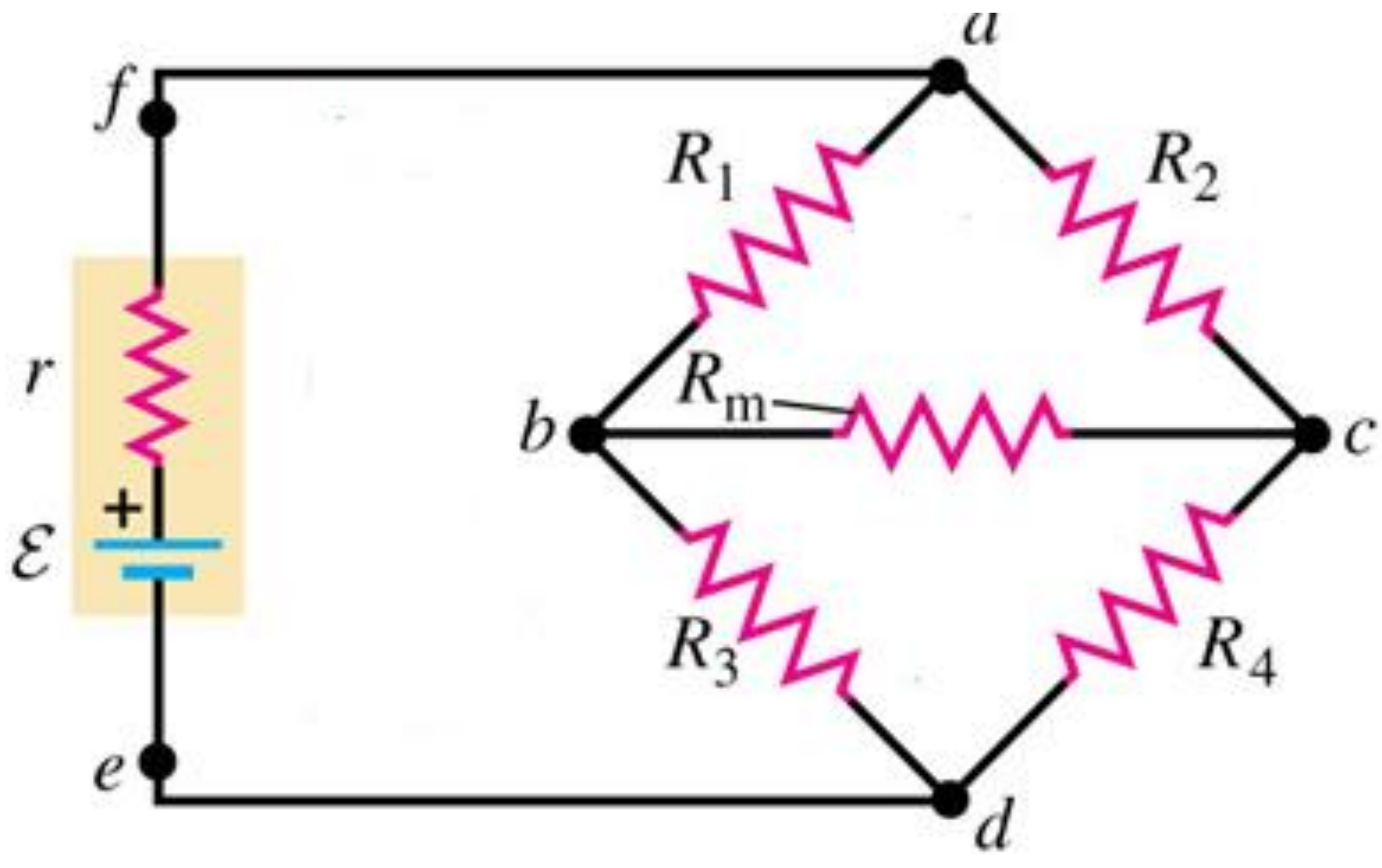
Lecture 26

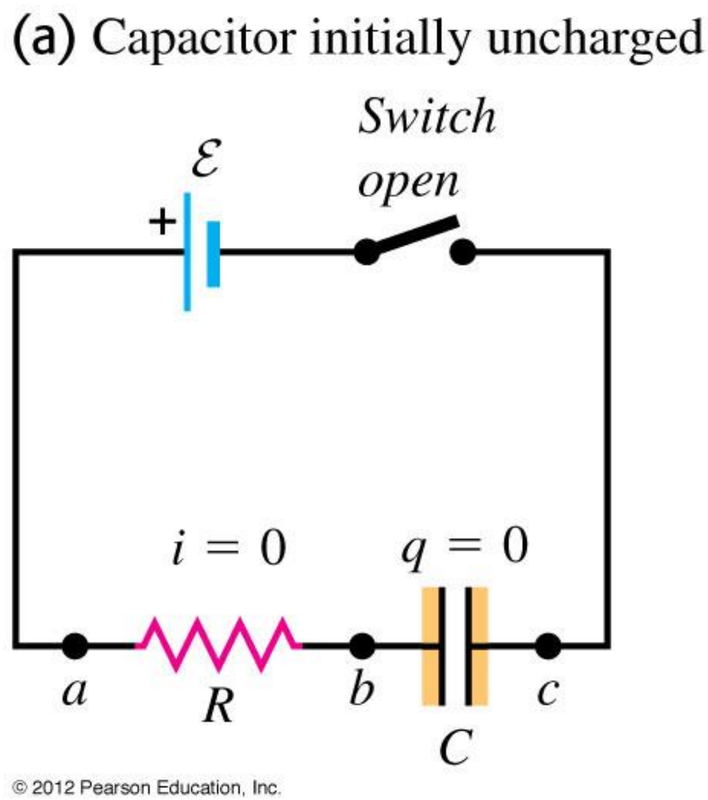
PHYC 161 Fall 2016

Back to our old “bridge” circuit:

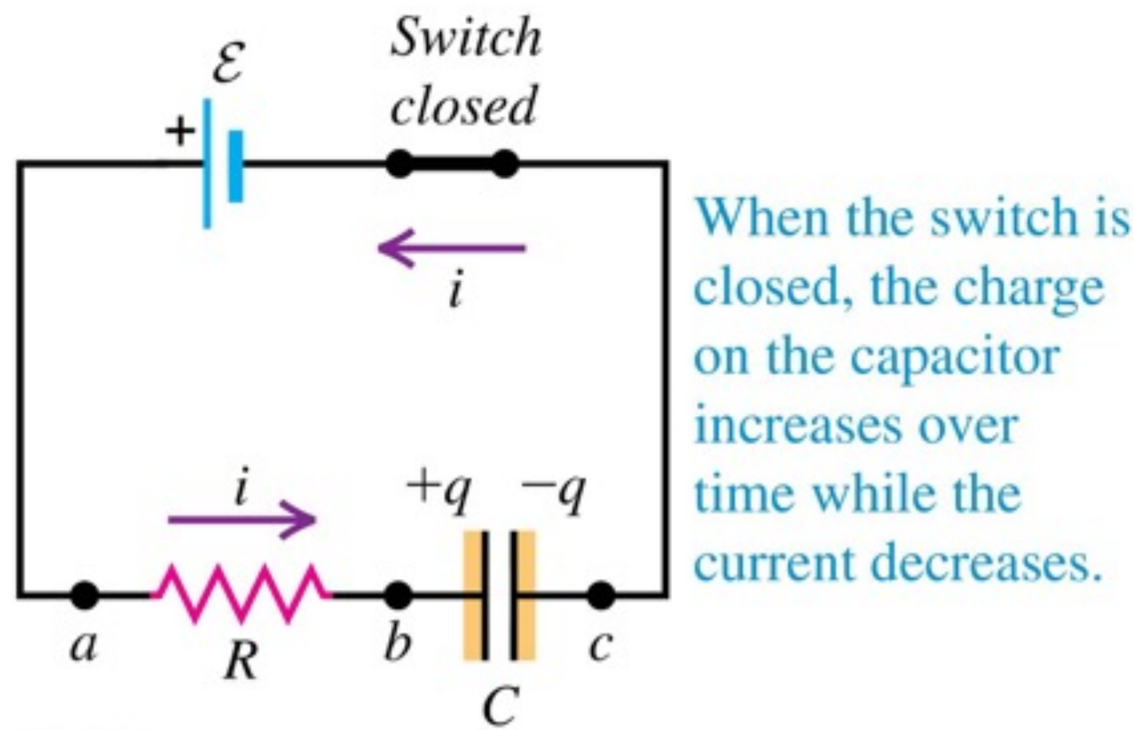


An R-C Circuit

- We are now going to explore the time dependence of the charge on a capacitor.
- If we start with a circuit with a resistor and a capacitor in series with a EMF, but with the circuit broken by an open switch, the capacitor is uncharged, since there is no potential difference across it.



- At some initial time $t = 0$ we close the switch, completing the circuit and permitting current around the loop to begin charging the capacitor.
- As t increases, the charge on the capacitor increases, while the current decreases.



CPS 26-1

A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the maximum charge stored on the capacitor?

A. the emf ε of the battery

B. the capacitance C of the capacitor

C. the resistance R of the resistor

D. both ε and C

E. all three of ε, C, and R

An R-C Circuit (Charging)

- Let's check our boundary values of q(t) at t=0 and as t goes to infinity:

$$q(t) = C\mathcal{E}\left(1 - e^{\frac{-t}{RC}}\right)$$

$$q(0) = C\mathcal{E}\left(1 - e^{\frac{-0}{RC}}\right) = C\mathcal{E}(1-1) = 0$$

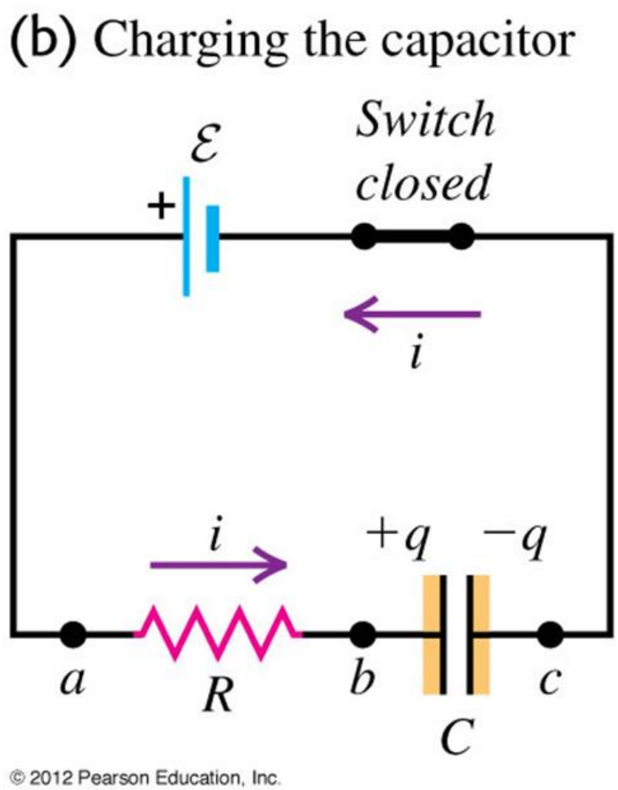
$$q(\infty) = C\mathcal{E}\left(1 - e^{\frac{-\infty}{RC}}\right) = C\mathcal{E}(1-0) = C\mathcal{E}$$

- And do the same for the current:

$$i(t) = \frac{\mathcal{E}}{R} e^{\frac{-t}{RC}} \Rightarrow$$

$$i(0) = \frac{\mathcal{E}}{R}\left(e^{\frac{-0}{RC}}\right) = \frac{\mathcal{E}}{R}(1) = \frac{\mathcal{E}}{R}$$

$$i(\infty) = \frac{\mathcal{E}}{R}\left(e^{\frac{-\infty}{RC}}\right) = \frac{\mathcal{E}}{R}(0) = 0$$



CPS 26-2

A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the rate at which the capacitor charges?

A. the emf ε of the battery

B. the capacitance C of the capacitor

C. the resistance R of the resistor

D. both C and R

E. all three of ε, C, and R

Example 26.12 Charging a capacitor

A 10-MΩ resistor is connected in series with a 1.0-μF capacitor and a battery with emf 12.0 V. Before the switch is closed at time $t = 0$, the capacitor is uncharged. (a) What is the time constant? (b) What fraction of the final charge Q_f is on the capacitor at $t = 46$ s? (c) What fraction of the initial current I_0 is still flowing at $t = 46$ s?

SOLUTION

IDENTIFY and SET UP: This is the same situation as shown in Fig. 26.20, with $R = 10$ MΩ, $C = 1.0\ \mu$F, and $\mathcal{E} = 12.0$ V. The charge q and current i vary with time as shown in Fig. 26.21. Our target variables are (a) the time constant τ, (b) the ratio q/Q_f at $t = 46$ s, and (c) the ratio i/I_0 at $t = 46$ s. Equation (26.14) gives τ. For a capacitor being charged, Eq. (26.12) gives q and Eq. (26.13) gives i.

EXECUTE: (a) From Eq. (26.14),

$$\tau = RC = (10 \times 10^6\ \Omega)(1.0 \times 10^{-6}\ \text{F}) = 10\ \text{s}$$

(b) From Eq. (26.12),

$$\frac{q}{Q_f} = 1 - e^{-t/RC} = 1 - e^{-(46\ \text{s})/(10\ \text{s})} = 0.99$$

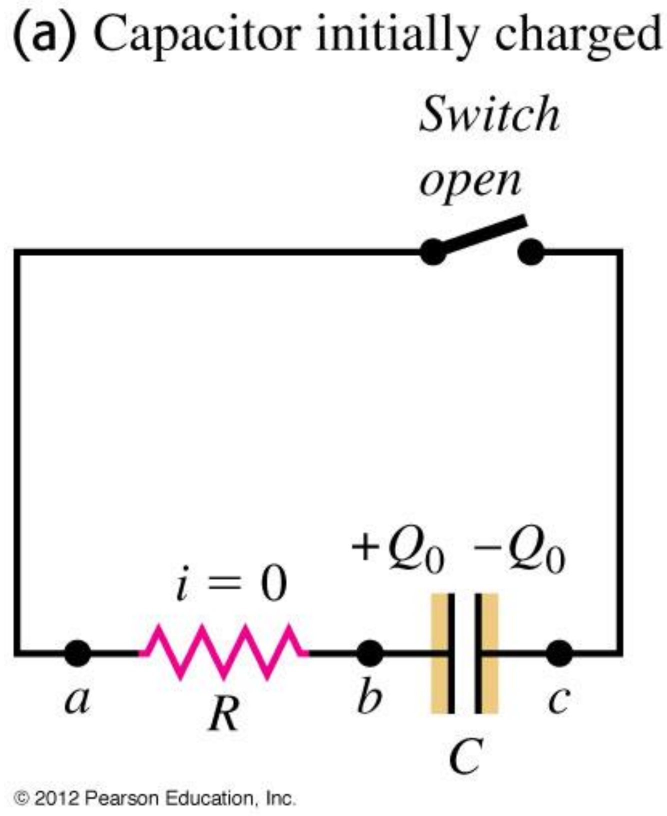
(c) From Eq. (26.13),

$$\frac{i}{I_0} = e^{-t/RC} = e^{-(46\ \text{s})/(10\ \text{s})} = 0.010$$

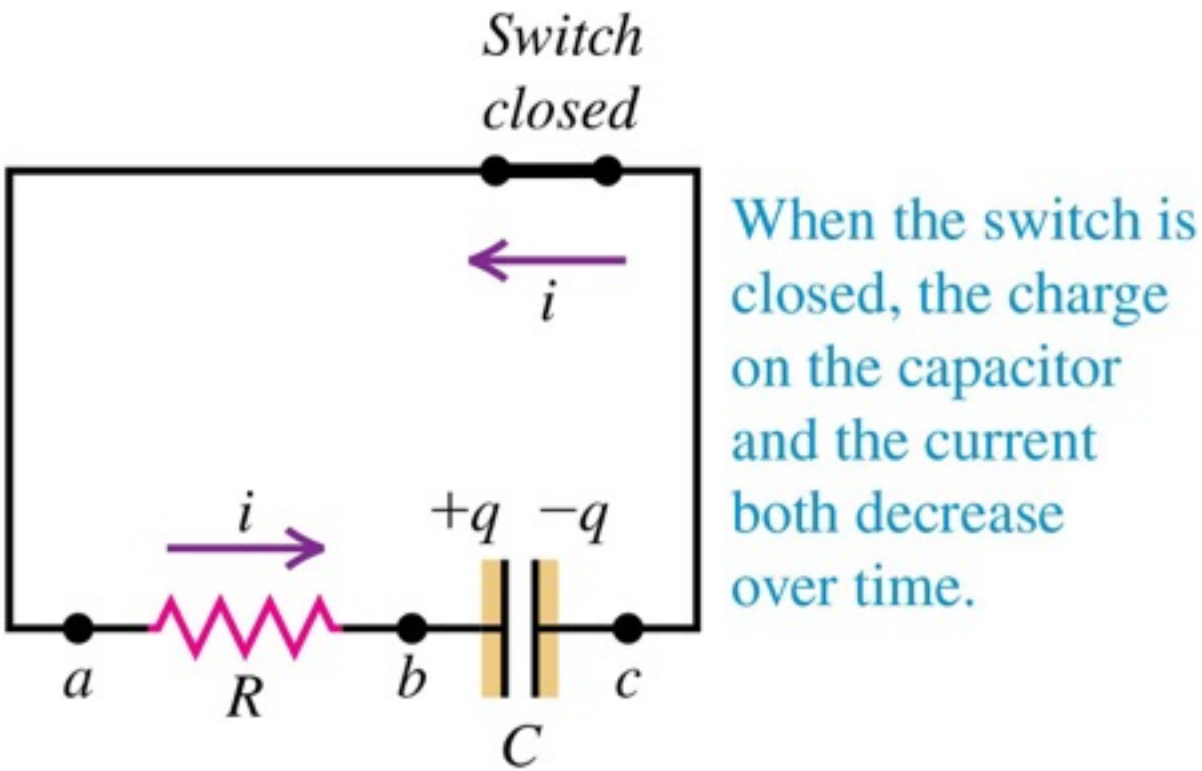
EVALUATE: After 4.6 time constants the capacitor is 99% charged and the charging current has decreased to 1.0% of its initial value. The circuit would charge more rapidly if we reduced the time constant by using a smaller resistance.

An R-C Circuit (Discharging)

- Now, let's start with a fully charged capacitor in an open circuit with no battery.
- Initially, there is charge Q_0 on the capacitor.
- At some time, t=0, we will close the switch, and charge will begin to flow around the circuit, through the resistor and back to the other side of the capacitor.

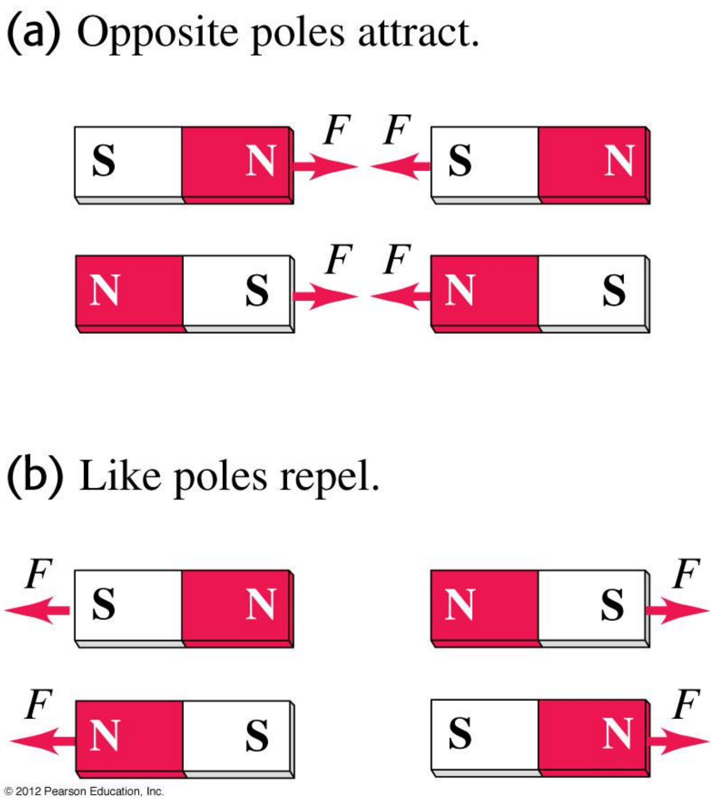


- At some initial time $t = 0$ we close the switch, allowing the capacitor to discharge through the resistor.
- As t increases, the magnitude of the current decreases, while the charge on the capacitor also decreases.



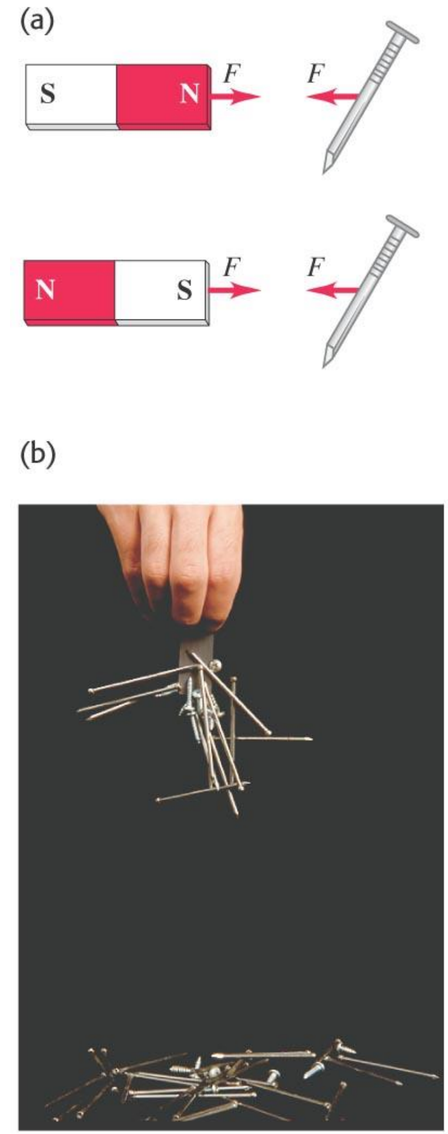
Magnets

- We can begin our exploration by reviewing the most basic experimental observations.
  - Magnets have two “poles”.
  - Opposite poles attract, and like poles repel.
- This makes us wonder if perhaps magnetic forces are just somehow caused by electric fields (since we have seen the whole like charges repel thing before).
- Well, it turns out that there is a relationship, but it is much more complicated.



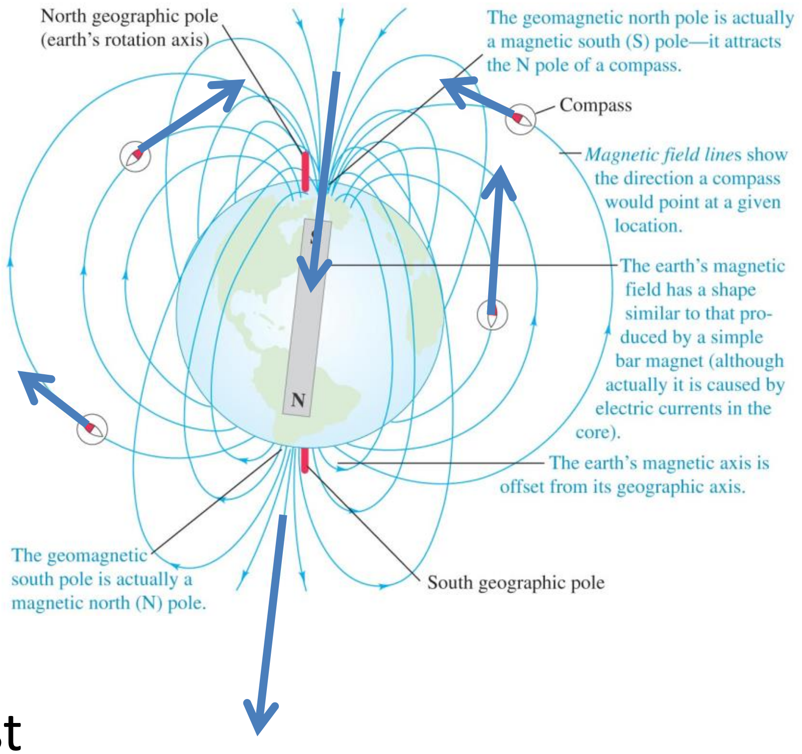
Magnets and Non-magnets

- But, if your refrigerator door isn’t made of a magnet, why do magnets stick to it?
- In a similar way that electric fields can polarize neutral objects and create a force…
- We will return to this later and be more quantitative.



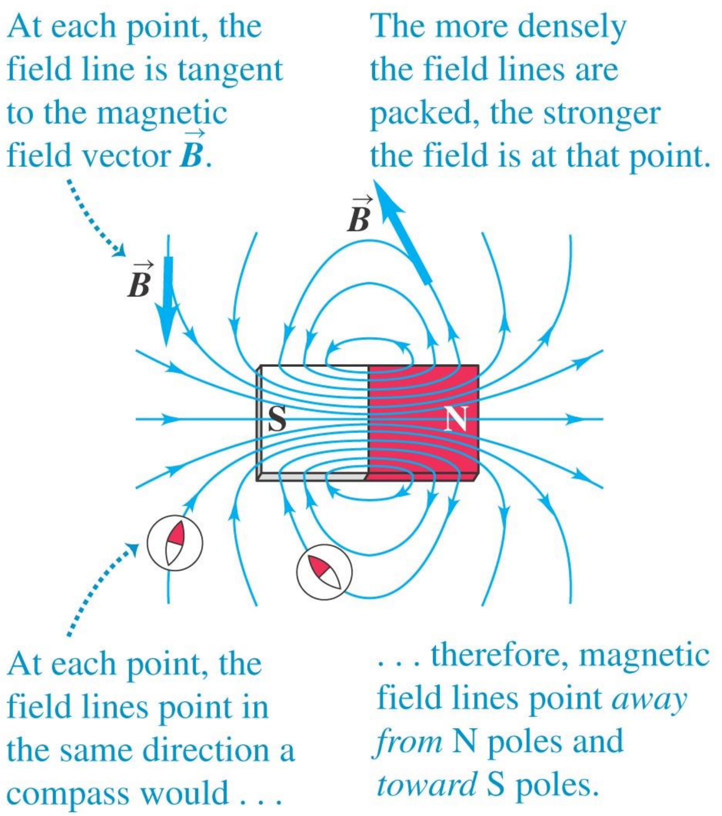
Magnetic Fields and Field Lines

- Magnetic fields are vector fields – at every point in space there is a direction and magnitude for the magnetic field.
- The earth has a magnetic field, and a compass will align itself to the direction of the field.
- Magnetic field lines follow the direction of the field (the field is always tangential to the lines), and the density of lines (how closely spaced they are) is an indication of the field strength.
- Remember that the field IS NOT THE LINES! The field is a set of vectors at every point in space. The lines are just a way of representing the field.



Magnetic Field and Field Lines

- This point is worth repeating, since many students are confused by this, and it is necessary to understand to be able to understand magnetic flux and the various phenomena associated with it.
- The magnetic field is a vector associated with every point in space.
- Magnetic field lines are just a pictorial representation of the field.



CPS 27-1

At which point is the magnitude of the magnetic field the largest?

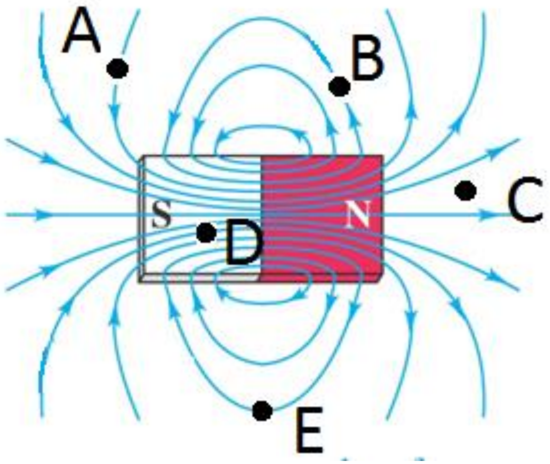
A.

B.

C.

D

E.

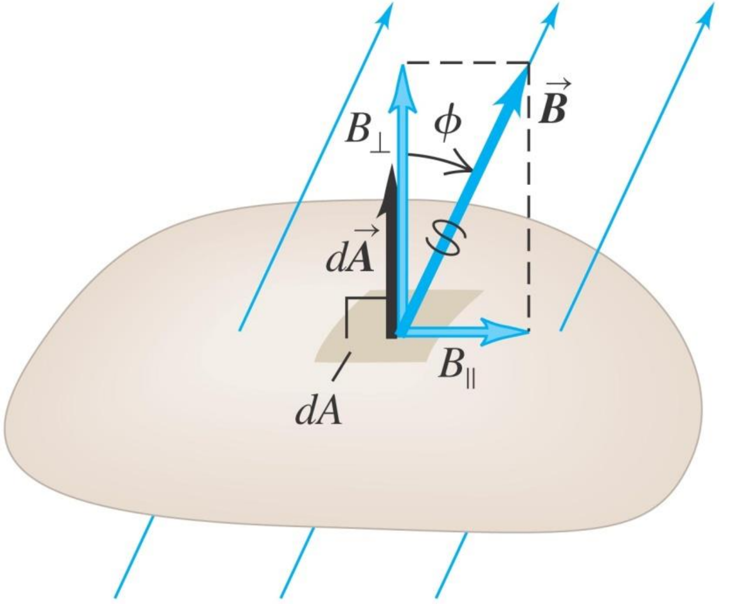


Magnetic Flux

- Yes, back to flux, which means back to surface integrals.
- We can define the magnetic flux in the same way that we defined the electric flux:

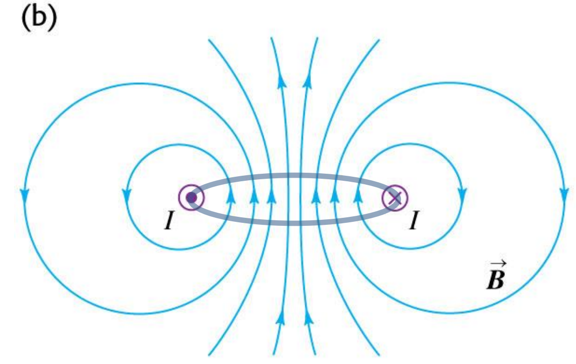
$$\Phi_B = \int_{Surface} \vec{B} \cdot d\vec{A}$$

- Let me go through an example or two…



Magnetic Field "Sources"

- The smallest source of a magnetic field is a magnetic dipole – there is still no beginning or end to the magnetic field lines!
- As we will learn in the next chapter, the source of magnetic fields is a moving charge, and in a magnet, it is the electrons in the atoms that cause the field.

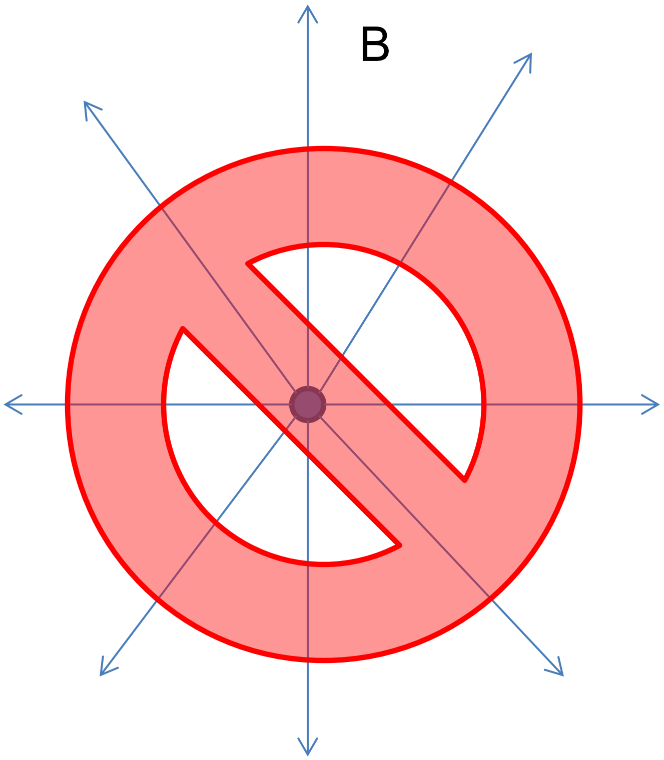


Gauss's Law for Magnetic Fields

- Then, given what we understand about Gauss's Law for the electric field, we can deduce that:

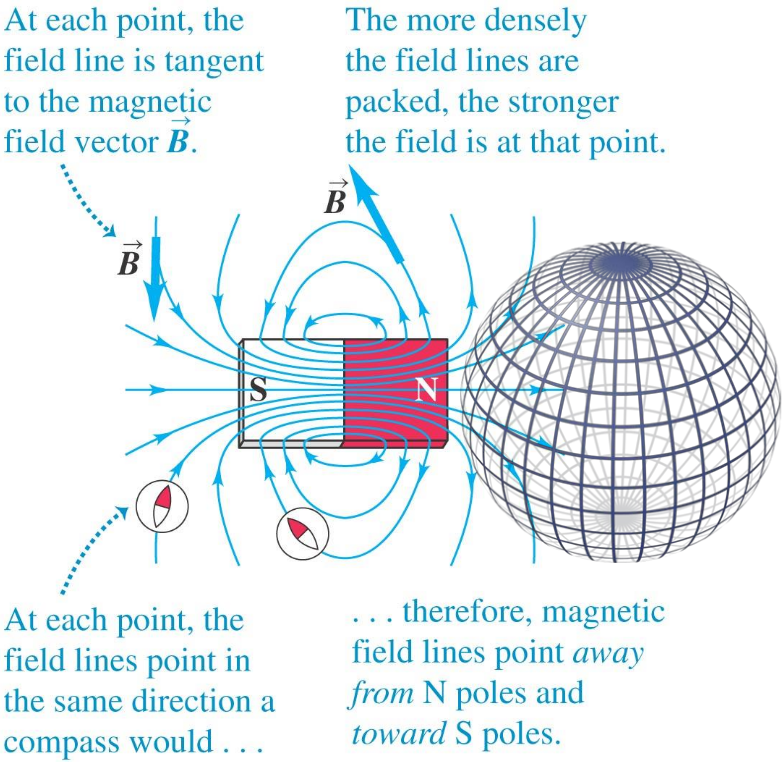
$$\oint \vec{B} \cdot d\vec{A} = 0$$

- In other words, there is no magnetic charge (magnetic monopoles).



Magnetic Field "Sources"

- So, if there is no source or sink of magnetic fields, there can be no *NET* flux through a closed surface (every field line that enters a closed surface must eventually exit the closed surface)!



Units of magnetic field and magnetic flux

- The SI unit of **magnetic field** B is called the tesla (1 T), in honor of Nikola Tesla:

$$1 \text{ tesla} = 1 \text{ T} = 1 \text{ N/A} \cdot \text{m}$$

- Another unit of B, the gauss ($1 \text{ G} = 10^{-4} \text{ T}$), is also in common use.
- The magnetic field of the earth is on the order of 10^{-4} T or 1 G.
- The SI unit of **magnetic flux** Φ_B is called the weber (1 Wb), in honor of Wilhelm Weber:

$$1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$$