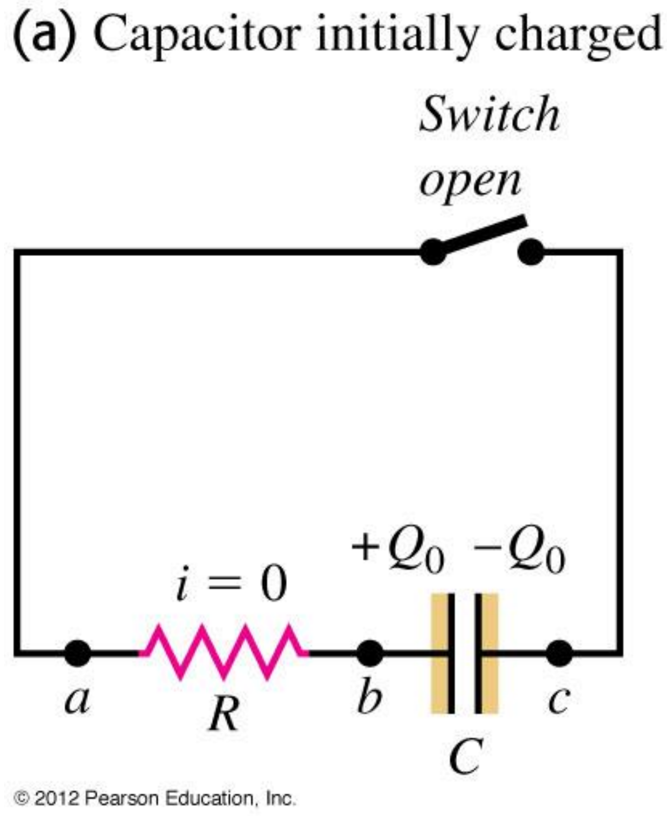
Lecture 27

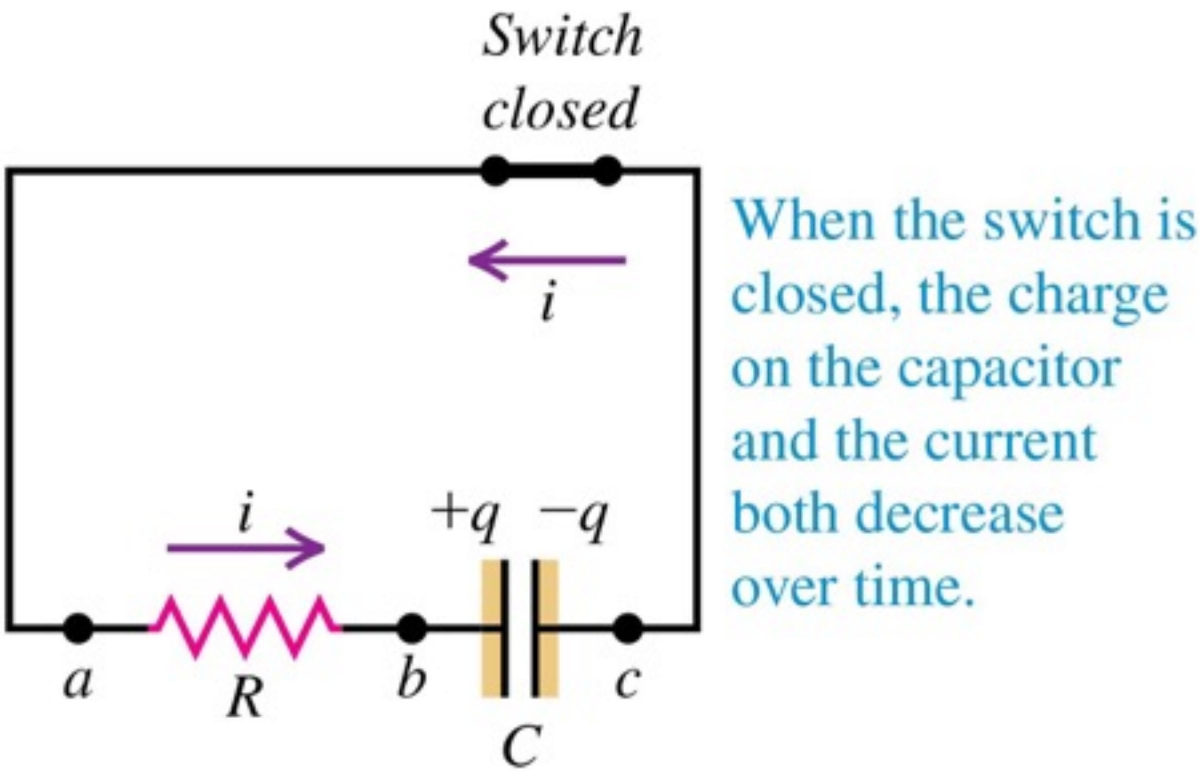
PHYC 161 Fall 2016

An R-C Circuit (Discharging)

- Now, let's start with a fully charged capacitor in an open circuit with no battery.
- Initially, there is charge Q_0 on the capacitor.
- At some time, t=0, we will close the switch, and charge will begin to flow around the circuit, through the resistor and back to the other side of the capacitor.

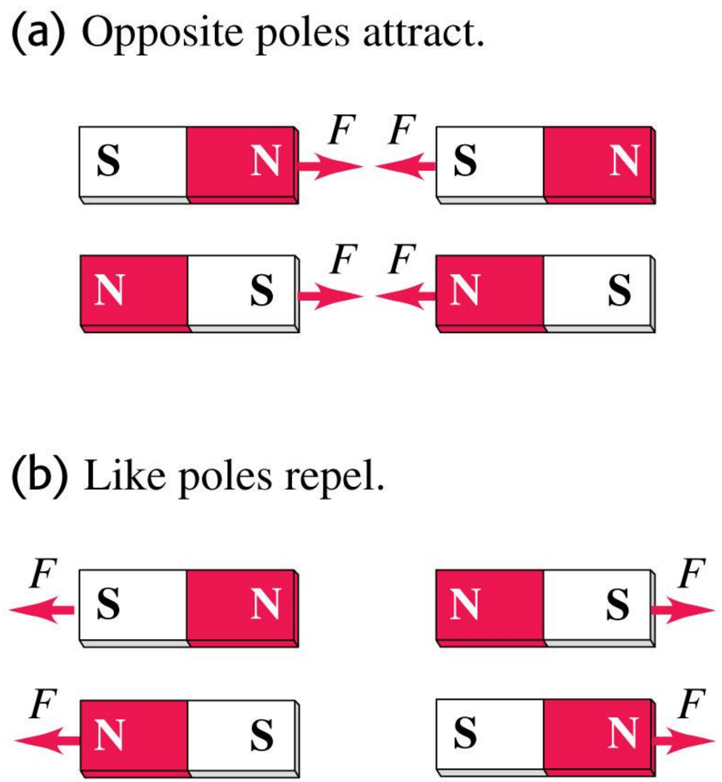


- At some initial time $t = 0$ we close the switch, allowing the capacitor to discharge through the resistor.
- As t increases, the magnitude of the current decreases, while the charge on the capacitor also decreases.



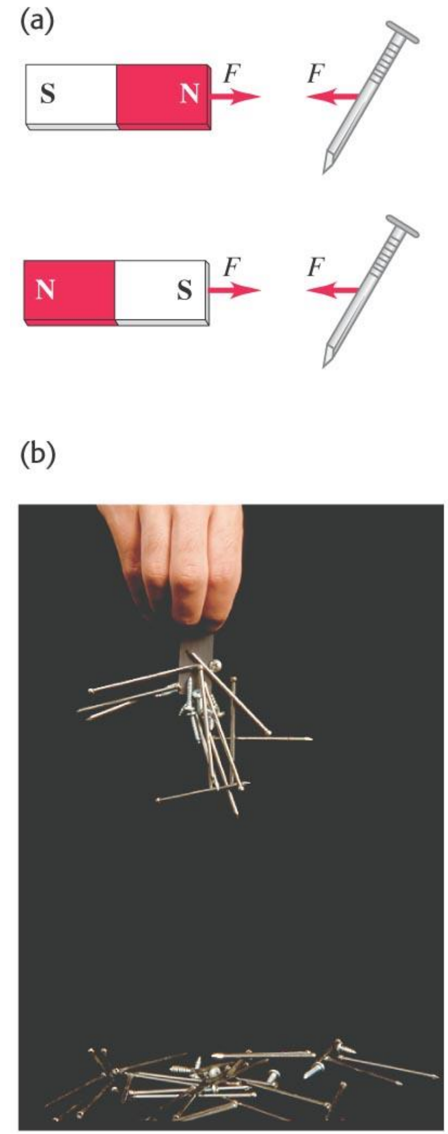
Magnets

- We can begin our exploration by reviewing the most basic experimental observations.
  - Magnets have two “poles”.
  - Opposite poles attract, and like poles repel.
- This makes us wonder if perhaps magnetic forces are just somehow caused by electric fields (since we have seen the whole like charges repel thing before).
- Well, it turns out that there is a relationship, but it is much more complicated.



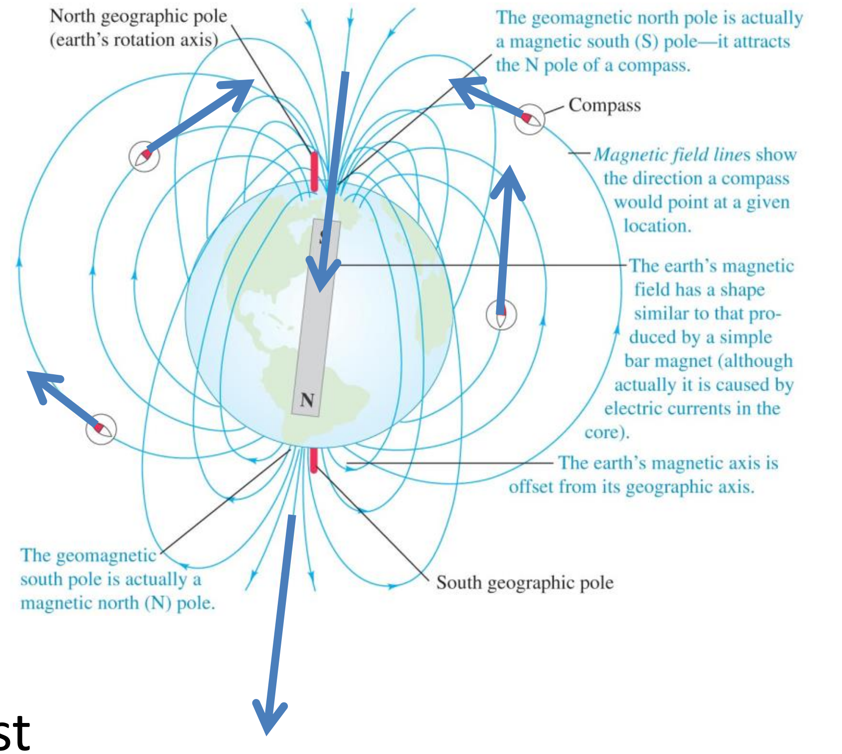
Magnets and Non-magnets

- But, if your refrigerator door isn't made of a magnet, why do magnets stick to it?
- In a similar way that electric fields can polarize neutral objects and create a force…
- We will return to this later and be more quantitative.



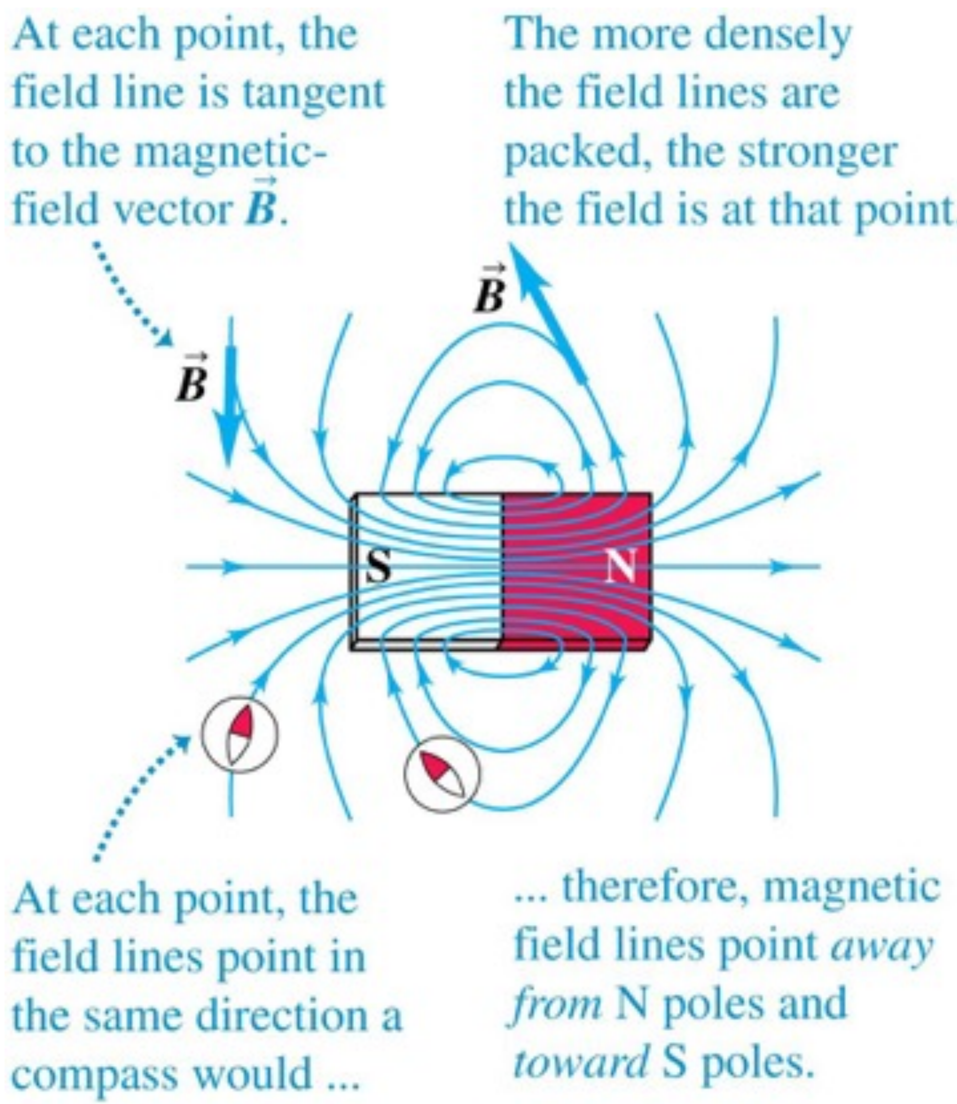
Magnetic Fields and Field Lines

- Magnetic fields are vector fields – at every point in space there is a direction and magnitude for the magnetic field.
- The earth has a magnetic field, and a compass will align itself to the direction of the field.
- Magnetic field lines follow the direction of the field (the field is always tangential to the lines), and the density of lines (how closely spaced they are) is an indication of the field strength.
- Remember that the field IS NOT THE LINES! The field is a set of vectors at every point in space. The lines are just a way of representing the field.



Magnetic field lines

- We can represent any magnetic field by **magnetic field lines**.
- We draw the lines so that the line through any point is tangent to the magnetic field vector at that point.
- Field lines never intersect.



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At which point is the magnitude of the magnetic field the largest?

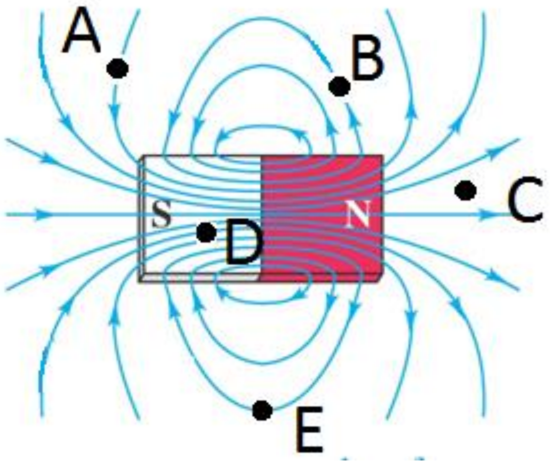
A.

B.

C.

D

E.

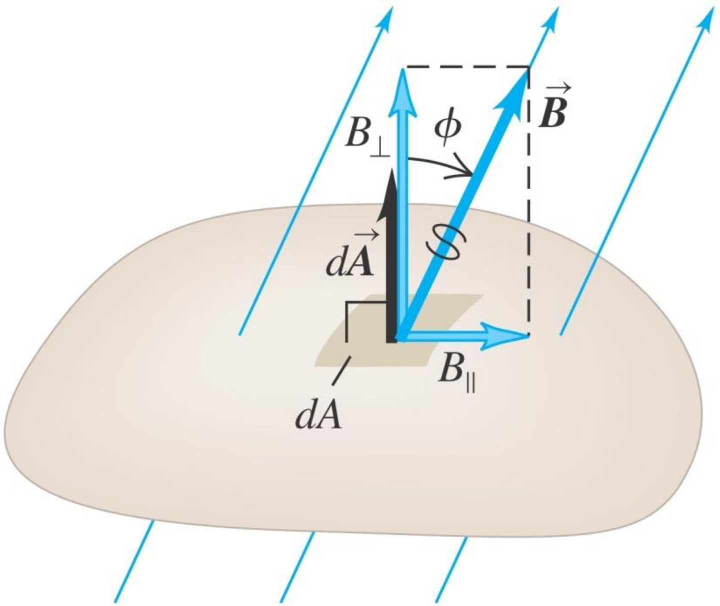


Magnetic Flux

- Yes, back to flux, which means back to surface integrals.
- We can define the magnetic flux in the same way that we defined the electric flux:

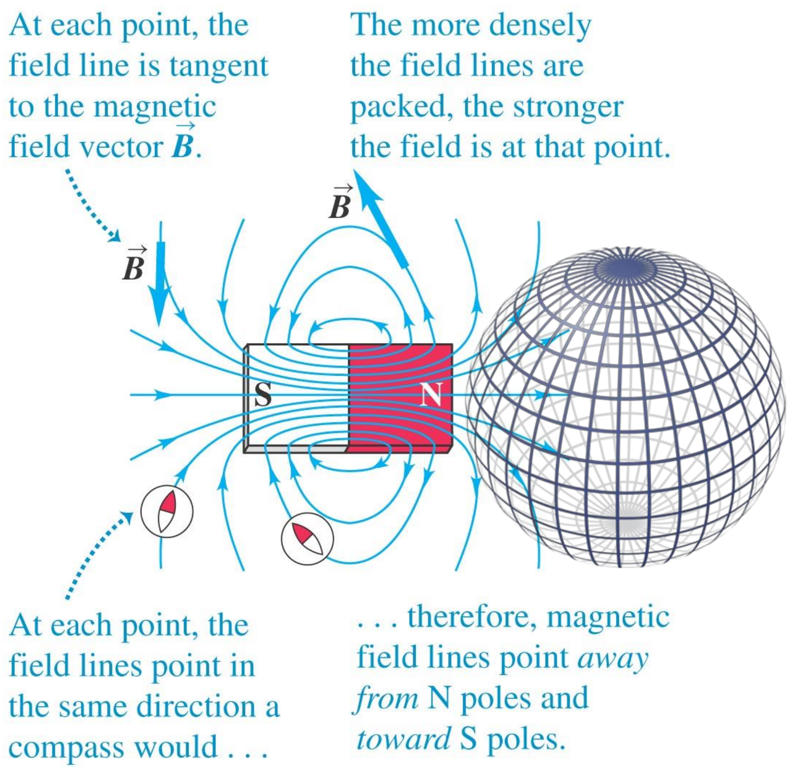
$$\Phi_B = \int_{Surface} \vec{B} \cdot d\vec{A}$$

- Let me go through an example or two…



Magnetic Field “Sources”

- So, if there is no source or sink of magnetic fields, there can be no *NET* flux through a closed surface (every field line that enters a closed surface must eventually exit the closed surface)!

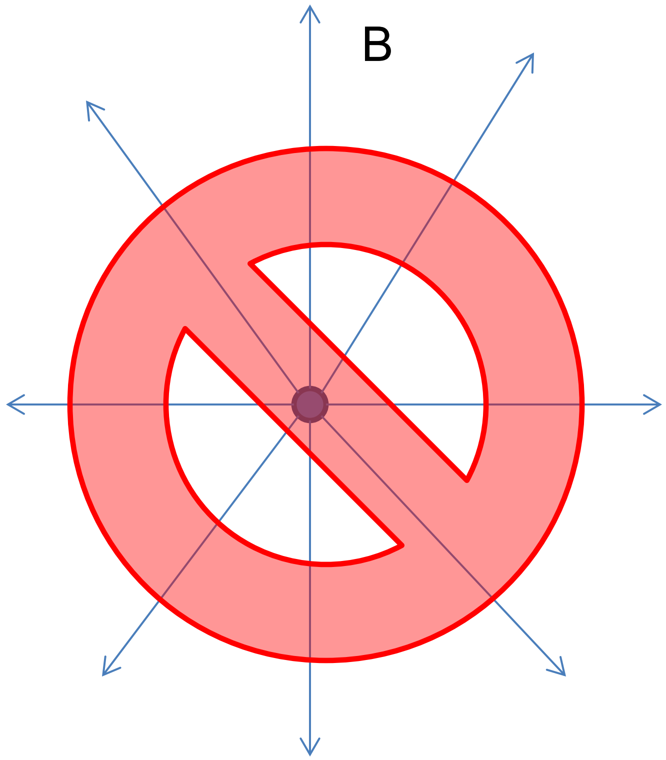


Gauss's Law for Magnetic Fields

- Then, given what we understand about Gauss's Law for the electric field, we can deduce that:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- In other words, there is no magnetic charge (magnetic monopoles).



Units of magnetic field and magnetic flux

- The SI unit of **magnetic field** B is called the tesla (1 T), in honor of Nikola Tesla:

$$1 \text{ tesla} = 1 \text{ T} = 1 \text{ N/A} \cdot \text{m}$$

- Another unit of B, the gauss ($1 \text{ G} = 10^{-4} \text{ T}$), is also in common use.
- The magnetic field of the earth is on the order of 10^{-4} T or 1 G.
- The SI unit of **magnetic flux** Φ_B is called the weber (1 Wb), in honor of Wilhelm Weber:

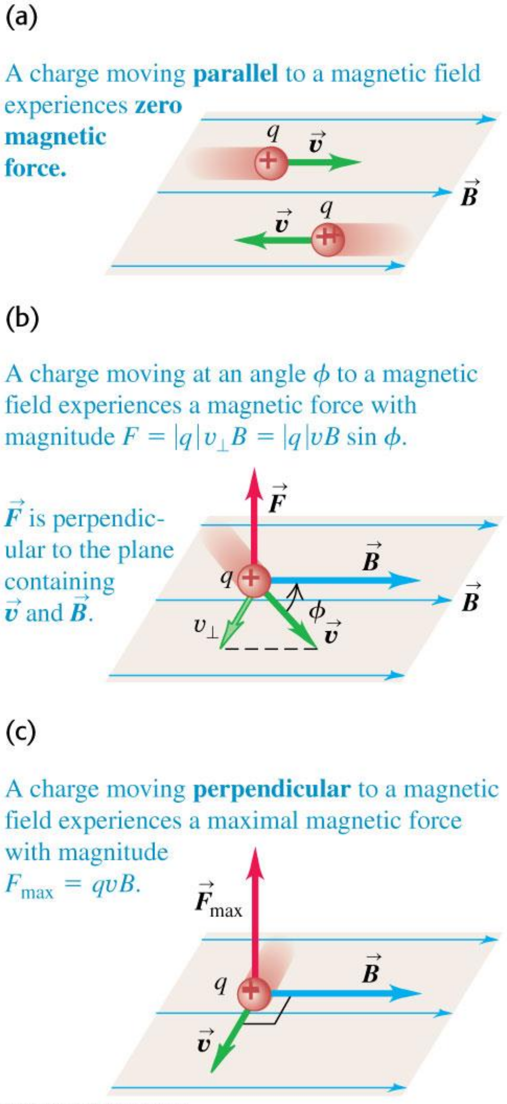
$$1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$$

Magnetic Force

- Magnetic fields have an affect on *moving* charges.

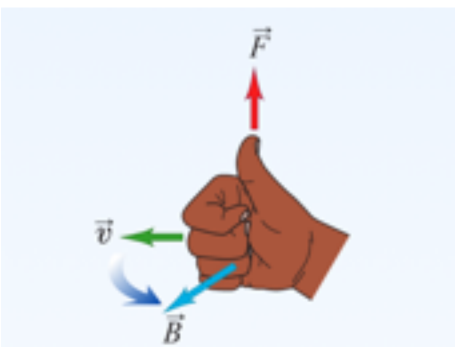
$$\vec{F} = q\vec{v} \times \vec{B}$$

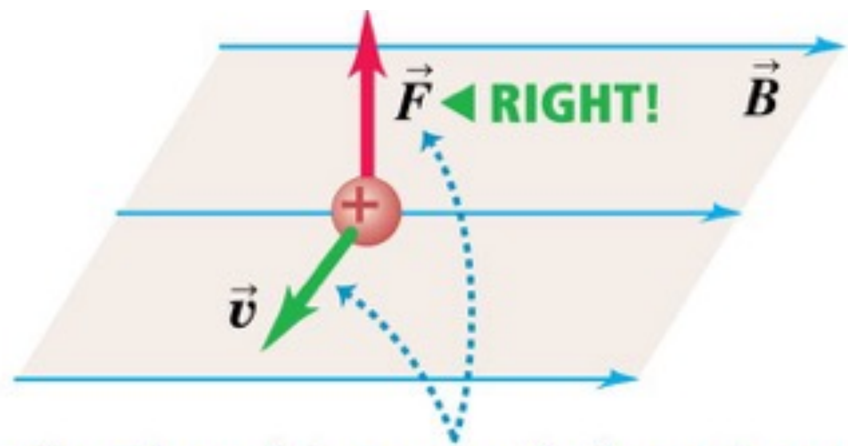
- Since it has been a while since we dealt with cross products, let's take a few minutes to review what this means.
- There is no force on a charge moving in the same direction as the magnetic field.
- The force on a moving charge in a magnetic field is perpendicular to both the field direction and the direction of motion.
- The direction of the force on a negative charge is opposite to that on a positive charge moving in the same direction.
- Since the force is perpendicular to the direction of motion, no work is done by the magnetic force.



Magnetic field lines are *not* lines of force

- It is important to remember that magnetic field lines are *not* lines of magnetic force.
- The force on a charged particle is not along the direction of a field line.



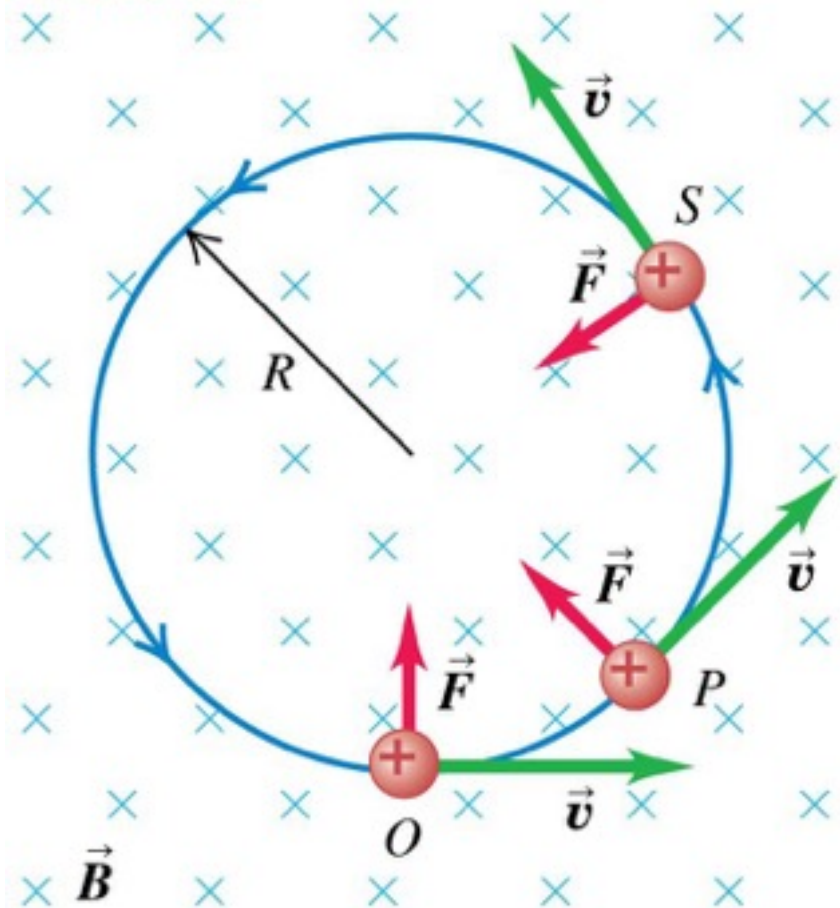


The direction of the magnetic force depends on the velocity $\vec{v}$, as expressed by the magnetic force law $\vec{F} = q\vec{v} \times \vec{B}$.

Motion of charged particles in a magnetic field

- When a charged particle moves in a magnetic field, it is acted on by the magnetic force.
- The force is always perpendicular to the velocity, so it cannot change the speed of the particle.

A charge moving at right angles to a uniform $\boldsymbol{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.



Consequences and Applications

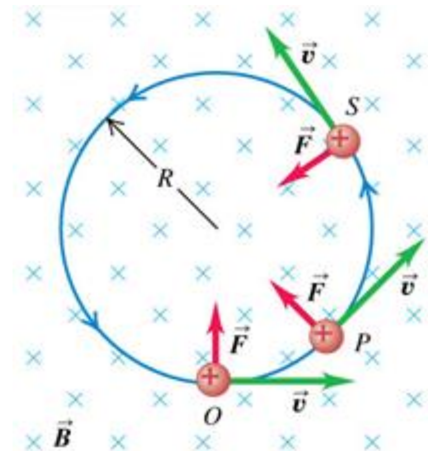
- A beam of charged particles will move in a circle at constant speed when they are sent into it perpendicular to a magnetic field.

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a} \Rightarrow$$

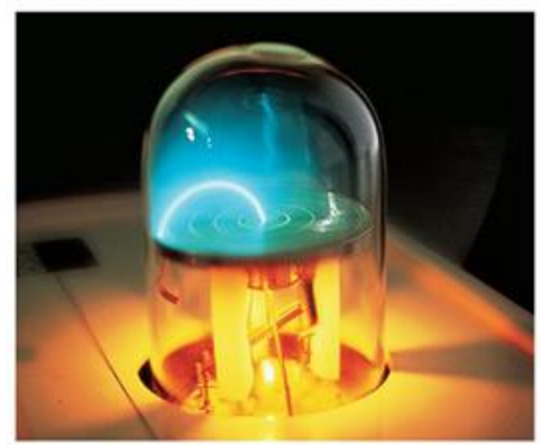
$$|\vec{a}| = \frac{|q\vec{v}\vec{B}|}{m} = \frac{v^2}{R} \Rightarrow$$

$$R = \frac{mv}{qB}$$

$$\left[\frac{kg \cdot \frac{m}{s}}{C \cdot T}\right] = \left[\frac{kg \cdot \frac{m}{s}}{C \cdot \frac{N}{A \cdot m}}\right] = \left[\frac{kg \cdot \frac{m}{s}}{C \cdot \frac{\frac{kg \cdot m}{s^2}}{\frac{C}{s} \cdot m}}\right] = [m]$$



(b) An electron beam (seen as a white arc) curving in a magnetic field



HW - Cyclotron

Charge Moving in a Cyclotron Orbit

Description: General problem, which goes through charged-particle motion perpendicular to a magnetic field; reviews cyclotron frequency derivation. Goes through kinematics and frequency invariance. rhr_B convention used.

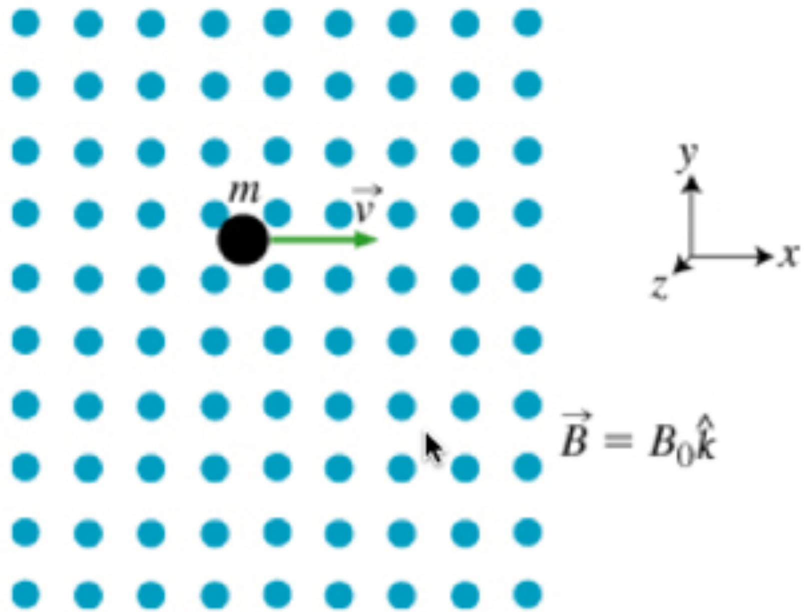
Learning Goal:

To understand why charged particles move in circles perpendicular to a magnetic field and why the frequency is an invariant.

A particle of charge q and mass m moves in a region of space where there is a uniform magnetic field $\vec{B} = B_0\hat{k}$ (i.e., a magnetic field of magnitude B_0 in the +z direction). In this problem, neglect any forces on the particle other than the magnetic

Which way is $\vec{F}$?

a) + x-hat
b) - x-hat
c) + y hat
d) - y hat
e) - k hat



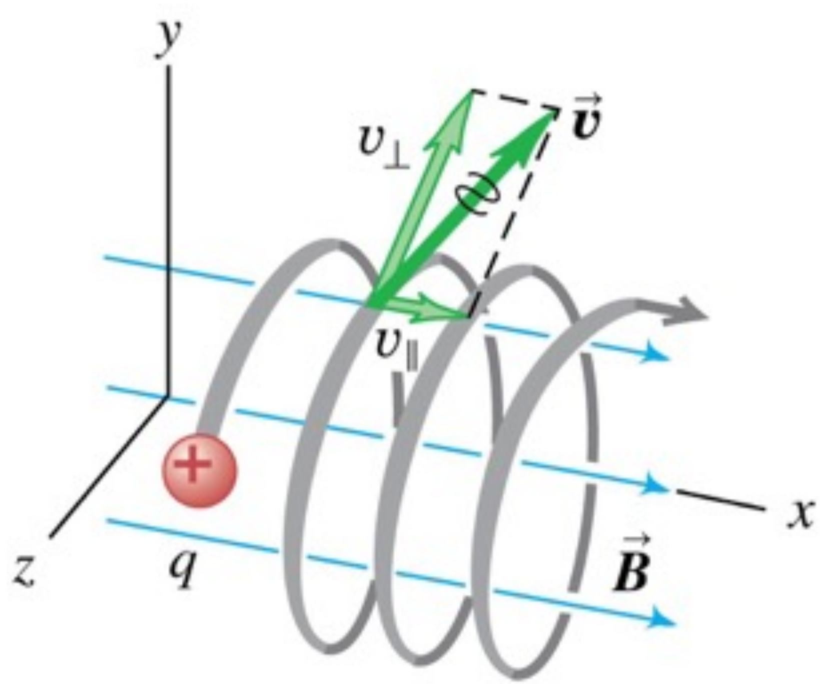
Part A

At a given moment the particle is moving in the +x direction (and the magnetic field is always in the +z direction). If q is positive, what is the direction of the force on the particle due to the magnetic field?

Helical motion

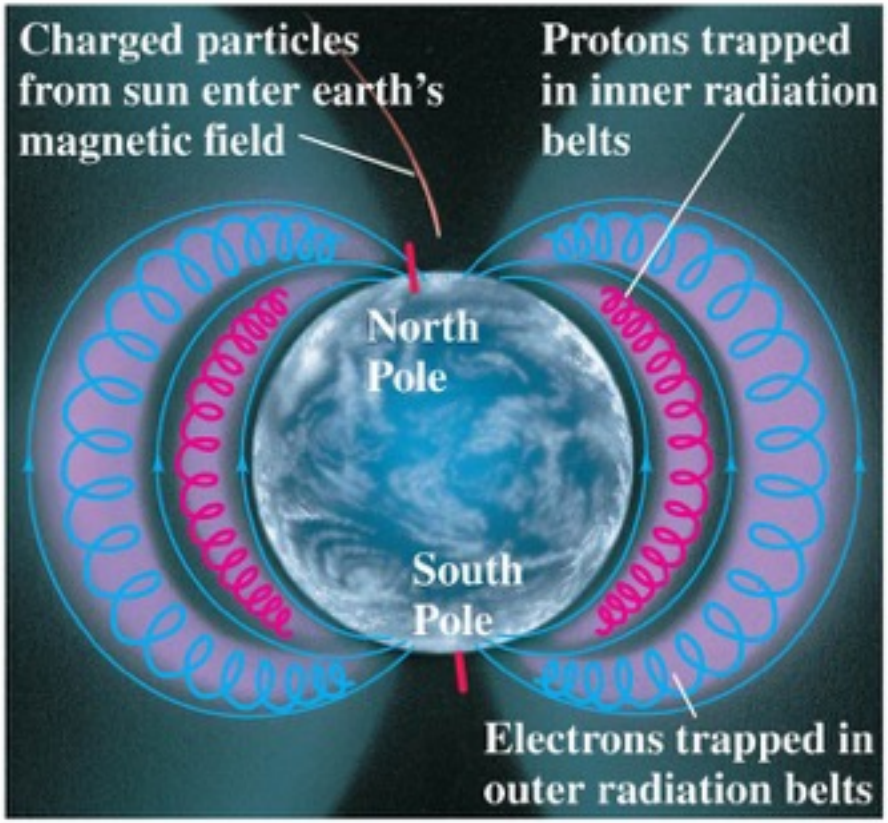
- If the particle has velocity components parallel to and perpendicular to the field, its path is a *helix*.
- The speed and kinetic energy of the particle remain constant.

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a helical path.

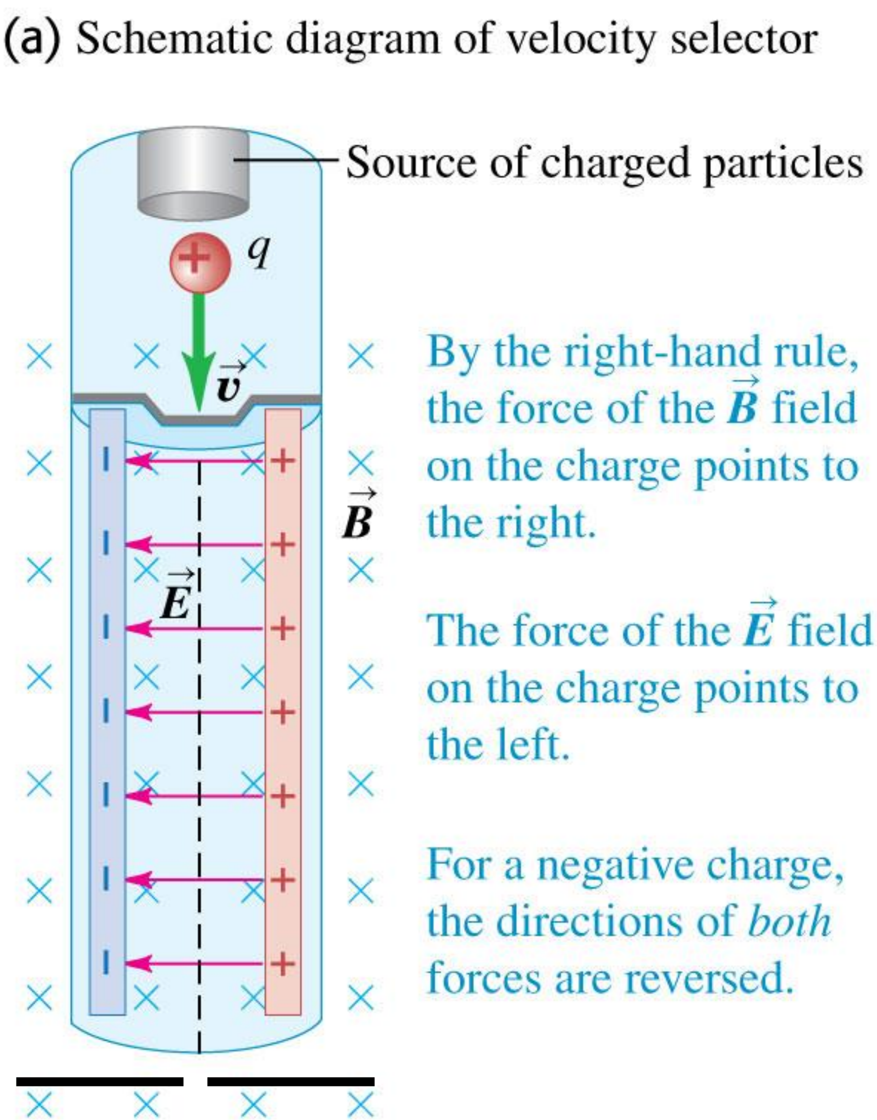


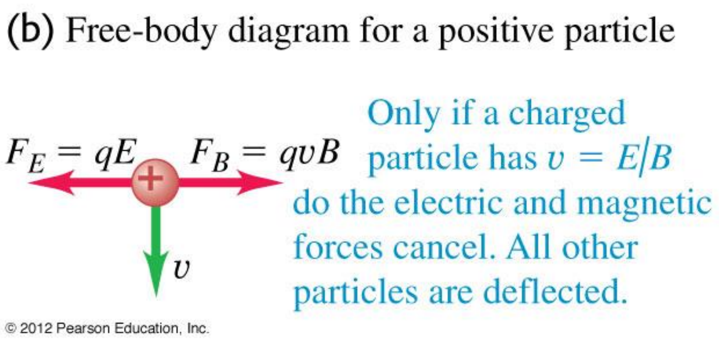
The Van Allen radiation belts

- Near the poles, charged particles from these belts can enter the atmosphere, producing the aurora borealis ("northern lights") and aurora australis ("southern lights").



When both electric and magnetic fields present





$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{F}_E = q\vec{E}$$

HW Velocity Filter

- Only charged particles of a certain velocity pass through

Electromagnetic Velocity Filter

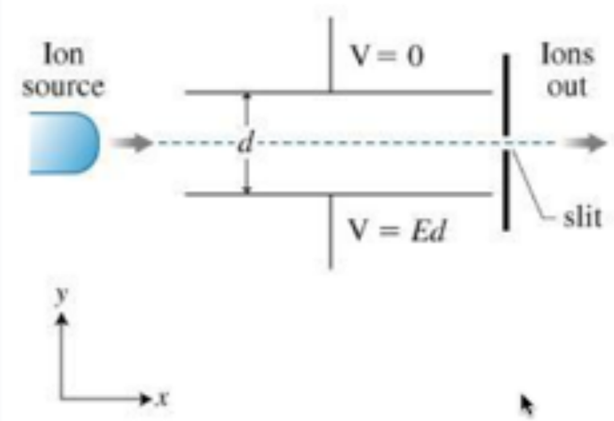
Description: Find the velocity of a charged particle that is undeflected in crossed electric and magnetic fields. Look at relation between mass, charge, and acceleration as charged particle traverses the fields.

When a particle with charge q moves across a magnetic field of magnitude B, it experiences a force to the side. If the proper electric field $\vec{E}$ is simultaneously applied, the electric force on the charge will be in such a direction as to cancel the magnetic force with the result that the particle will travel in a straight line. The balancing condition provides a relationship involving the velocity $\vec{v}$ of the particle. In this problem you will figure out how to arrange the fields to create this balance and then determine this relationship.

Part A

Consider the arrangement of ion source and electric field plates shown in the figure. The ion source sends particles with velocity $\vec{v}$ along the positive x axis. They encounter electric field plates spaced a distance d apart that generate a uniform electric field of magnitude E in the $+y$ direction. To cancel the resulting electric force with a magnetic force, a magnetic field (not shown) must be added in which direction? Using the right-hand rule, you can see that the positive z axis is directed out of the screen.

Choose the direction of $\hat{B}$.



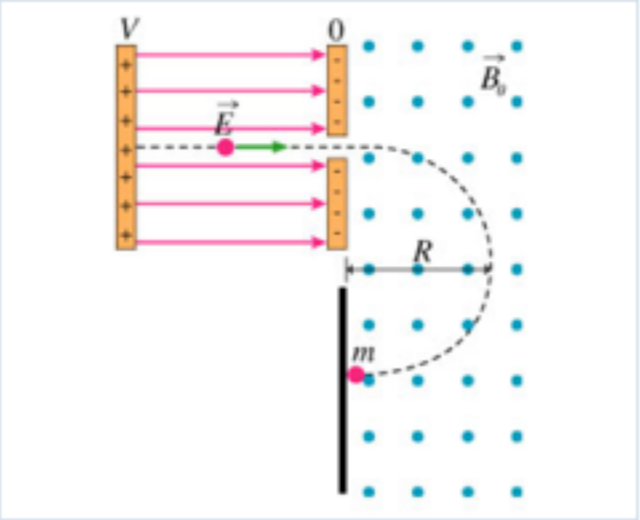
HW - Mass Spectrometer

Mass Spectrometer

Description: Potential V accelerates charged particle to speed u, then measure radius in magnetic field to find m/q.

J. J. Thomson is best known for his discoveries about the nature of cathode rays. Another important contribution of his was the invention, together with one of his students, of the mass spectrometer. The ratio of mass m to (positive) charge q of an ion may be accurately determined in a mass spectrometer. In essence, the spectrometer consists of two regions: one that accelerates the ion through a potential difference V and a second that measures its radius of curvature in a perpendicular magnetic field as shown in .

The ion begins at potential V and is accelerated toward zero potential. When the particle exits the region with the electric field it will have obtained a speed u.



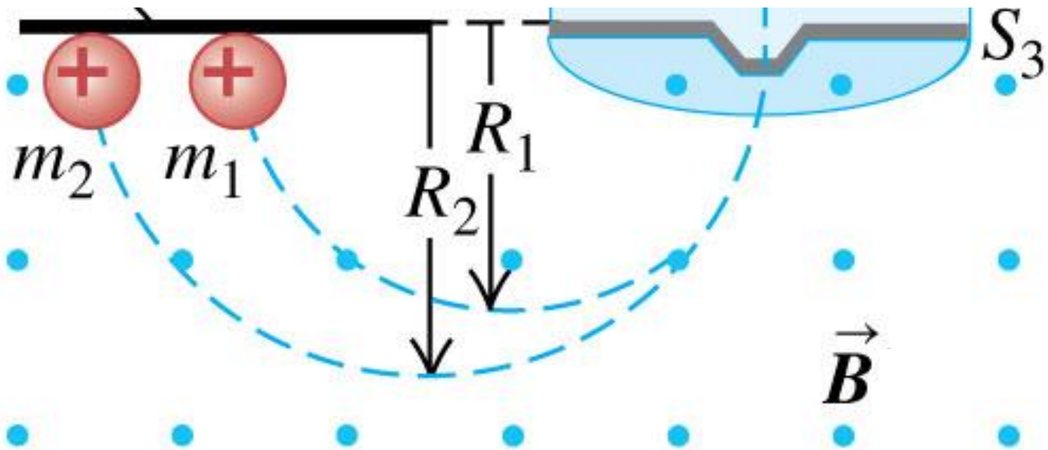
Part A

With what speed u does the ion exit the acceleration region?

Find the speed in terms of m, q, V, and any constants.

Mass-spec

- R=mv/qB



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

Magnetic Force on a Current Element

- In electronics, we rarely deal with "beams" of charged particles, but rather deal with current in a wire.
- But current is just moving charged particles.

$$\vec{F}_B = q\vec{v}_d \times \vec{B}$$

- The total force on the wire segment of length *dl* is just the sum of the forces on all the moving charges:

$$d\vec{F} = n\left(V\right)q\vec{v}_d \times \vec{B}$$

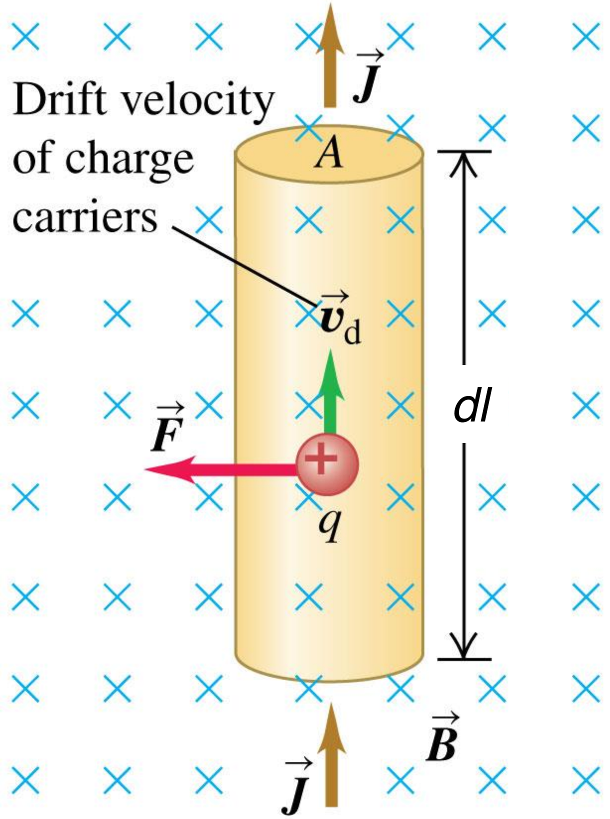
$$= n\left(Adl\right)q\vec{v}_d \times \vec{B}$$

- But nqv_d is just the current density, and the current density times the area is just the current:

$$d\vec{F} = n\left(Adl\right)q\vec{v}_d \times \vec{B}$$

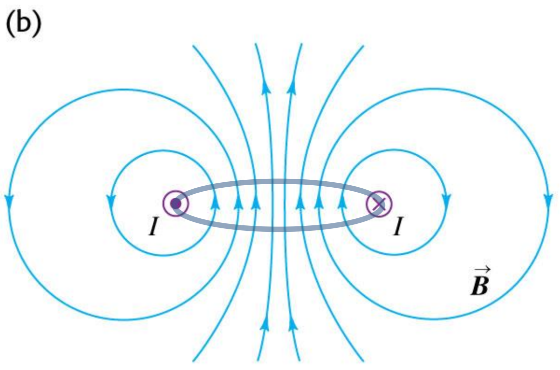
$$d\vec{F} = Id\vec{l} \times \vec{B}$$

- Where we have designated the direction of dl to be in the same direction as the current.



Magnetic Field "Sources"

- The smallest source of a magnetic field is a magnetic dipole – there is still no beginning or end to the magnetic field lines!
- As we will learn in the next chapter, the source of magnetic fields is a moving charge, and in a magnet, it is the electrons in the atoms that cause the field.



Magnetic field of a straight current-carrying wire