rotation per 24 hours).
- The moon's internal currents are much weaker; it is much smaller than the earth, has a predominantly solid interior, and spins slowly (one rotation per 27.3 days).
- Hence the moon's magnetic field is only about 10^{-4} as strong as that of the earth.



Magnetic fields of current-carrying wires

- Computer cables, or cables for audio-video equipment, create little or no magnetic field.
- This is because within each cable, closely spaced wires carry current in both directions along the length of the cable.
- The magnetic fields from these opposing currents cancel each other.

Force between parallel conductors

- The magnetic field of the lower wire exerts an *attractive* force on the upper wire.
- If the wires had currents in *opposite* directions, they would *repel* each other.