

# Lecture 19

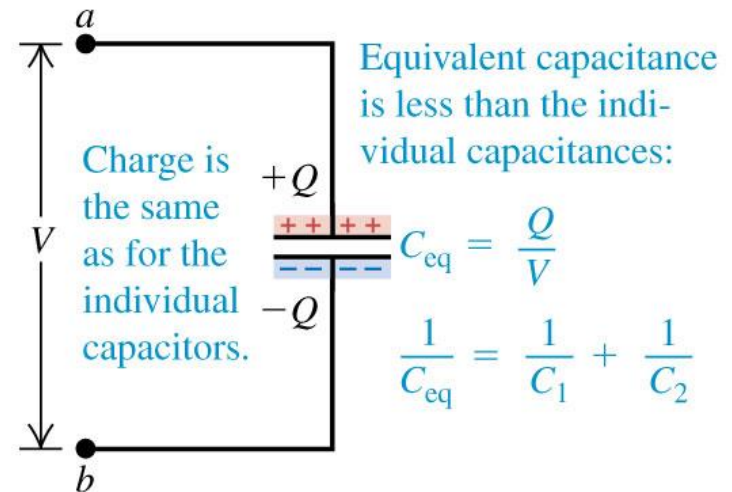
PHYC 161 Fall 2016

# Capacitors in Series

- If you wanted to replace these capacitors with just one equivalent capacitor:

$$C_{\text{eq}} = \frac{Q}{V} = \frac{Q}{V_1 + V_2} \Rightarrow$$
$$\frac{1}{C_{\text{eq}}} = \frac{V_1 + V_2}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow$$
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

(b) The equivalent single capacitor

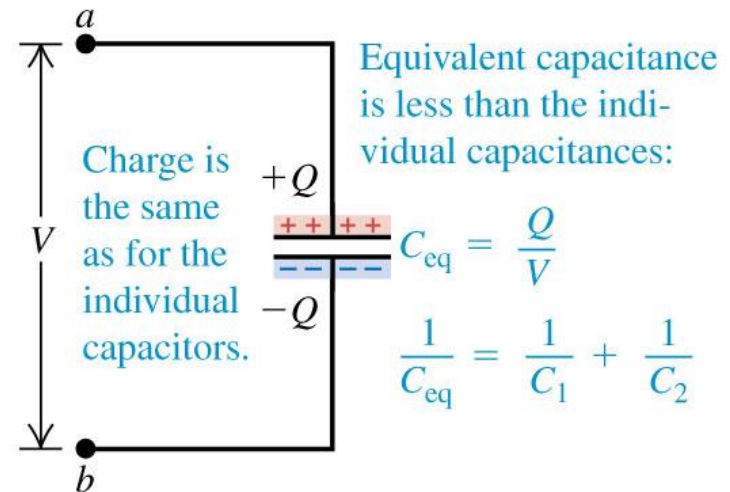


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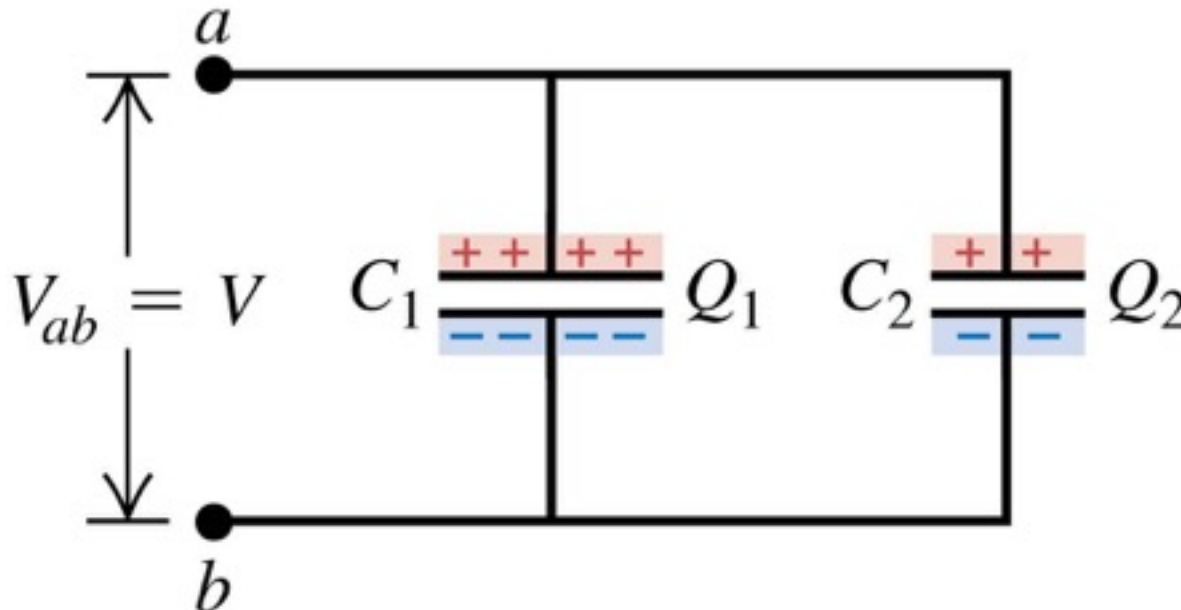


# Capacitors in parallel: Slide 1 of 3

- Capacitors are connected in *parallel* between  $a$  and  $b$  if the potential difference  $V_{ab}$  is the same for all the capacitors.

## Capacitors in parallel:

- The capacitors have the same potential  $V$ .
- The charge on each capacitor depends on its capacitance:  $Q_1 = C_1V$ ,  $Q_2 = C_2V$ .



# Capacitor in Parallel

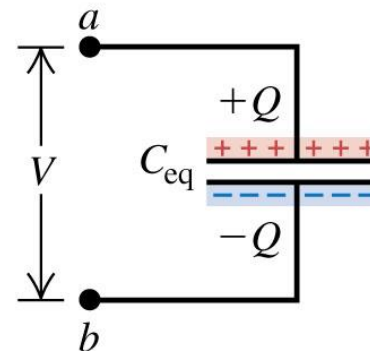
- If you wanted to replace these capacitors with just one equivalent capacitor:

$$C_{\text{eq}} = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} \Rightarrow$$

$$C_{\text{eq}} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2 \Rightarrow$$

$$C_{\text{eq}} = C_1 + C_2$$

(b) The equivalent single capacitor



Charge is the sum of the individual charges:

$$Q = Q_1 + Q_2$$

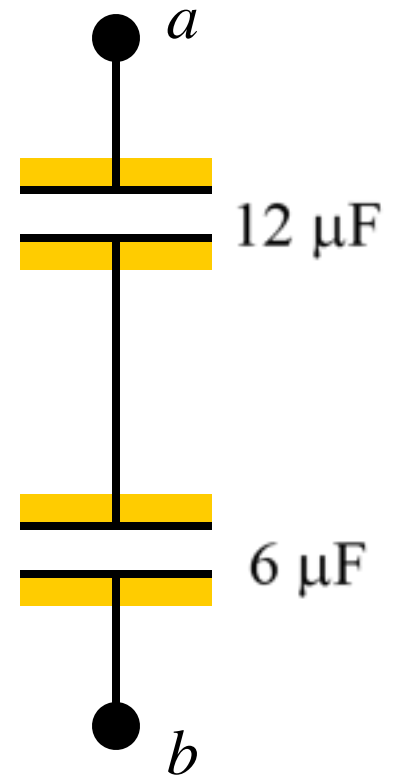
Equivalent capacitance:

$$C_{\text{eq}} = C_1 + C_2$$

## Q24.3

A  $12\text{-}\mu\text{F}$  capacitor and a  $6\text{-}\mu\text{F}$  capacitor are connected together as shown. What is the equivalent capacitance of the two capacitors as a unit?

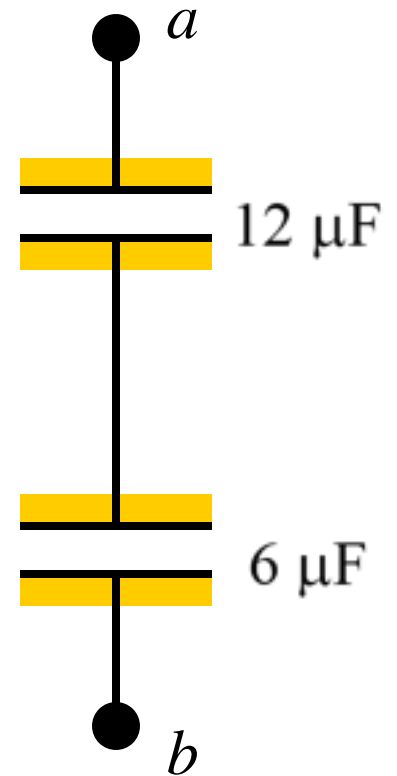
- A.  $C_{\text{eq}} = 18\ \mu\text{F}$
- B.  $C_{\text{eq}} = 9\ \mu\text{F}$
- C.  $C_{\text{eq}} = 6\ \mu\text{F}$
- D.  $C_{\text{eq}} = 4\ \mu\text{F}$
- E.  $C_{\text{eq}} = 2\ \mu\text{F}$



## Q24.4

A  $12\text{-}\mu\text{F}$  capacitor and a  $6\text{-}\mu\text{F}$  capacitor are connected together as shown. If the charge on the  $12\text{-}\mu\text{F}$  capacitor is  $24$  microcoulombs ( $24\text{ }\mu\text{C}$ ), the charge on the capacitor?

$6\text{-}\mu\text{F}$

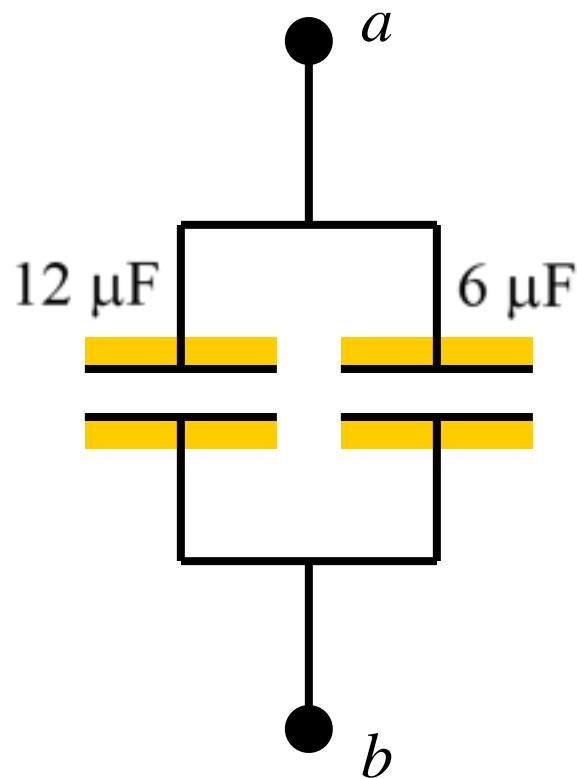


- A.  $48\ \mu\text{C}$
- B.  $36\ \mu\text{C}$
- C.  $24\ \mu\text{C}$
- D.  $12\ \mu\text{C}$
- E.  $6\ \mu\text{C}$

## Q24.5

A  $12\text{-}\mu\text{F}$  capacitor and a  $6\text{-}\mu\text{F}$  capacitor are connected together as shown. What is the equivalent capacitance of the two capacitors as a unit?

- A.  $C_{\text{eq}} = 18\ \mu\text{F}$
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- C.  $C_{\text{eq}} = 6\ \mu\text{F}$
- D.  $C_{\text{eq}} = 4\ \mu\text{F}$
- E.  $C_{\text{eq}} = 2\ \mu\text{F}$



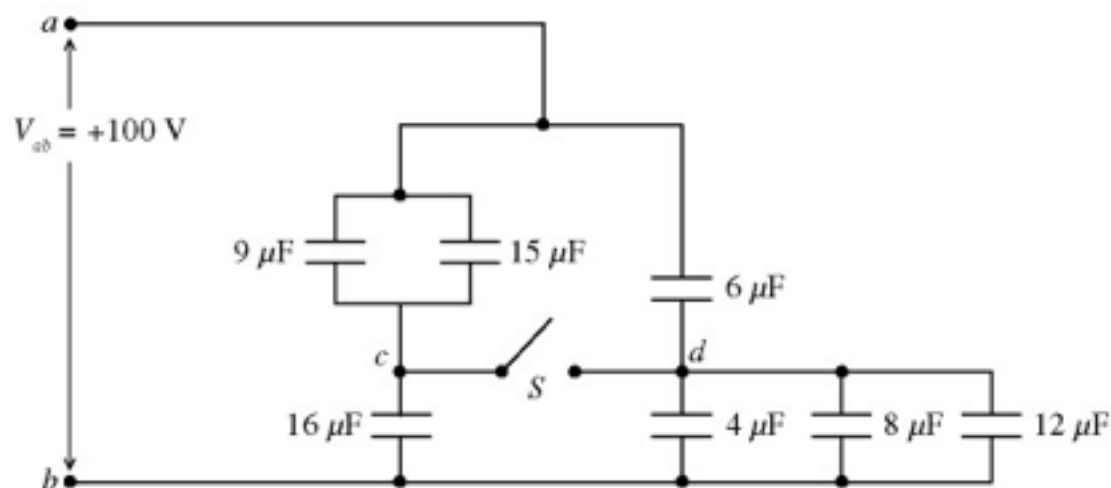


## Problem 24.15

**Description:** (a) The capacitive network shown in the figure is assembled with initially uncharged capacitors. A potential difference,  $V_{ab} = +100\text{V}$ , is applied across the network. The switch  $S$  in the network is kept open. Assume that all the capacitances shown...

### Part A

The capacitive network shown in the figure is assembled with initially uncharged capacitors. A potential difference,  $V_{ab} = +100\text{V}$ , is applied across the network. The switch  $S$  in the network is kept open. Assume that all the capacitances shown are accurate to two significant figures. What is potential difference  $V_{cd}$  across the open switch  $S$ ?



# Energy stored in a capacitor

- The potential energy stored in a capacitor is:

Potential energy stored in a capacitor  $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

Magnitude of charge on each plate (points to  $Q^2$ )

Capacitance (points to  $C$ )

Potential difference between plates (points to  $V$ )

- The capacitor energy is stored in the *electric field* between the plates.
- The *energy density* is:

Electric energy density in a vacuum  $u = \frac{1}{2}\epsilon_0 E^2$

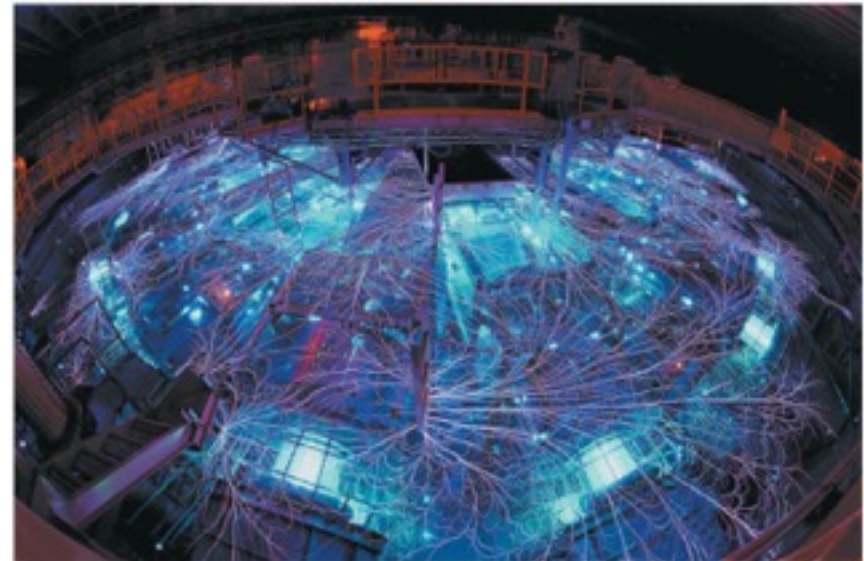
Magnitude of electric field (points to  $E^2$ )

Electric constant (points to  $\epsilon_0$ )

# Energy stored in a capacitor

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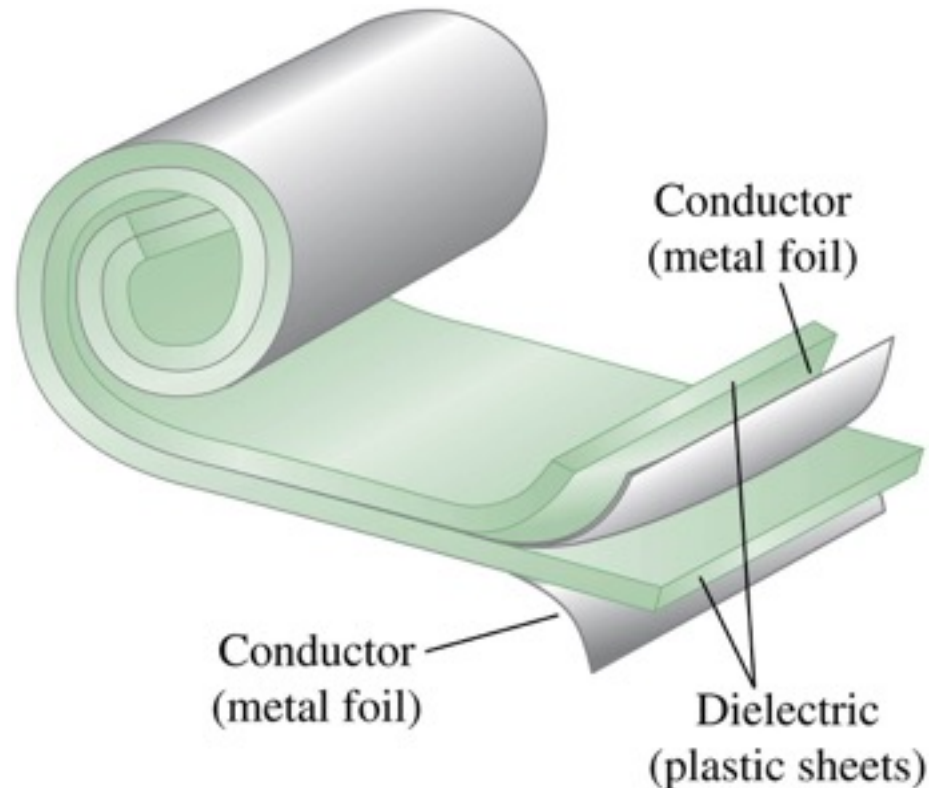
- A practical application of capacitors is their ability to store energy and release it quickly.
- An extreme example of the same principle is the Z machine at Sandia National Laboratories in New Mexico, which is used in experiments in controlled nuclear fusion.
- The Z machine uses a large number of capacitors in parallel to give a tremendous equivalent capacitance.
- The arcs shown here are produced when the capacitors discharge their energy into a target, which is heated to a temperature higher than  $2 \times 10^9$  K.



# Dielectrics

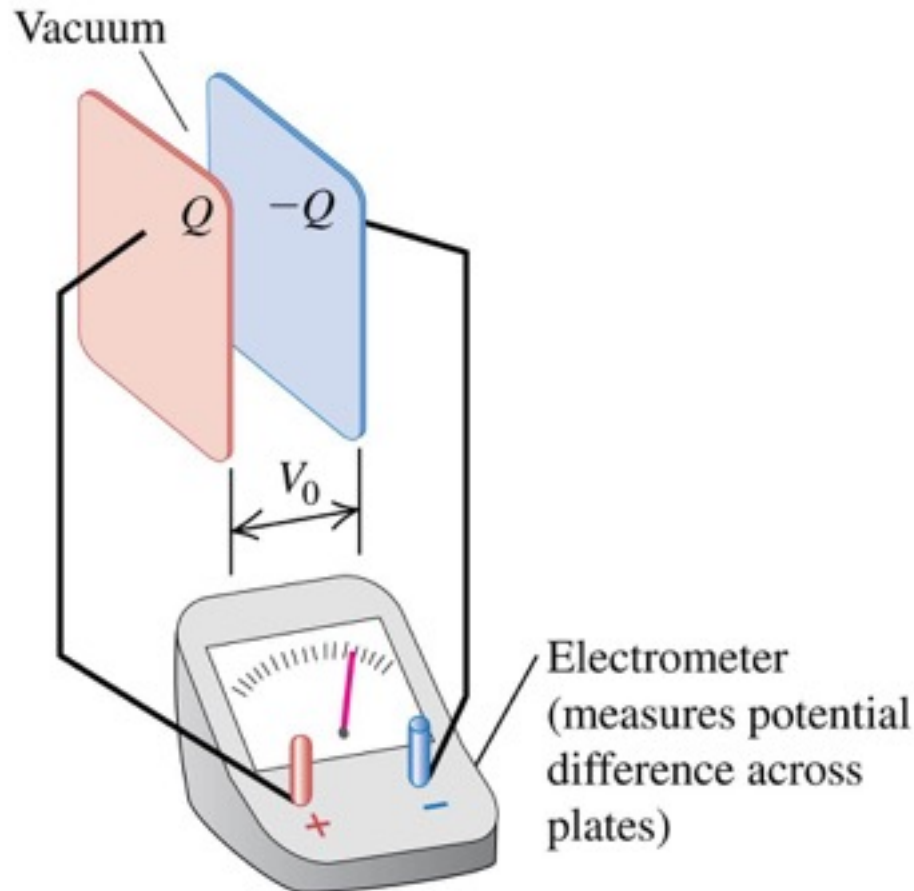
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- Most capacitors have a nonconducting material, or dielectric, between their conducting plates.
- A common type of capacitor uses long strips of metal foil for the plates, separated by strips of plastic sheet such as Mylar.



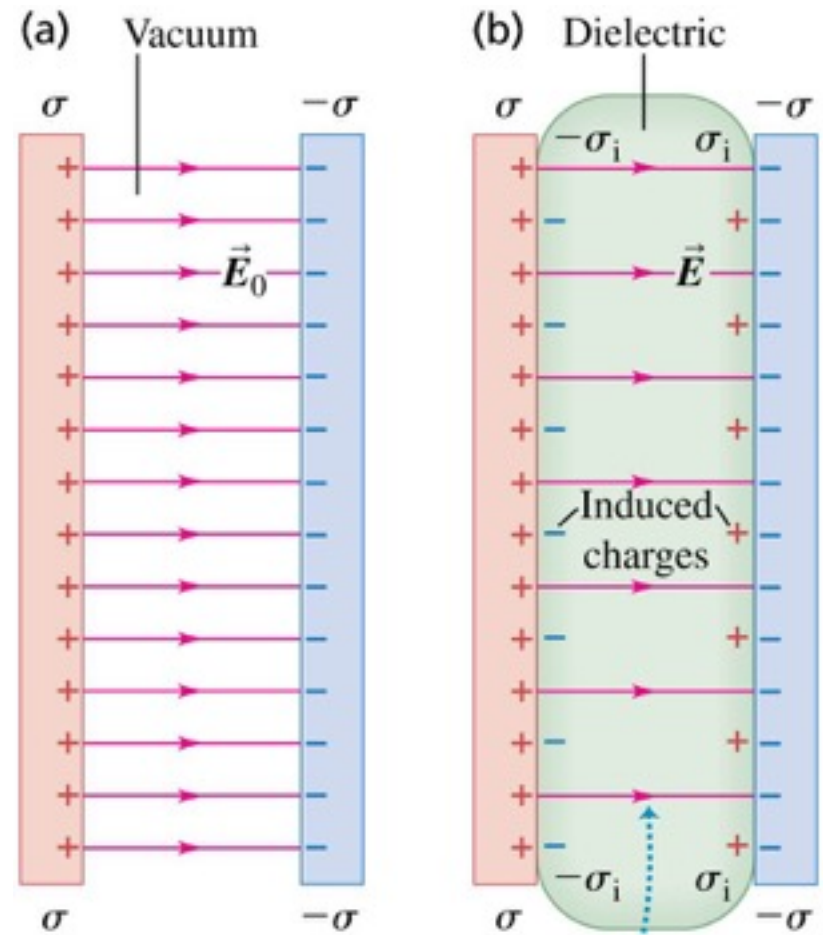
# Dielectrics increase capacitance: Slide 1 of 2

- Consider an electrometer connected across a charged capacitor, with magnitude of charge  $Q$  on each plate and potential difference  $V_0$ .



# Dielectrics

- When a dielectric is inserted between the plates of a capacitor, the electric field *decreases*.
- This is due to **polarization** of the charge within the dielectric, which results in induced surface charges, as shown.



For a given charge density  $\sigma$ , the induced charges on the dielectric's surfaces reduce the electric field between the plates.

# The dielectric constant

- When an insulating material is inserted between the plates of a capacitor whose original capacitance is  $C_0$ , the new capacitance is greater by a factor  $K$ , where  $K$  is the **dielectric constant** of the material.

Capacitance of a parallel-plate capacitor, dielectric between plates

$$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

Dielectric constant

Area of each plate

Permittivity =  $K\epsilon_0$

Capacitance without dielectric

Electric constant

Distance between plates

- The energy density in the capacitor also increases:

Electric energy density in a dielectric

$$u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2$$

Dielectric constant

Permittivity =  $K\epsilon_0$

Electric constant

Magnitude of electric field

## Table 24.1—Some dielectric constants

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<b>Material</b>	<b><math>K</math></b>
<i>Vacuum</i>	<i>1</i>
<i>Air (1 atm)</i>	<i>1.00059</i>
<i>Teflon</i>	<i>2.1</i>
<i>Mylar</i>	<i>3.1</i>
<i>Glass</i>	<i>5 – 10</i>
<i>Glycerin</i>	<i>42.5</i>
<i>Water</i>	<i>80.4</i>



# Dielectric breakdown

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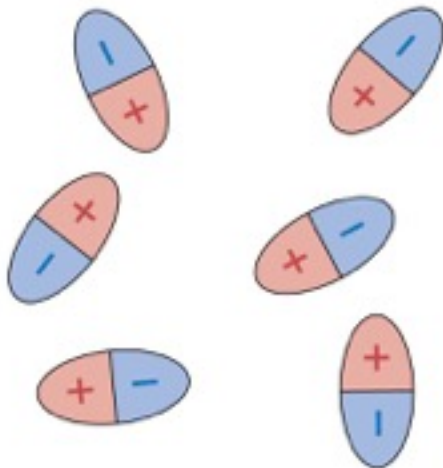
- If the electric field is strong enough, **dielectric breakdown** occurs and the dielectric becomes a conductor.
- The **dielectric strength** is the maximum electric field the material can withstand before breakdown occurs.
- For example, Pyrex glass has a dielectric constant of  $K = 4.7$ , and a dielectric strength of  $E_m = 1 \times 10^7$  V/m.
- Dry air has a dielectric constant of  $K = 1.00059$  and a dielectric strength of  $E_m = 3 \times 10^6$  V/m.



# Molecular model of induced charge

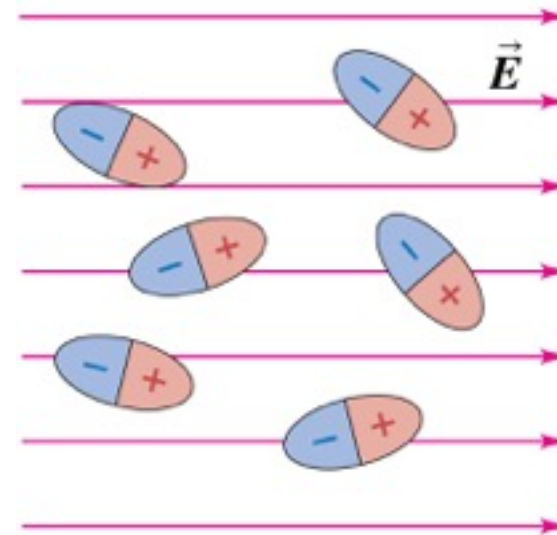
- Some molecules, such as  $\text{H}_2\text{O}$  and  $\text{N}_2\text{O}$ , form natural electric dipoles, and the molecule is called a **polar molecule**.
- When no electric field is present in a gas or liquid with polar molecules, the molecules are oriented randomly (a).
- In an electric field, however, they tend to orient themselves as in (b).

(a)



In the absence of an electric field, polar molecules orient randomly.

(b)

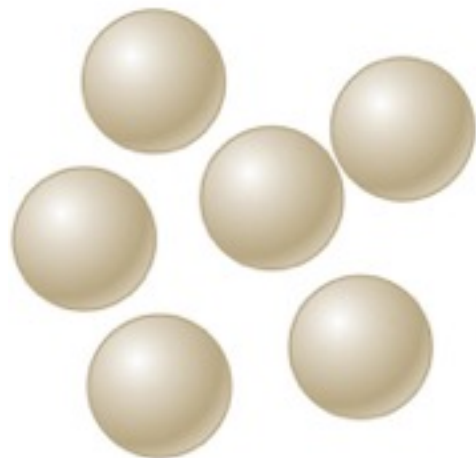


When an electric field is applied, the molecules tend to align with it.

# Molecular model of induced charge

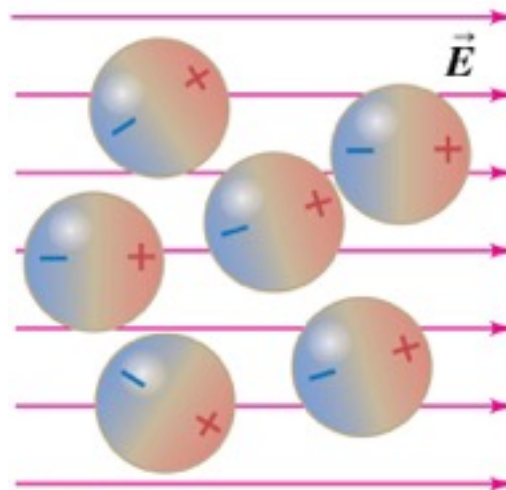
- Even a molecule that is not ordinarily polar (a) becomes a dipole when it is placed in an electric field because the field pushes the positive charges in the molecules in the direction of the field and pushes the negative charges in the opposite direction (b).
- Such dipoles are called *induced* dipoles.

(a)



In the absence of an electric field, nonpolar molecules are not electric dipoles.

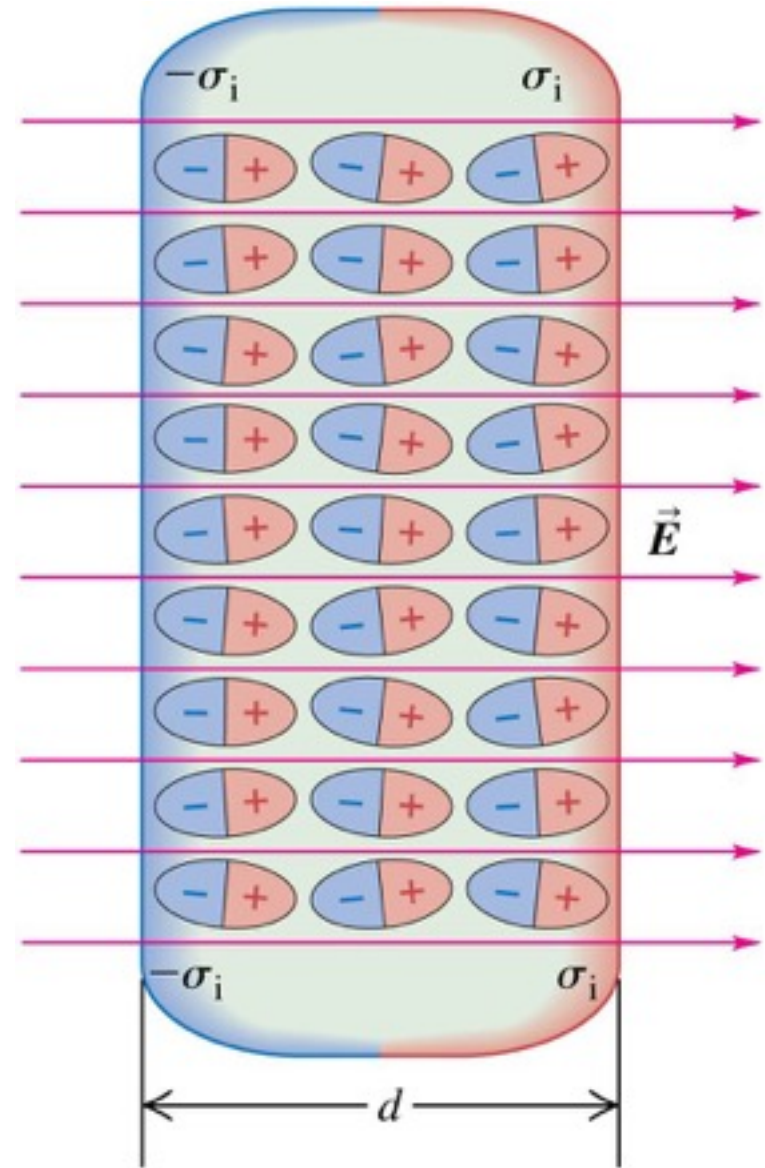
(b)



An electric field causes the molecules' positive and negative charges to separate slightly, making the molecule effectively polar.

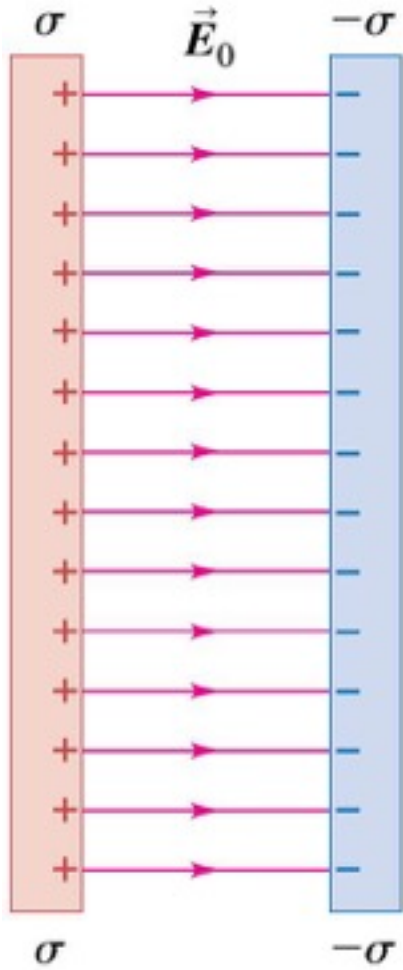
# Molecular model of induced charge

- The polarization of molecules within a dielectric leads to the formation of a layer of charge on each surface of the dielectric material.
- These layers have a surface charge density of magnitude  $\sigma_i$ .

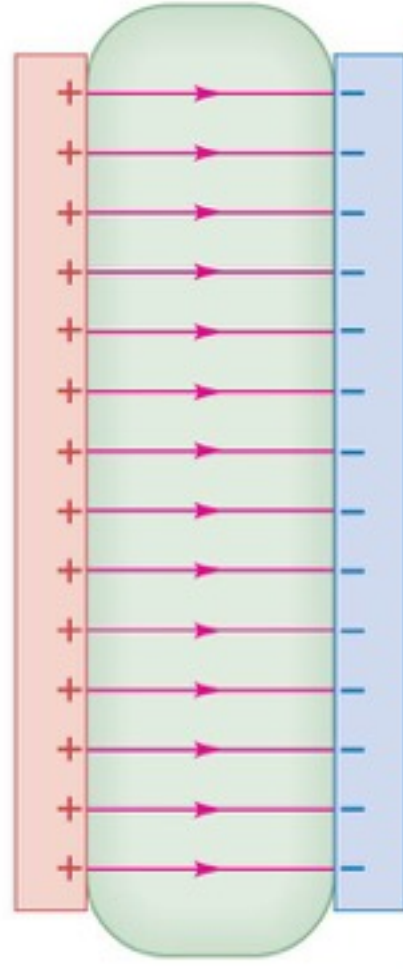


# Behavior of a dielectric in four steps

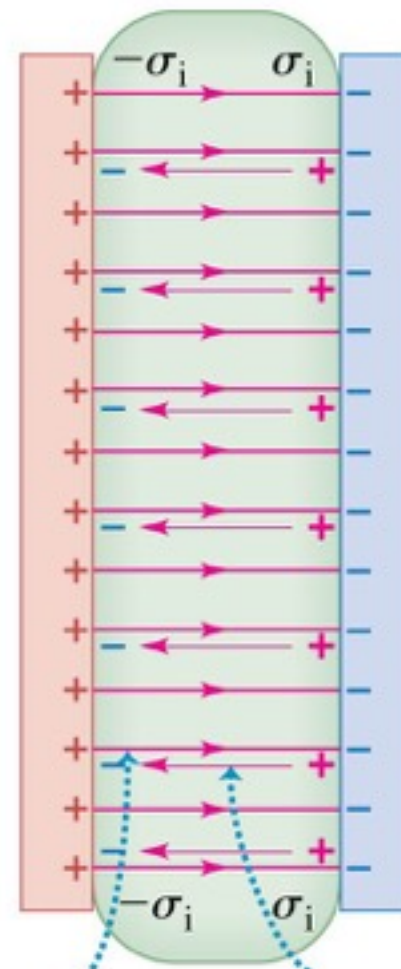
(a) No dielectric



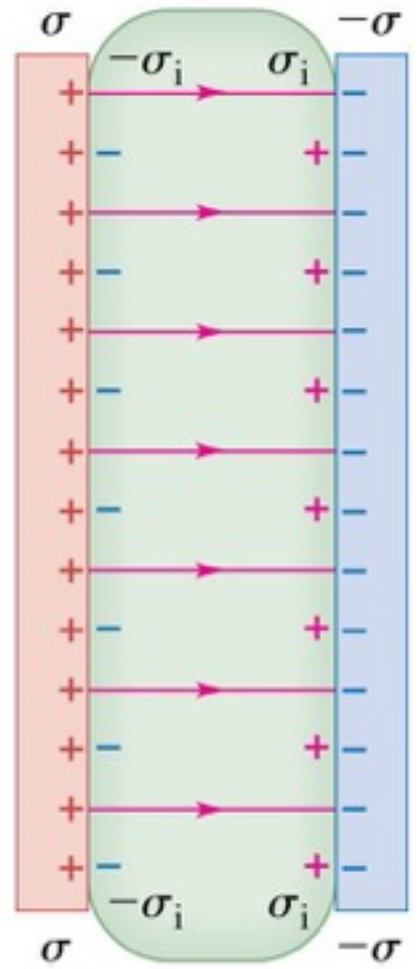
(b) Dielectric just inserted



(c) Induced charges create electric field



(d) Resultant field



Original  
electric field

Weaker field in dielectric  
due to induced (bound) charges