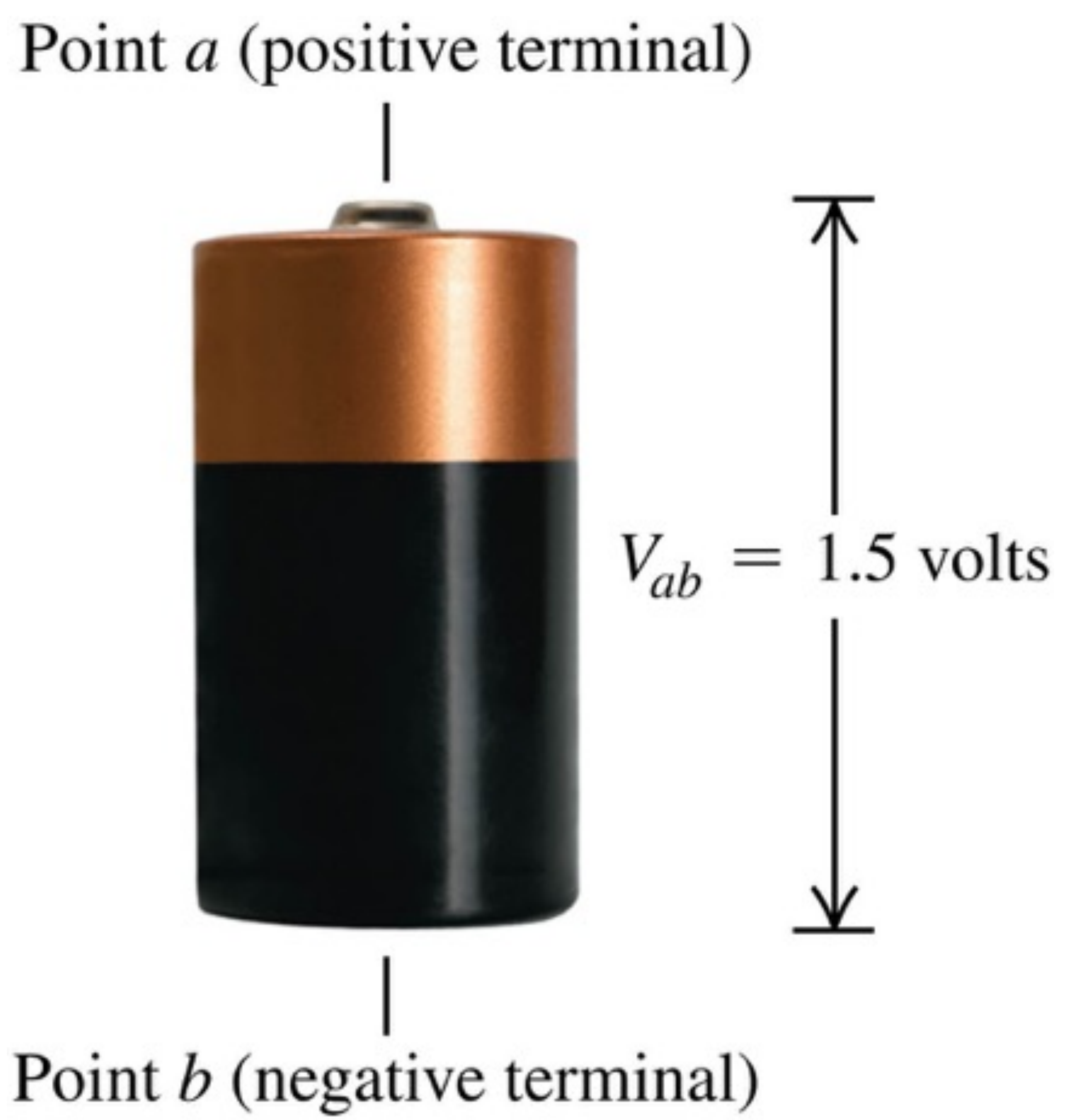
Lecture 17

PHYC 161 Fall 2016

Electric potential

- **Potential** is *potential energy per unit charge*.
- The potential of a with respect to b ($V_{ab} = V_a - V_b$) equals the work done by the electric force when a *unit* charge moves from a to b.



Electric potential

- The potential due to a single point charge is:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

Electric potential due to a point charge: V; Value of point charge: q; Distance from point charge to where potential is measured: r; Electric constant: ϵ_0

- Like electric field, potential is independent of the test charge that we use to define it.
- For a collection of point charges:

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Electric potential due to a collection of point charges: V; Value of ith point charge: q_i; Distance from ith point charge to where potential is measured: r_i; Electric constant: ϵ_0

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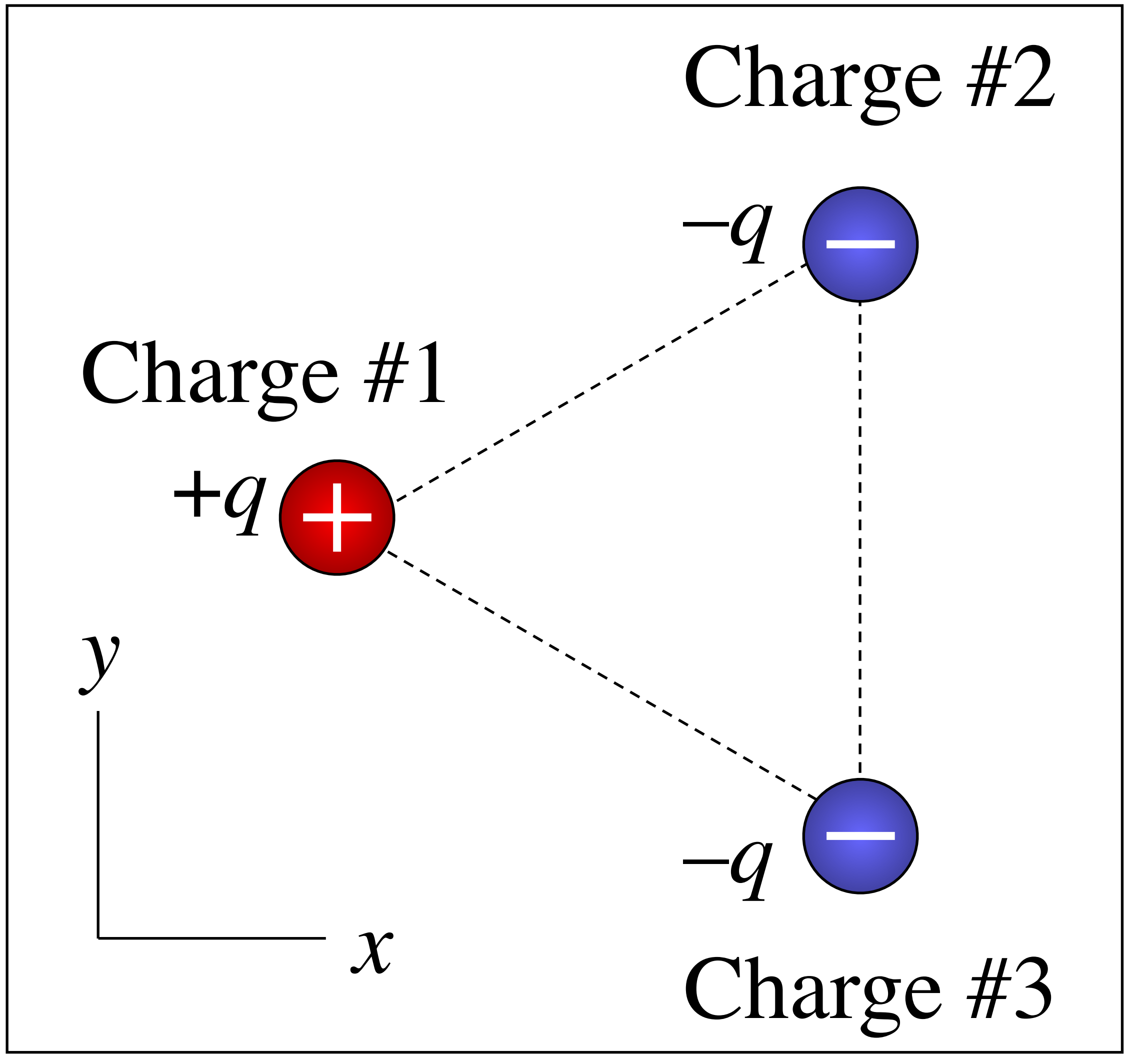
The electric potential due to a point charge approaches zero as you move farther away from the charge. If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential at the center of the triangle is



A. positive.

B. negative.

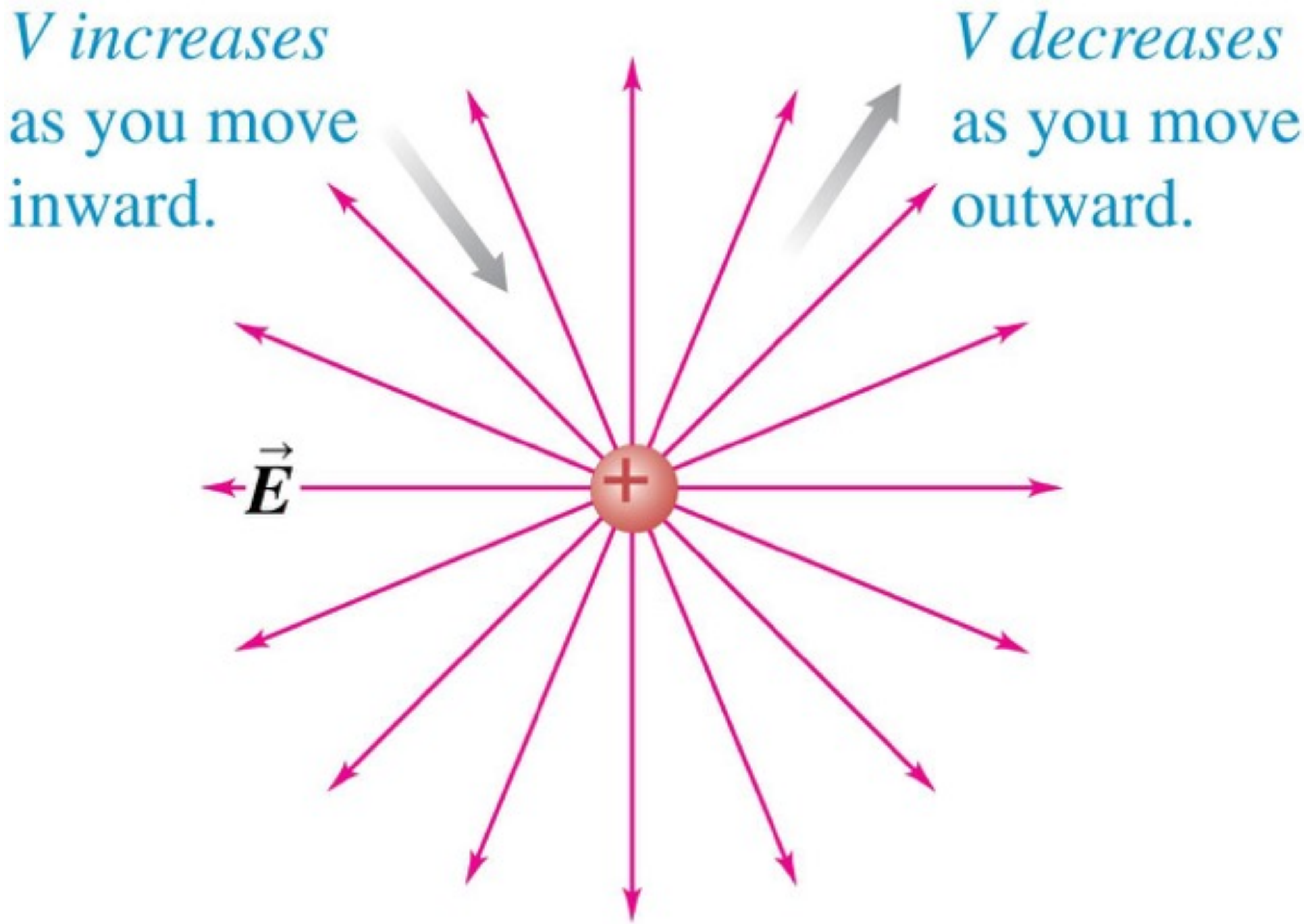
C. zero.

D. either positive or negative.

E. either positive, negative, or zero.

Finding electric potential from the electric field

- If you move in the direction of the electric field, the electric potential *decreases*, but if you move opposite the field, the potential *increases*.



Electric potential and electric field

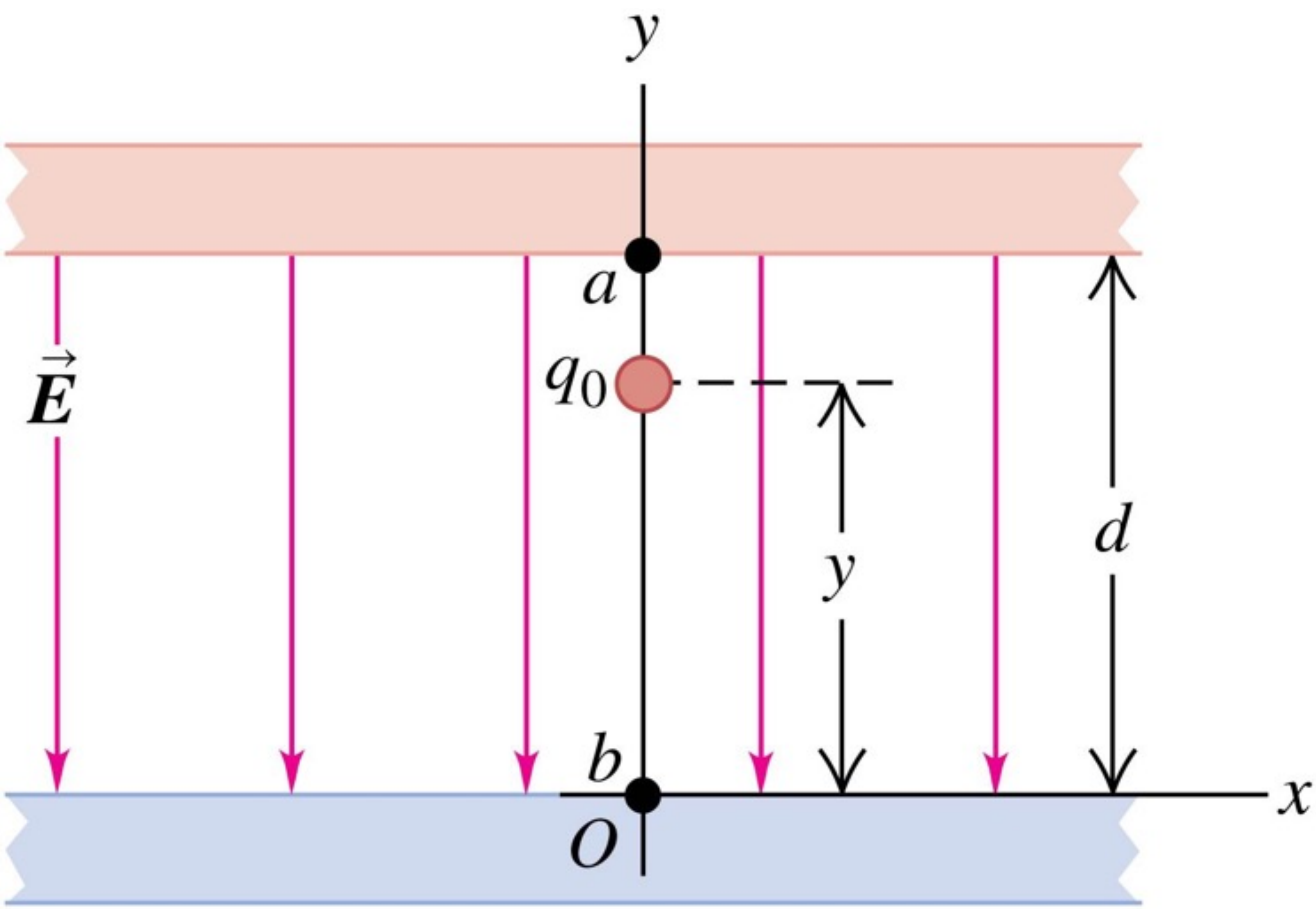
- Moving with the direction of the electric field means moving in the direction of decreasing V, and vice versa.
- To move a unit charge slowly against the electric force, we must apply an external force per unit charge equal and opposite to the electric force per unit charge.
- The electric force per unit charge is the electric field.
- The potential difference $V_a - V_b$ equals the work done per unit charge by this external force to move a unit charge from b to a:

$$V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{l}$$

- The unit of electric 1 N/C = 1 V/m.

Oppositely charged parallel plates

- The potential at any height y between the two large oppositely charged parallel plates is $V = Ey$.



The electron volt

- When a particle with charge q moves from a point where the potential is V_b to a point where it is V_a, the change in the potential energy U is

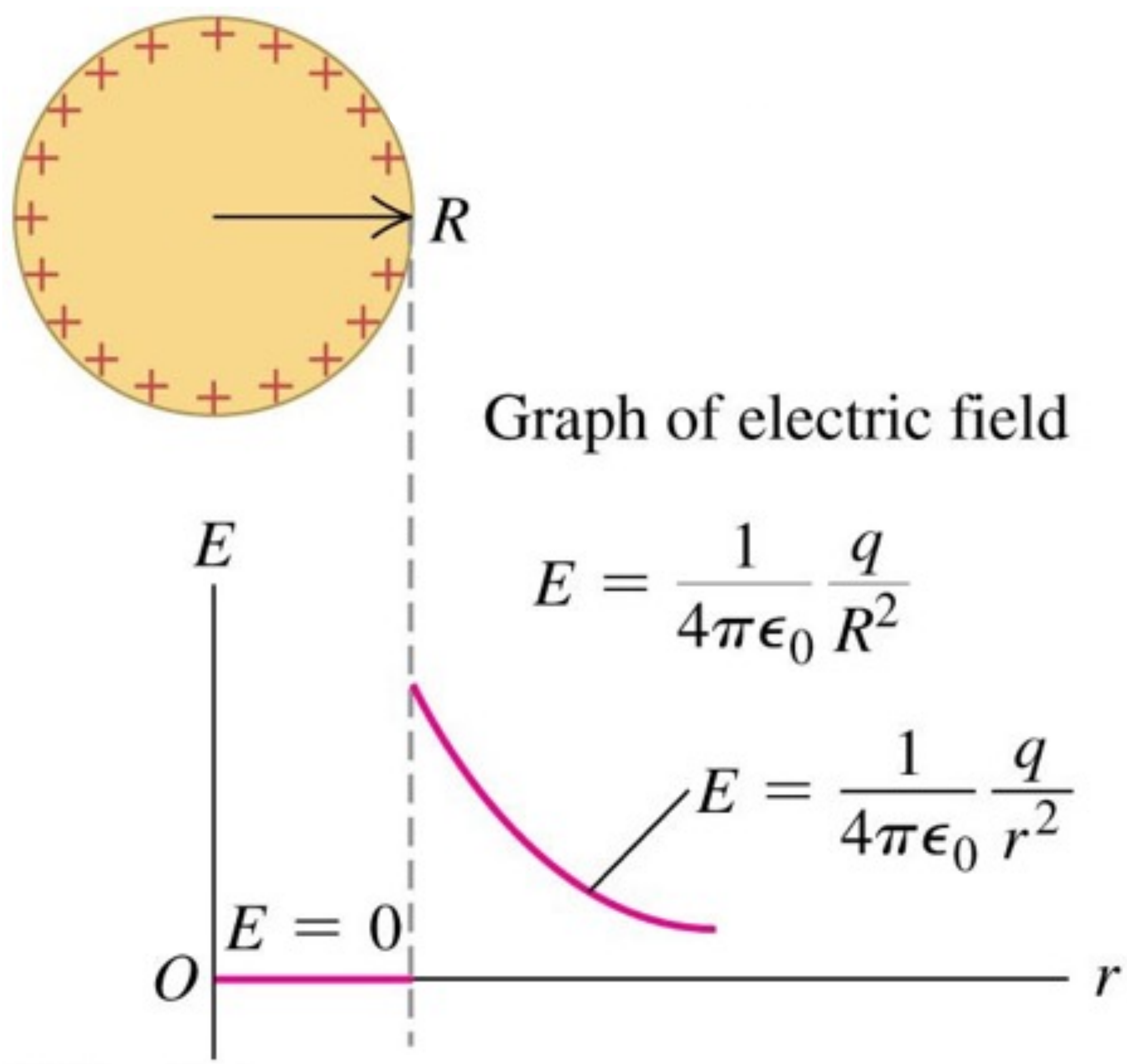
$$U_a - \mathrm{U}_b = q(V_a - V_b)$$

- If charge q equals the magnitude e of the electron charge, and the potential difference is 1 V, the change in energy is defined as one electron volt (eV):

$$1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$$

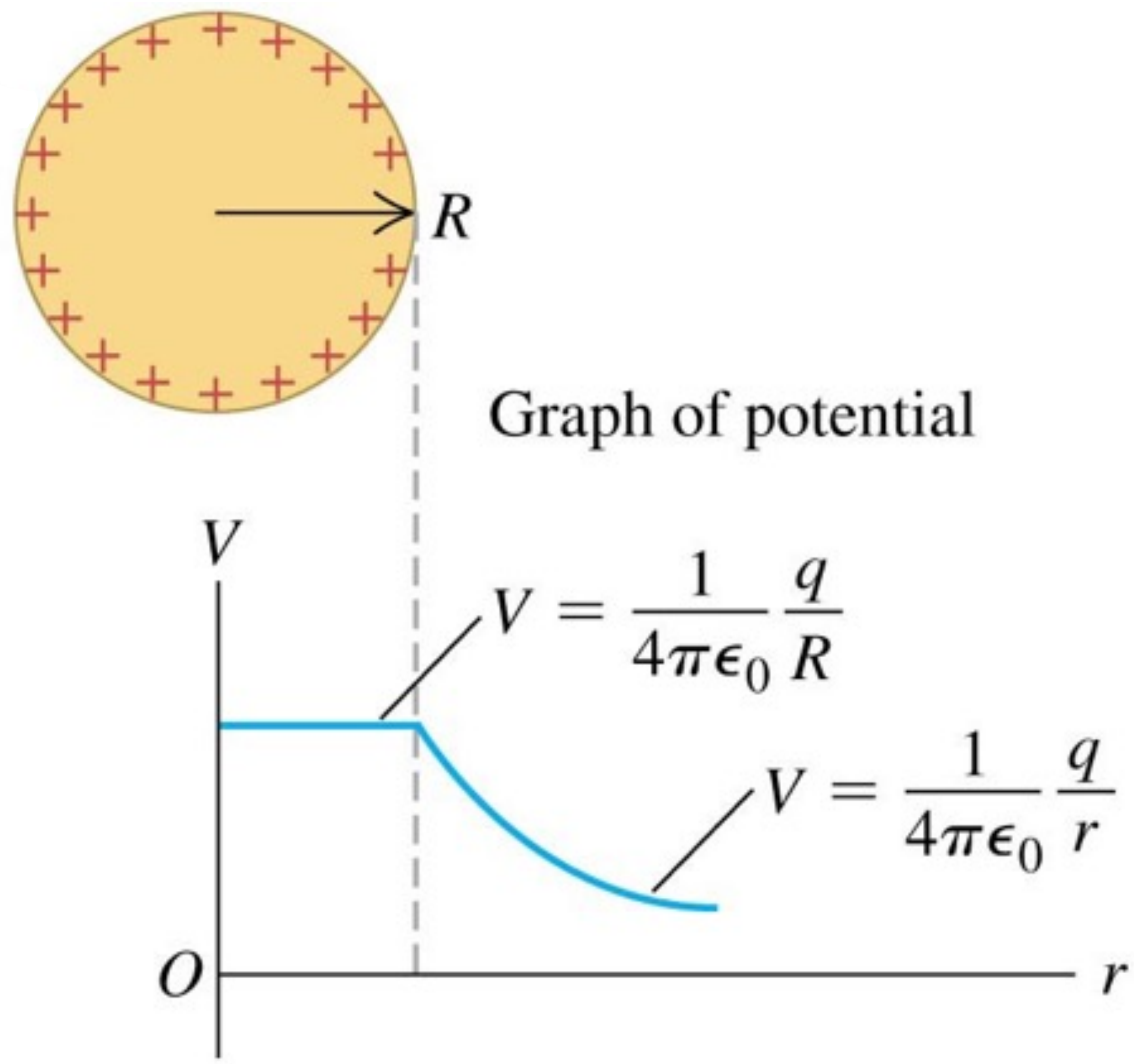
Electric potential and field of a charged conductor

- A solid conducting sphere of radius R has a total charge q.
- The electric field *inside* the sphere is zero everywhere.



Electric potential and field of a charged conductor

- The potential is the *same* at every point inside the sphere and is equal to its value at the surface.

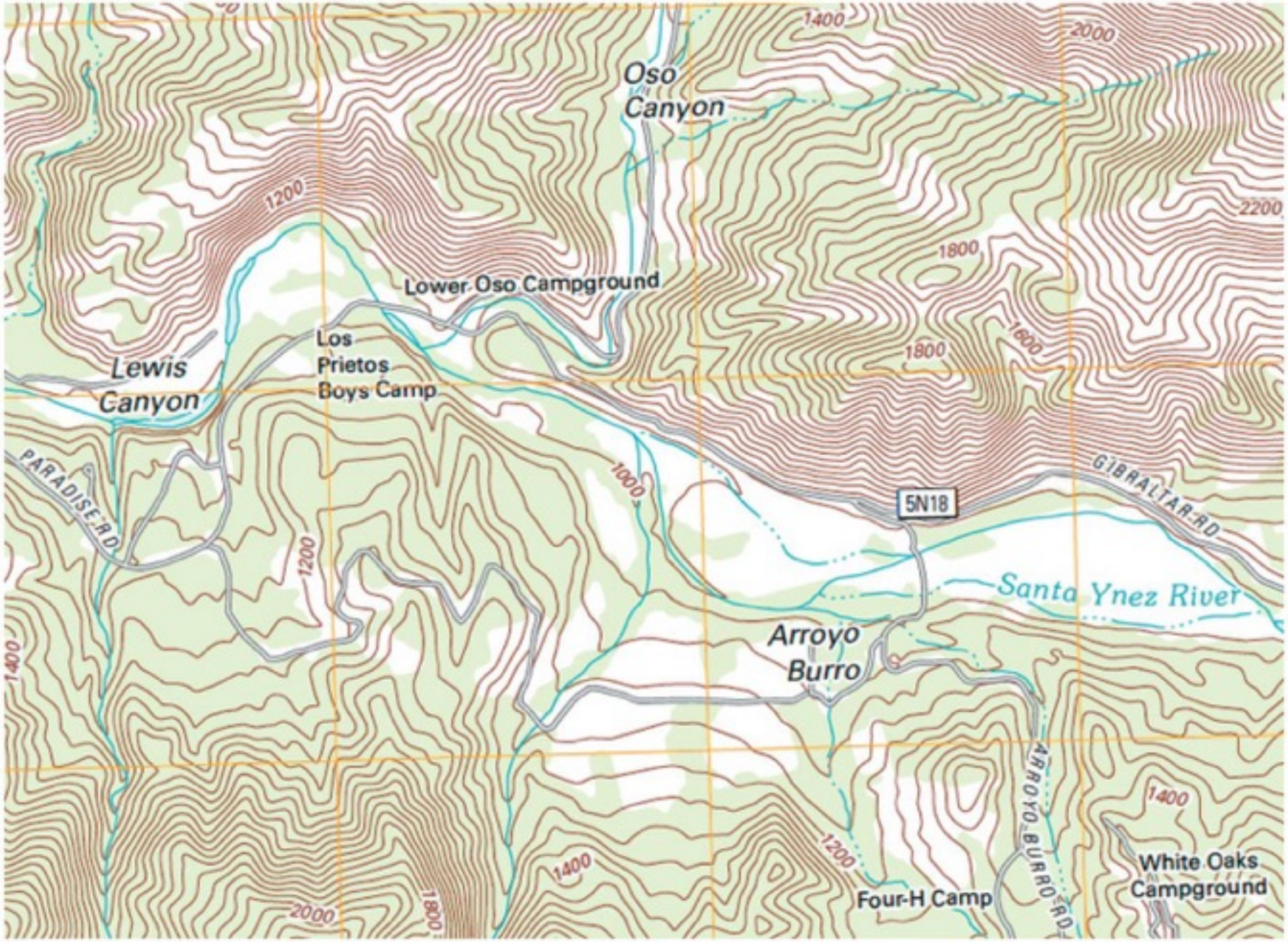


Ionization and corona discharge

- At an electric-field magnitude of about 3×10^6 V/m or greater, air molecules become ionized, and air becomes a conductor.
- For a charged conducting sphere, $V_{\text{surface}} = E_{\text{surface}} R$.
- Thus, if E_{m} is the electric-field magnitude at which air becomes conductive (known as the **dielectric strength** of air), then the maximum potential V_{m} to which a spherical conductor can be raised is $V_{\text{m}} = RE_{\text{m}}$.

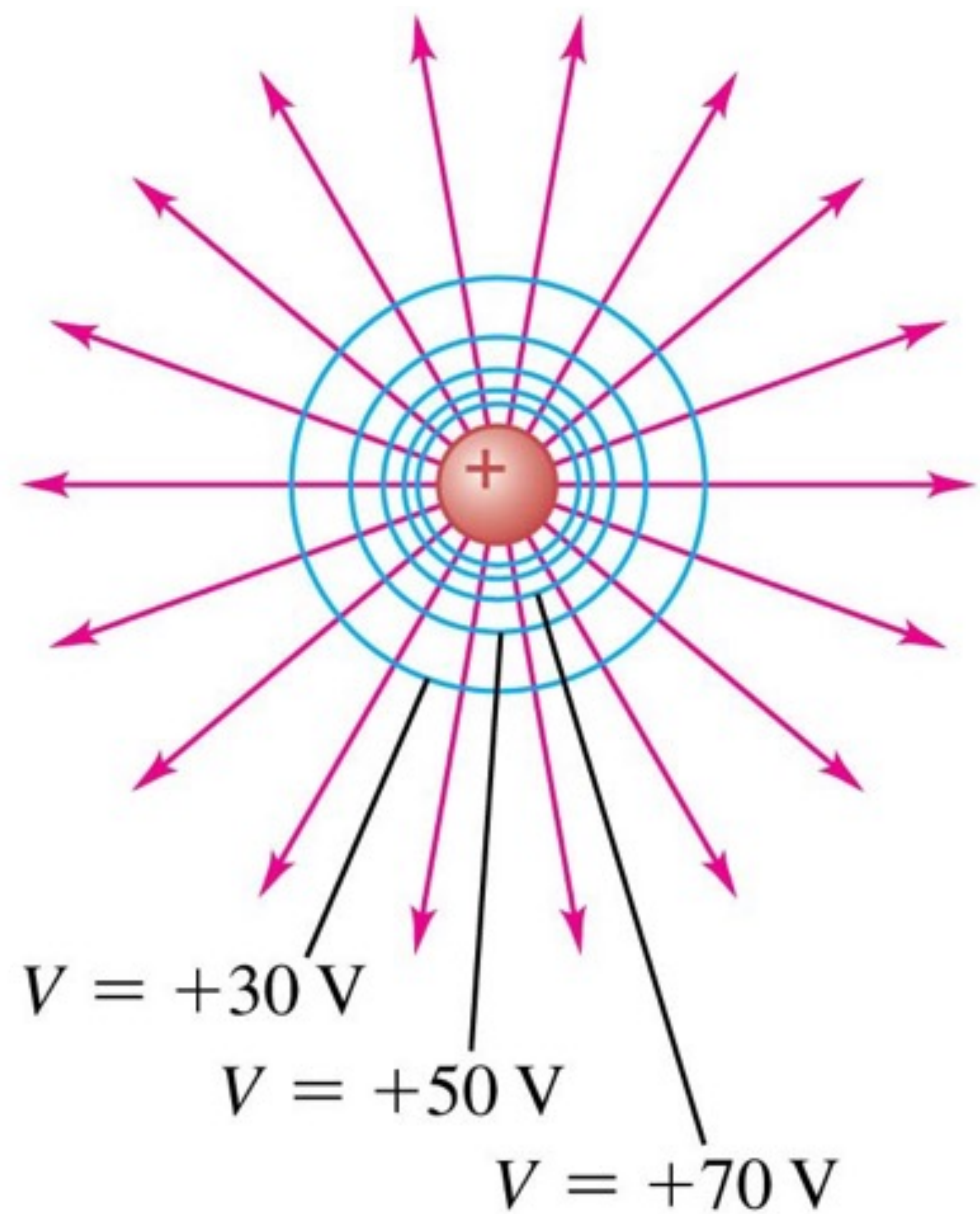
Equipotential surfaces

- Contour lines on a topographic map are curves of constant elevation and hence of constant gravitational potential energy.

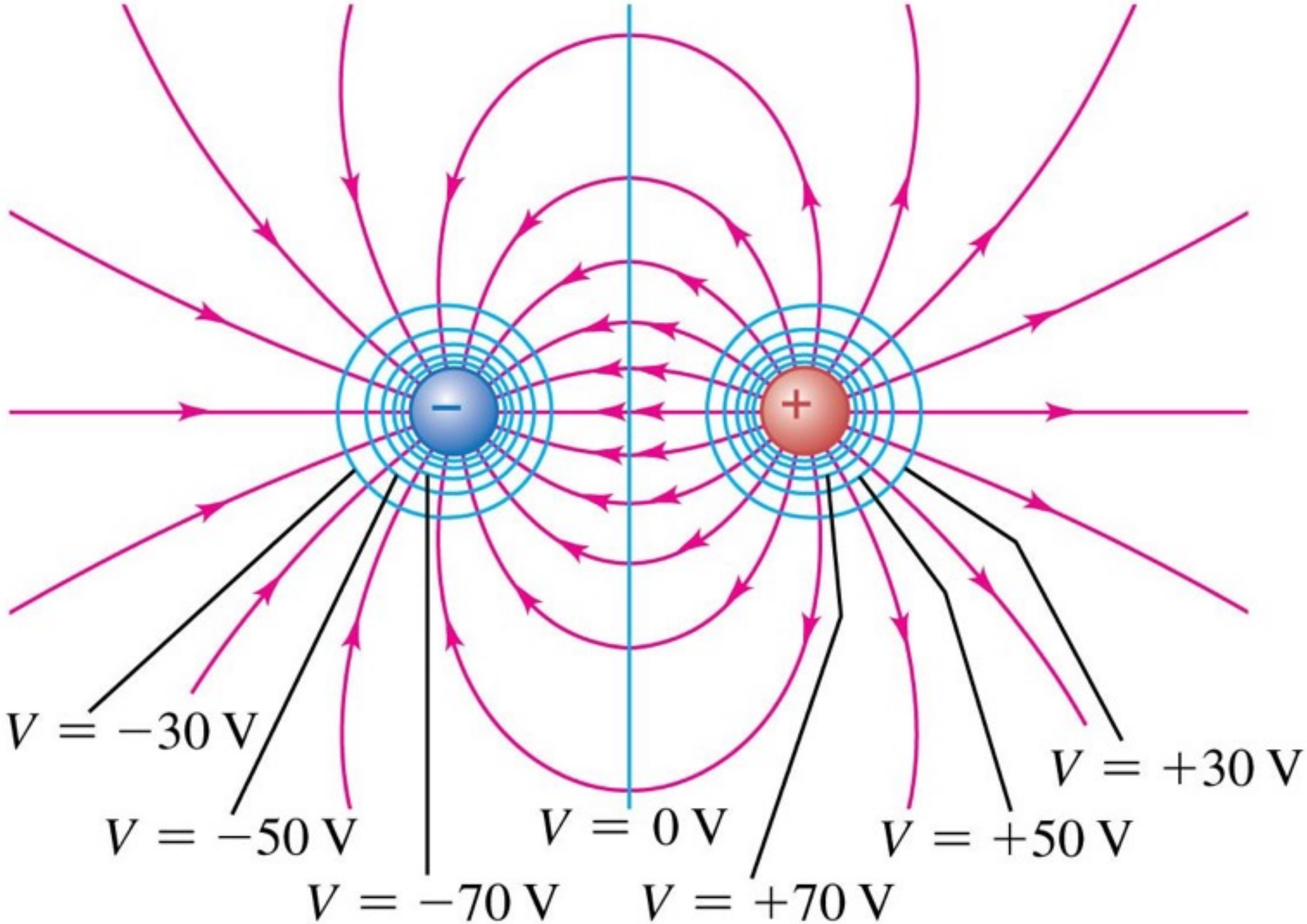


Equipotential surfaces and field lines

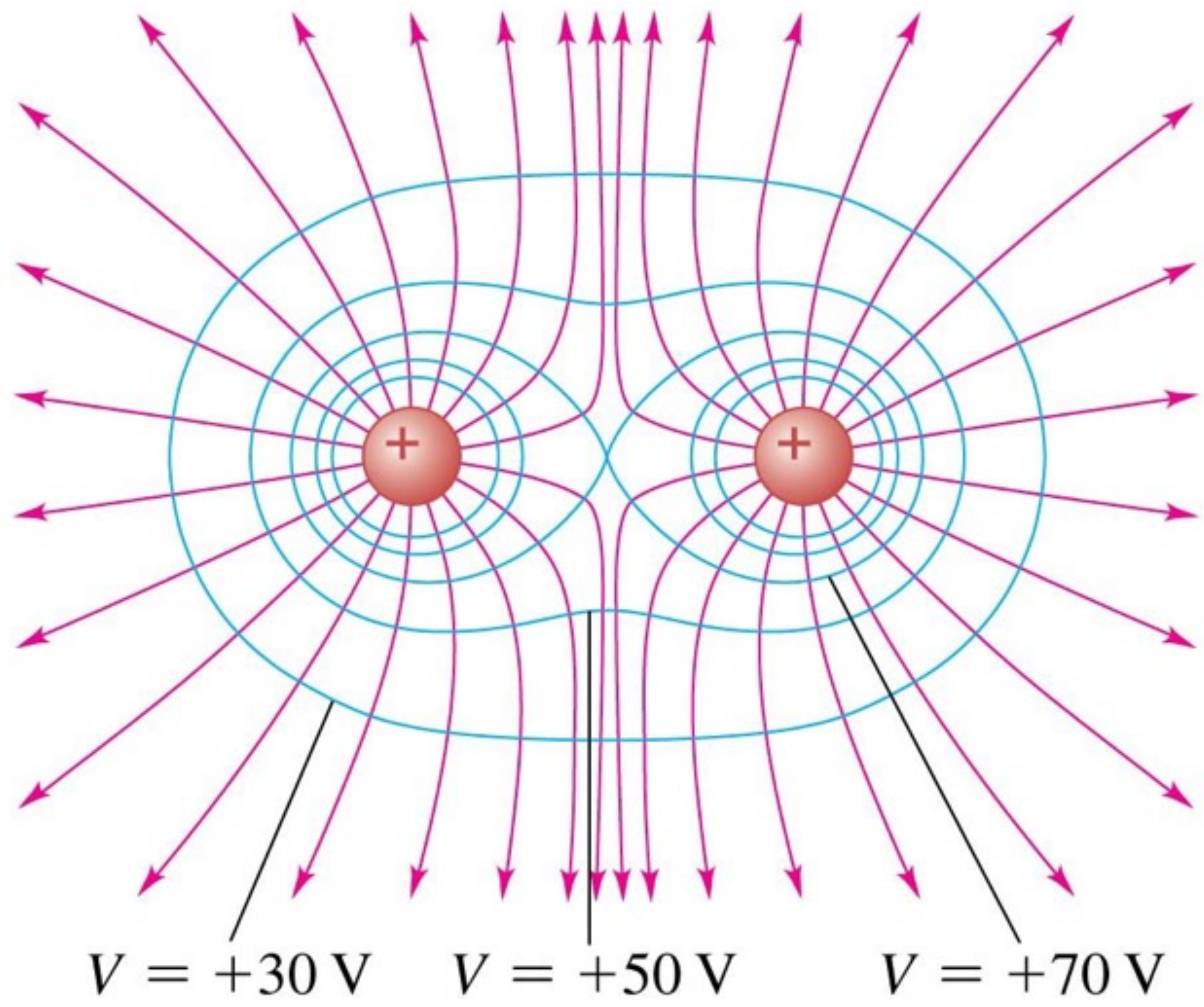
- An **equipotential surface** is a surface on which the electric potential is the same at every point.
- Field lines and equipotential surfaces are always mutually perpendicular.
- Shown are cross sections of equipotential surfaces (blue lines) and electric field lines (red lines) for a single positive charge.



Equipotential surfaces and field lines for a dipole

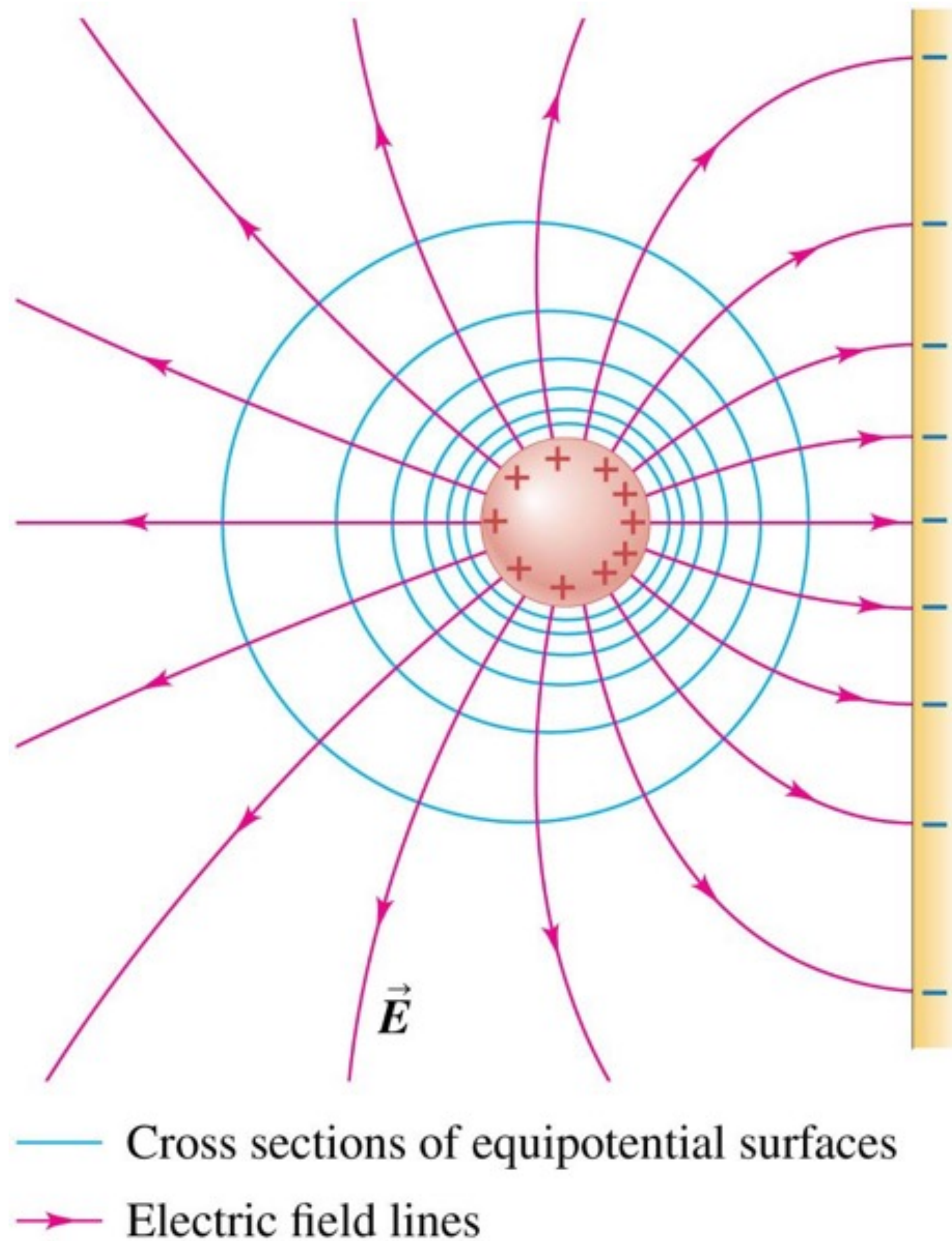


Field and potential of two equal positive charges



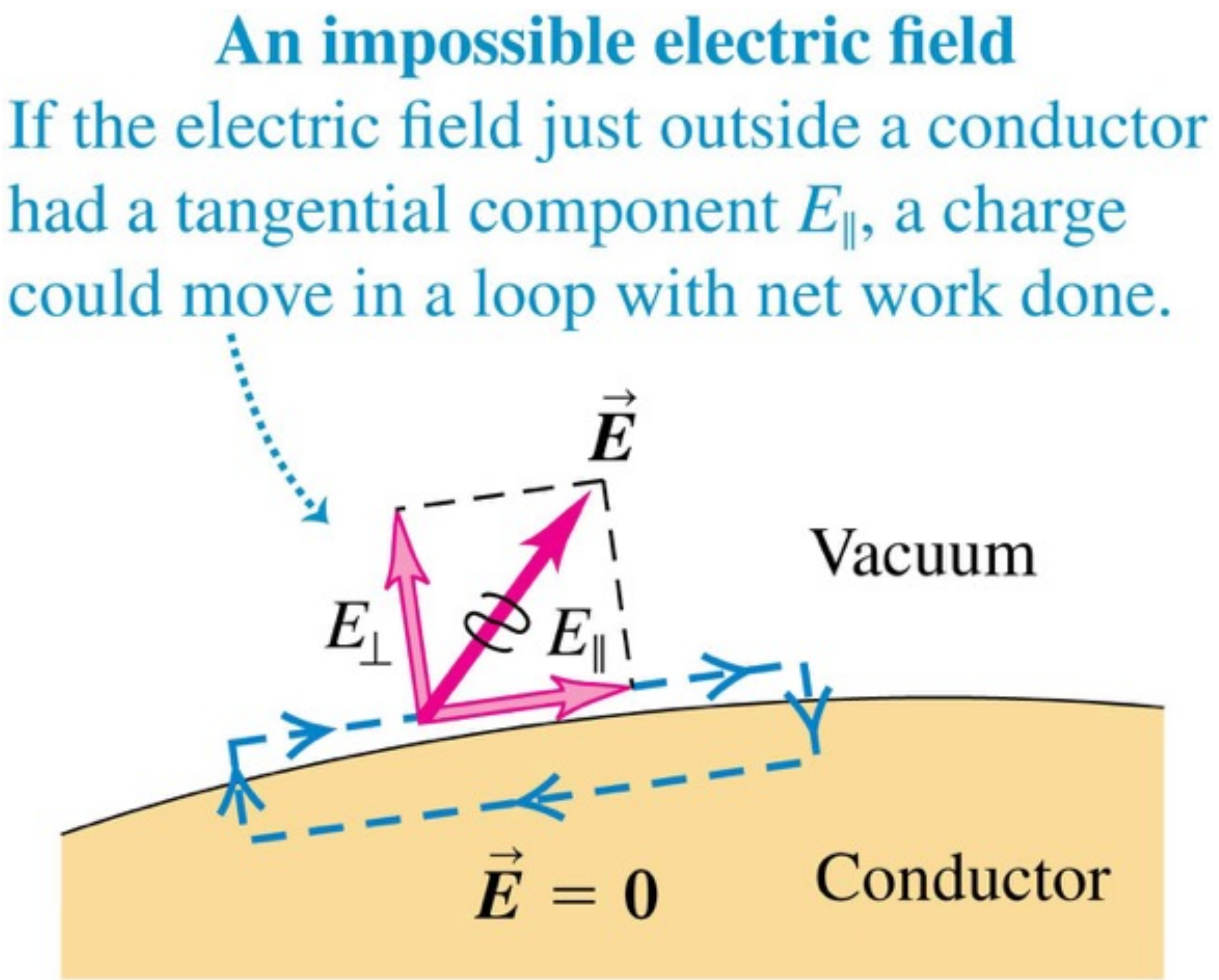
Equipotentials and conductors

- When all charges are at rest:
  - the surface of a conductor is always an equipotential surface.
  - the electric field just outside a conductor is always perpendicular to the surface.



Equipotentials and conductors

- If the electric field had a tangential component at the surface of a conductor, a net amount of work would be done on a test charge by moving it around a loop as shown here—which is impossible because the electric force is conservative.



Potential gradient

- The components of the electric field can be found by taking partial derivatives of the electric potential:

Electric field components found from potential: $E_x = -\dfrac{\partial V}{\partial x} \qquad E_y = -\dfrac{\partial V}{\partial y} \qquad E_z = -\dfrac{\partial V}{\partial z}$

Each electric field component ...

... equals the negative of the corresponding partial derivative of electric potential function V.

- The electric field is the negative gradient of the potential:

$$\vec{\boldsymbol{E}} = -\vec{\nabla} V$$

Electric Field from the Potential

- Let's say that somehow we have determined the electric potential everywhere in space from a charge distribution.

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{r} \Rightarrow$$

$$\int_a^b dV = -\int_a^b \vec{E} \cdot d\vec{r} \Rightarrow$$

$$dV = -\vec{E} \cdot d\vec{r}$$

$$dV = -\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right) \cdot \left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$dV = -E_x dx - E_y dy - E_z dz$$

Electric Field from the Potential

- If we now hold y and z constant (so that dy and dz are zero) then,

$$dV = -E_x dx - \cancel{E_y dy} - \cancel{E_z dz} \Rightarrow$$

$$dV = -E_x dx \Rightarrow$$

$$E_x = -\left.\frac{dV}{dx}\right|_{\text{y and z constant}} \equiv -\frac{\partial V}{\partial x}$$

- Likewise then, for the other components of the electric field,

$$E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

Potential gradient

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Electric field components found from potential:

Each electric field component ...

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