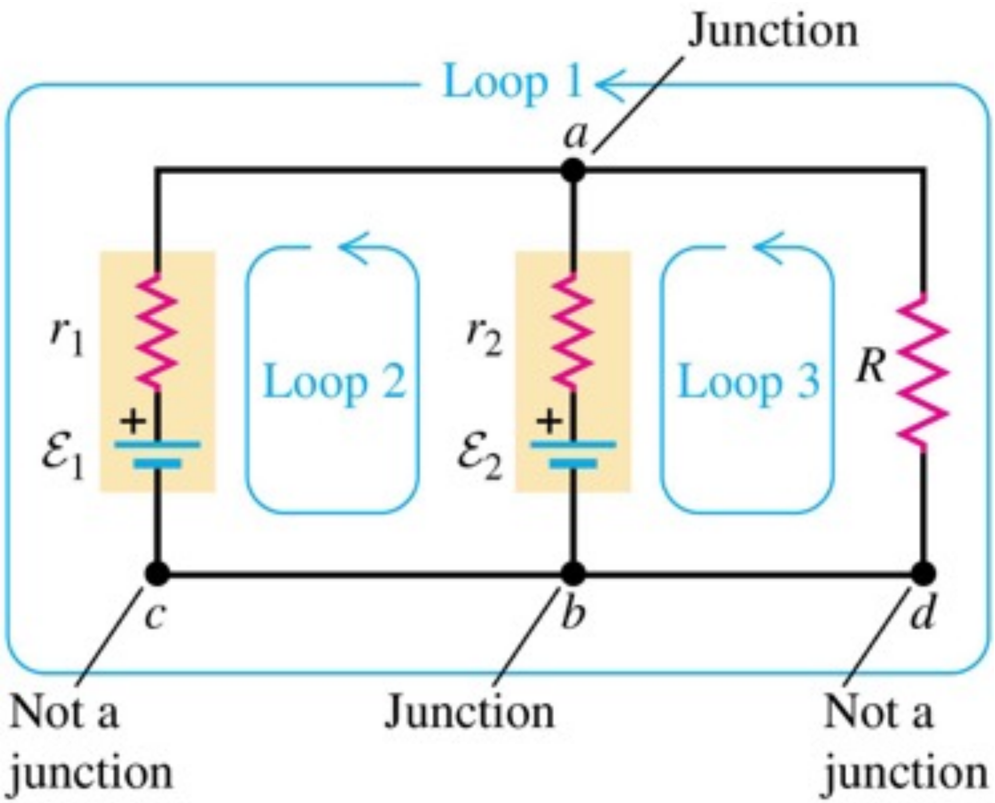
Lecture 25

PHYC 161 Fall 2016

Kirchhoff's rules

- Many practical resistor networks cannot be reduced to simple series-parallel combinations.
- To analyze these networks, we'll use the techniques developed by Kirchhoff.

Define:
Loop
Junction



Kirchhoff's loop rule

- A **loop** is any closed conducting path.
- Kirchhoff's loop rule (valid for any closed loop) is:

Kirchhoff's loop rule (valid for any closed loop):

The sum of the potential differences around any loop ...

$$\sum V = 0$$

... equals zero.

- Conservative force —> potential at a point has a definite value

Kirchhoff's junction rule

- A **junction** is a point where three or more conductors meet.

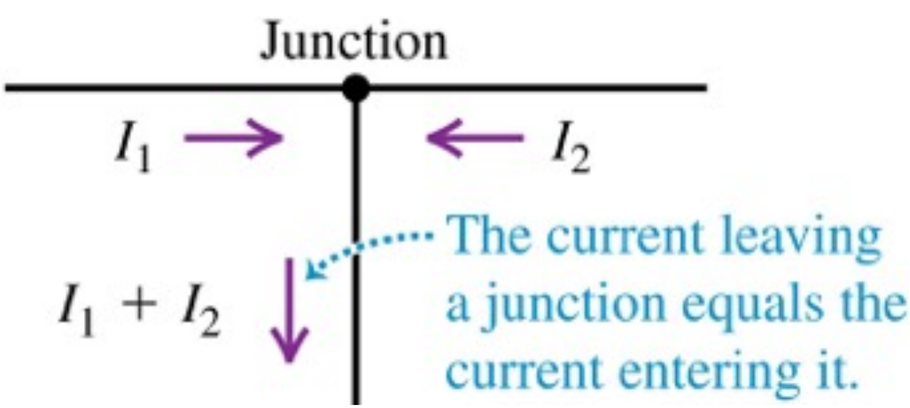
Kirchhoff's junction rule (valid at any junction):

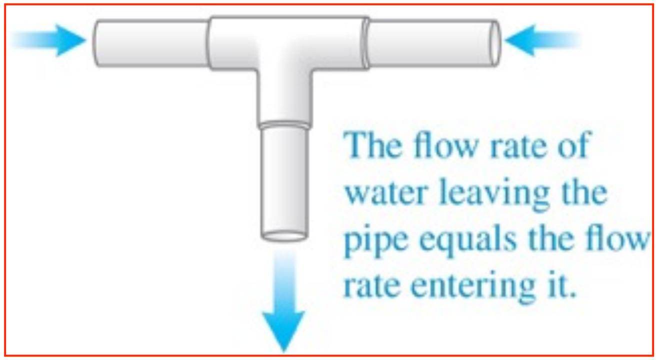
The sum of the currents into any junction ...

$$\sum I = 0$$

... equals zero.

Conservation of charge

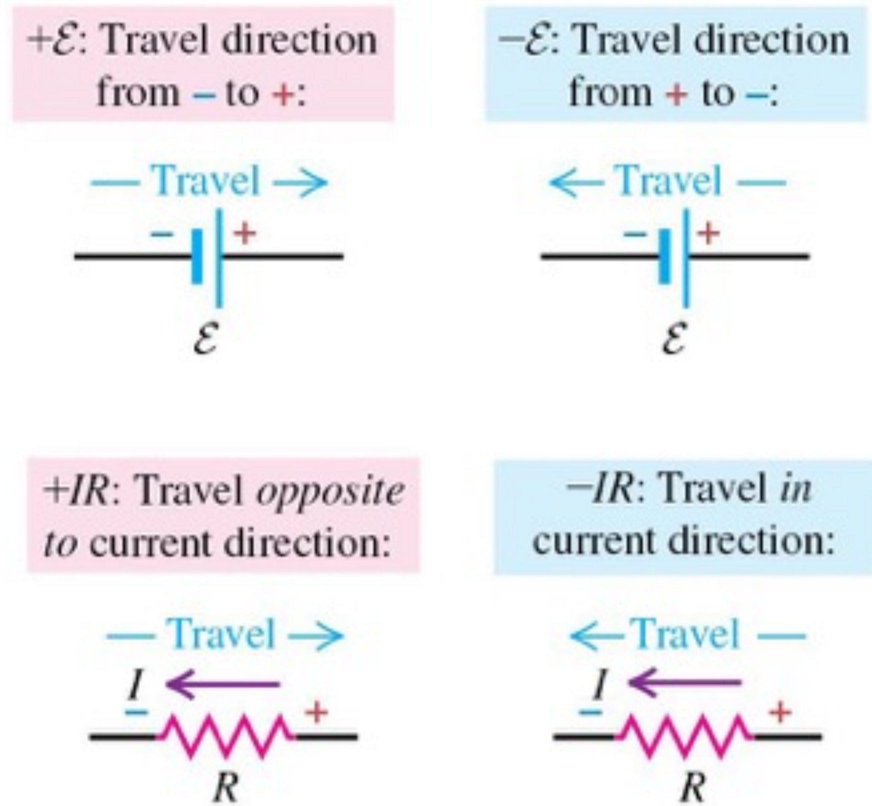




Sign conventions for the loop rule

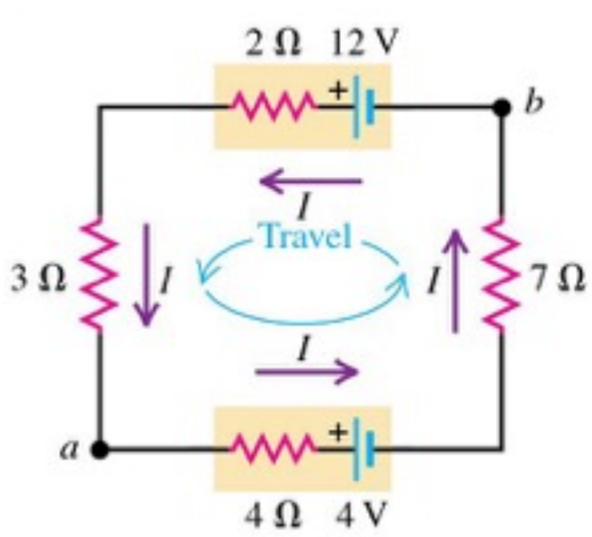
- Use these sign conventions when you apply Kirchhoff's loop rule.

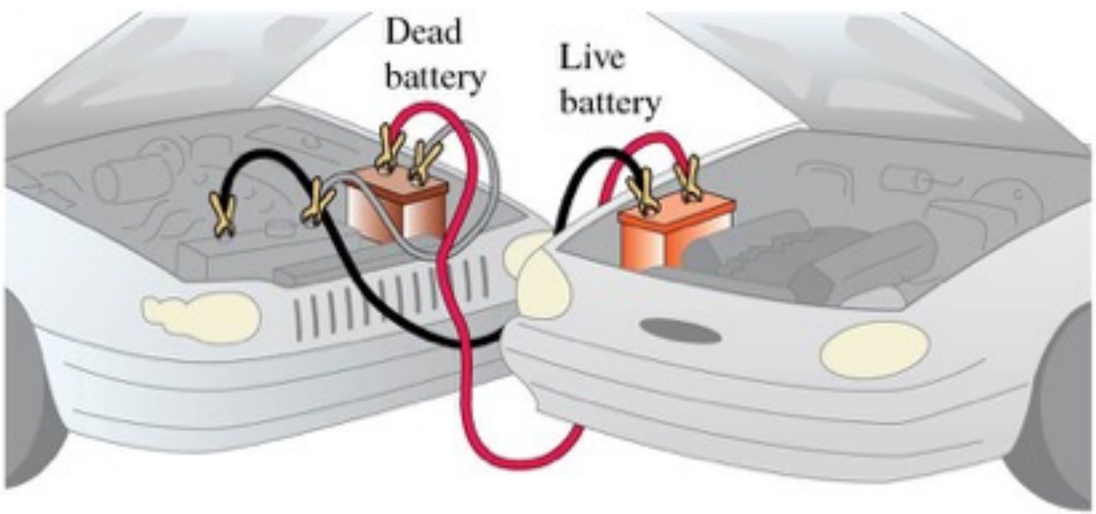
- In each part of the figure, "Travel" is the direction that we imagine going around the loop, which is not necessarily the direction of the current.



A single-loop circuit

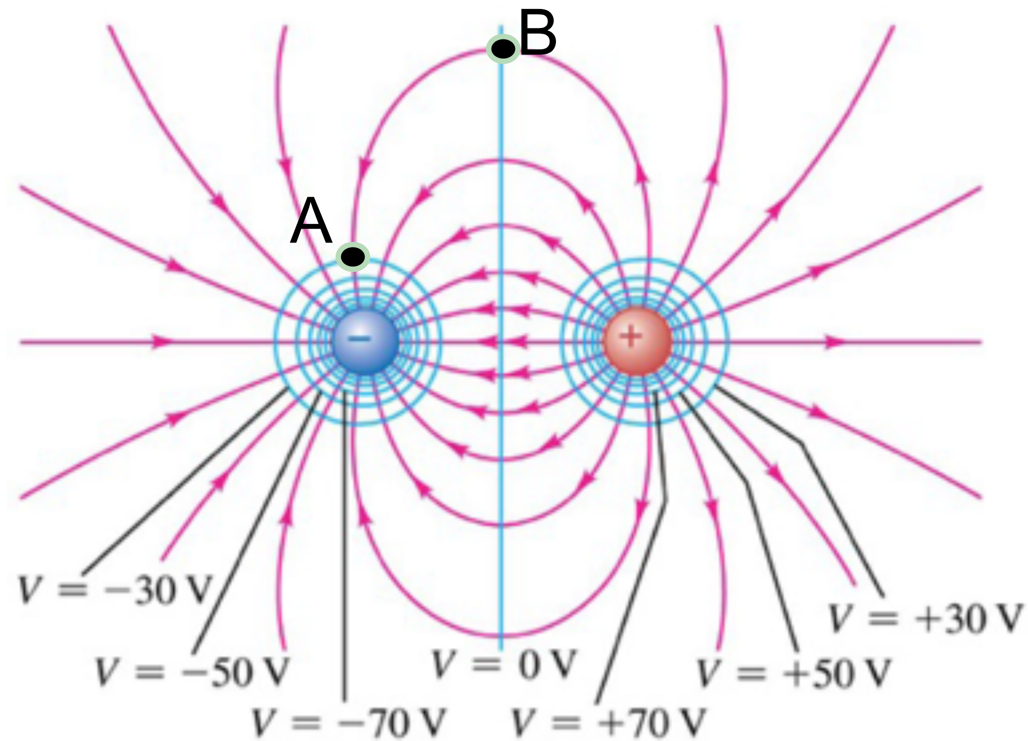
- The circuit shown contains two batteries, each with an emf and an internal resistance, and two resistors.
- Using Kirchhoff's rules, you can find the current in the circuit, the potential difference V_{ab}, and the power output of the emf of each battery.





Energy and potential

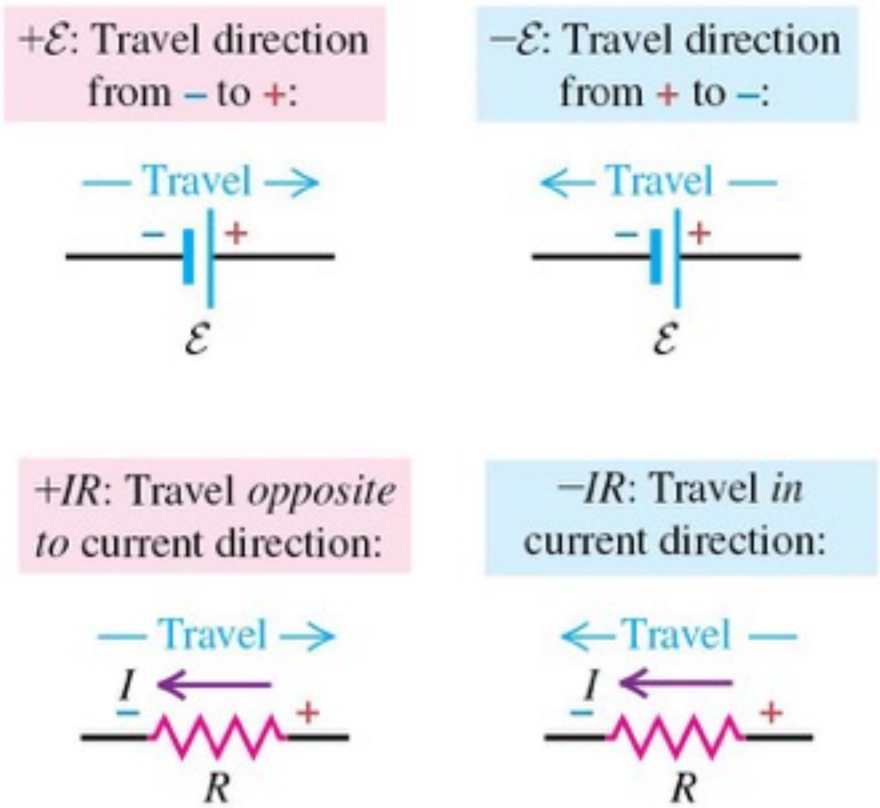
- An electron goes from **rest** at point A to B across a potential difference Vab
- Its kinetic energy at B is:
- a. 90 eV
- b. 20 eV
- c. 30 eV
- d. -30 eV
- e. none of the above



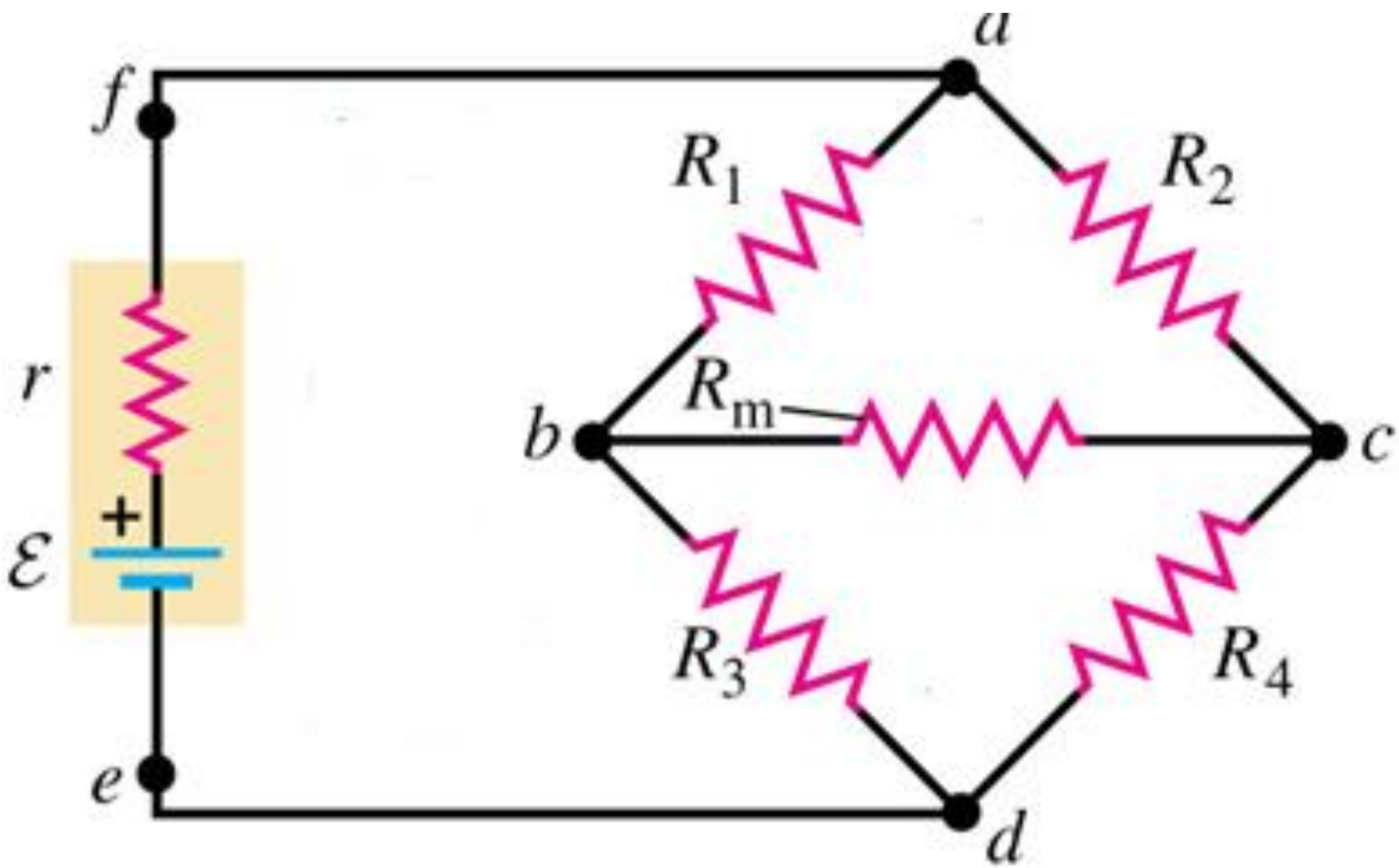
Red = electric field lines
Blue = equipotential surfaces

Sign conventions for the loop rule

- Use these sign conventions when you apply Kirchhoff's loop rule.
- In each part of the figure, "Travel" is the direction that we imagine going around the loop, which is not necessarily the direction of the current.

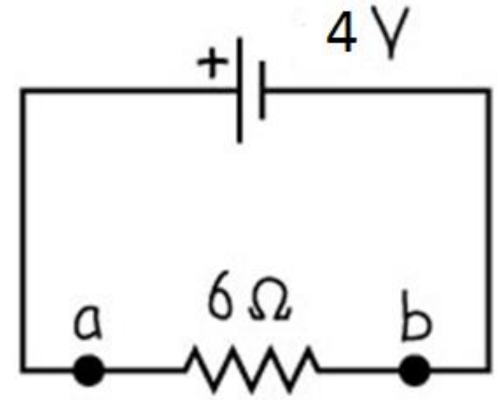


Back to our old “bridge” circuit:



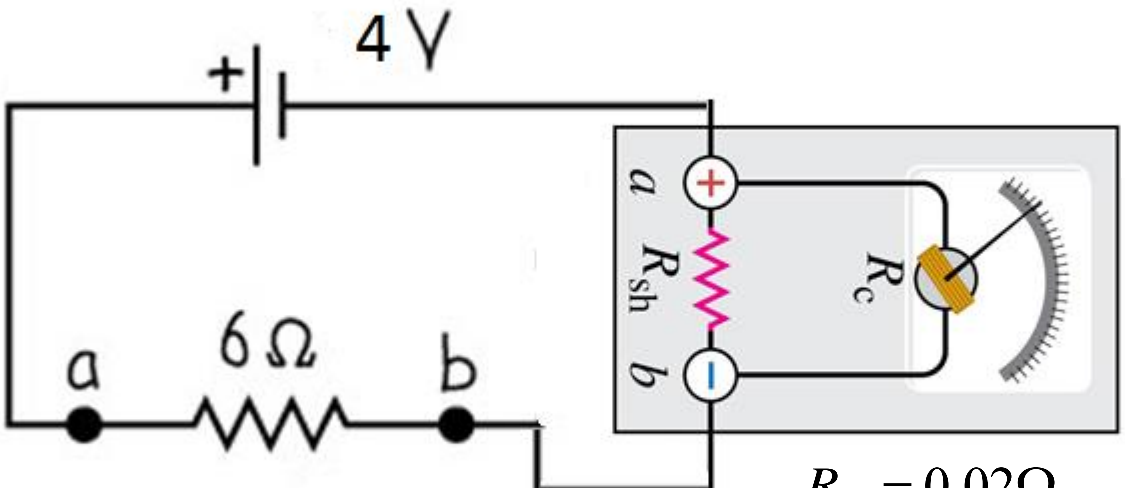
NON ideal Instruments: Ammeter

- Note that until now, we have considered ammeters to have *zero* resistance.
- Since this is not the case in reality, we have to take this into account when we place an ammeter into a circuit.
- Let’s consider two different circuits:



$$V_{ab} = 4V = IR = I(6\Omega) \Rightarrow$$

$$I = 0.667A$$



$$R_{eq} = 0.02\Omega$$

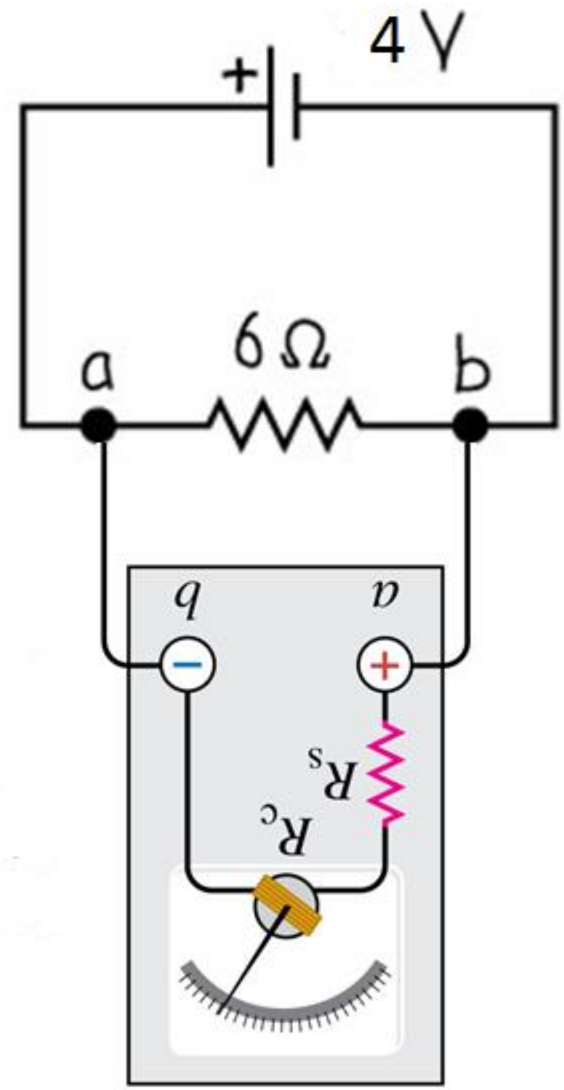
$$V_{ab} \neq 4V$$

$$4V - I(6\Omega) - I(0.02\Omega)$$

$$I = 0.664A$$

NON ideal Instruments: Voltmeter

- Note that until now, we have considered ammeters to have *infinite* resistance.
- Since this is not the case in reality, we have to take this into account when we place an voltmeter into a circuit.
- Let's consider two different circuits:



$$\frac{1}{R_{eq}} = \frac{1}{R_{volt}} + \frac{1}{R} = \frac{1}{10k\Omega} + \frac{1}{6\Omega} \Rightarrow$$

$$R_{eq} = 5.996\Omega$$

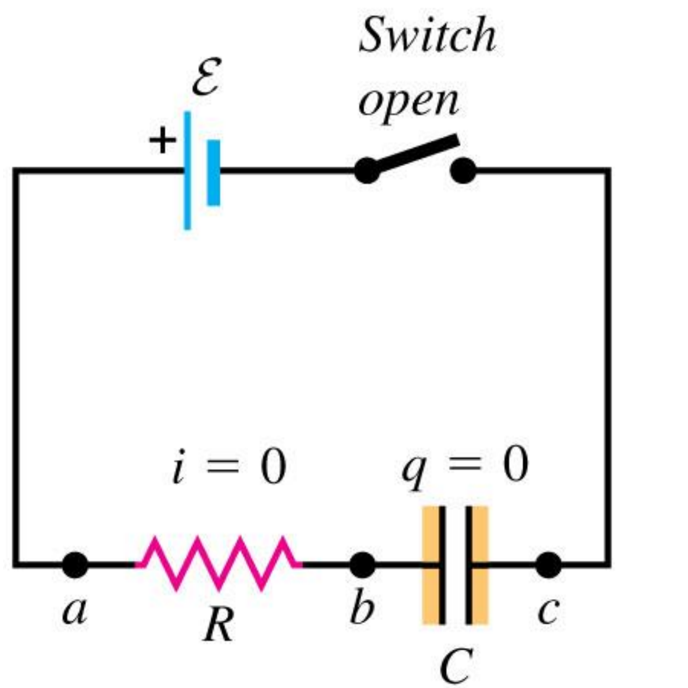
$$V_{ab} = 4V = I\left(5.996\Omega\right) \Rightarrow$$

$$I = 0.66707A$$

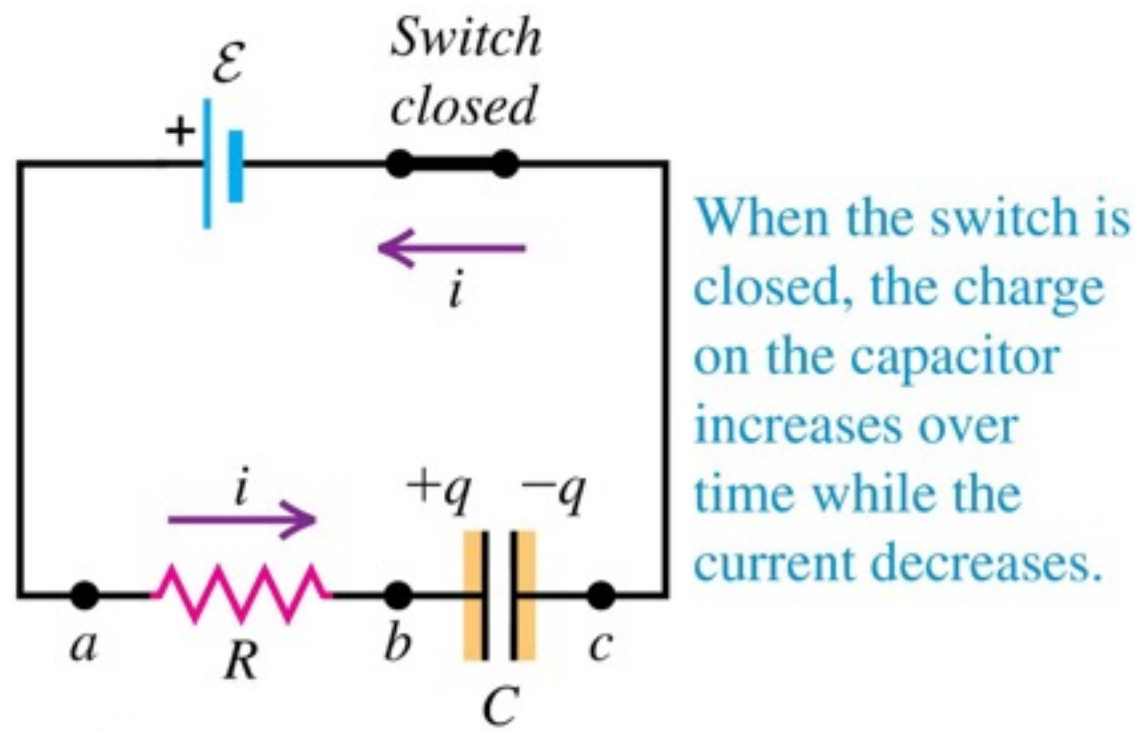
An R-C Circuit

- We are now going to explore the time dependence of the charge on a capacitor.
- If we start with a circuit with a resistor and a capacitor in series with a EMF, but with the circuit broken by an open switch, the capacitor is uncharged, since there is no potential difference across it.

(a) Capacitor initially uncharged



- At some initial time $t = 0$ we close the switch, completing the circuit and permitting current around the loop to begin charging the capacitor.
- As t increases, the charge on the capacitor increases, while the current decreases.



An R-C Circuit (Charging)

- Let's check our boundary values of q(t) at t=0 and as t goes to infinity:

$$q(t) = C\mathcal{E}\left(1 - e^{\frac{-t}{RC}}\right)$$

$$q(0) = C\mathcal{E}\left(1 - e^{\frac{-0}{RC}}\right) = C\mathcal{E}(1-1) = 0$$

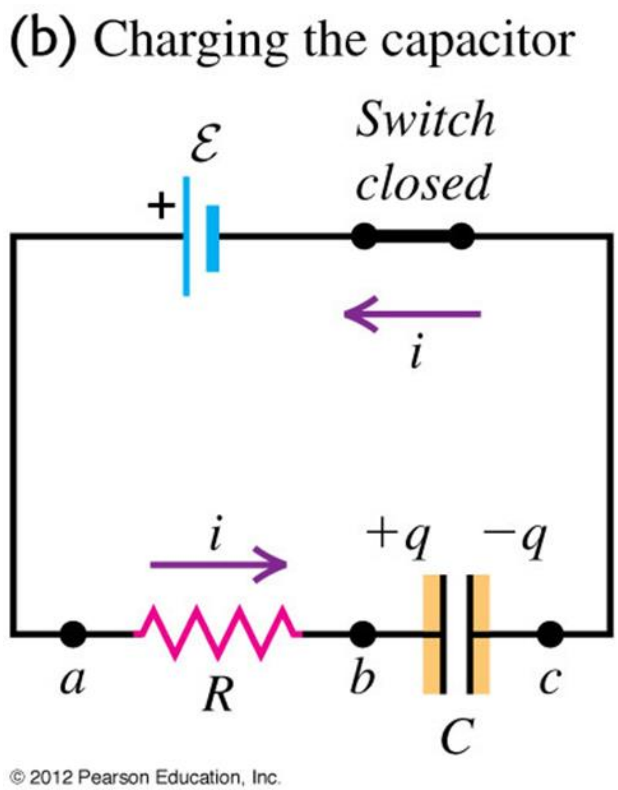
$$q(\infty) = C\mathcal{E}\left(1 - e^{\frac{-\infty}{RC}}\right) = C\mathcal{E}(1-0) = C\mathcal{E}$$

- And do the same for the current:

$$i(t) = \frac{\mathcal{E}}{R} e^{\frac{-t}{RC}} \Rightarrow$$

$$i(0) = \frac{\mathcal{E}}{R}\left(e^{\frac{-0}{RC}}\right) = \frac{\mathcal{E}}{R}(1) = \frac{\mathcal{E}}{R}$$

$$i(\infty) = \frac{\mathcal{E}}{R}\left(e^{\frac{-\infty}{RC}}\right) = \frac{\mathcal{E}}{R}(0) = 0$$



CPS 26-1

A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the maximum charge stored on the capacitor?

A. the emf ε of the battery

B. the capacitance C of the capacitor

C. the resistance R of the resistor

D. both ε and C

E. all three of ε, C, and R

CPS 26-2

A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the rate at which the capacitor charges?

A. the emf ε of the battery

B. the capacitance C of the capacitor

C. the resistance R of the resistor

D. both C and R

E. all three of ε, C, and R

Example 26.12 Charging a capacitor

A 10-MΩ resistor is connected in series with a 1.0-μF capacitor and a battery with emf 12.0 V. Before the switch is closed at time $t = 0$, the capacitor is uncharged. (a) What is the time constant? (b) What fraction of the final charge Q_f is on the capacitor at $t = 46$ s? (c) What fraction of the initial current I_0 is still flowing at $t = 46$ s?

SOLUTION

IDENTIFY and SET UP: This is the same situation as shown in Fig. 26.20, with $R = 10$ MΩ, $C = 1.0$ μF, and $\mathcal{E} = 12.0$ V. The charge q and current i vary with time as shown in Fig. 26.21. Our target variables are (a) the time constant τ, (b) the ratio q/Q_f at $t = 46$ s, and (c) the ratio i/I_0 at $t = 46$ s. Equation (26.14) gives τ. For a capacitor being charged, Eq. (26.12) gives q and Eq. (26.13) gives i.

EXECUTE: (a) From Eq. (26.14),

$$\tau = RC = (10 \times 10^6\ \Omega)(1.0 \times 10^{-6}\ \text{F}) = 10\ \text{s}$$

(b) From Eq. (26.12),

$$\frac{q}{Q_f} = 1 - e^{-t/RC} = 1 - e^{-(46\ \text{s})/(10\ \text{s})} = 0.99$$

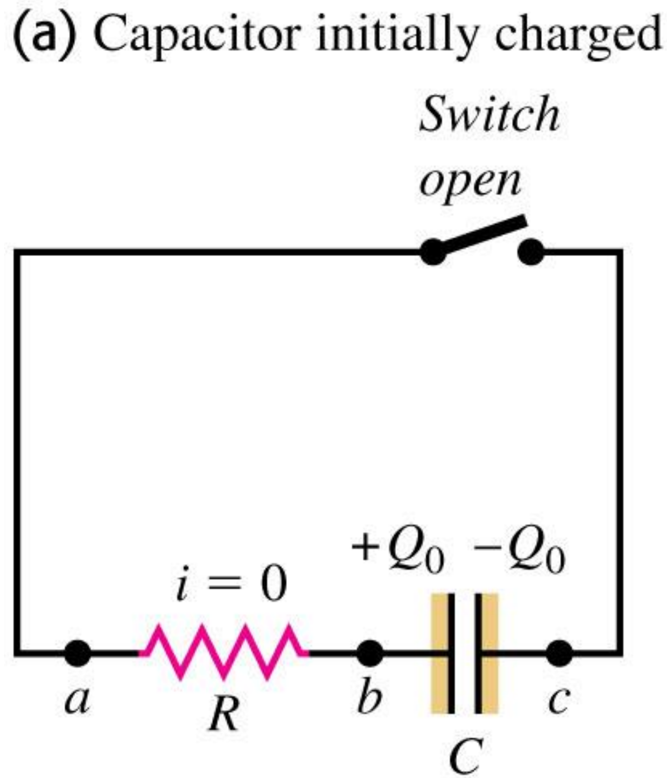
(c) From Eq. (26.13),

$$\frac{i}{I_0} = e^{-t/RC} = e^{-(46\ \text{s})/(10\ \text{s})} = 0.010$$

EVALUATE: After 4.6 time constants the capacitor is 99% charged and the charging current has decreased to 1.0% of its initial value. The circuit would charge more rapidly if we reduced the time constant by using a smaller resistance.

An R-C Circuit (Discharging)

- Now, let's start with a fully charged capacitor in an open circuit with no battery.
- Initially, there is charge Q_0 on the capacitor.
- At some time, t=0, we will close the switch, and charge will begin to flow around the circuit, through the resistor and back to the other side of the capacitor.



- At some initial time $t = 0$ we close the switch, allowing the capacitor to discharge through the resistor.
- As t increases, the magnitude of the current decreases, while the charge on the capacitor also decreases.