Lecture 13

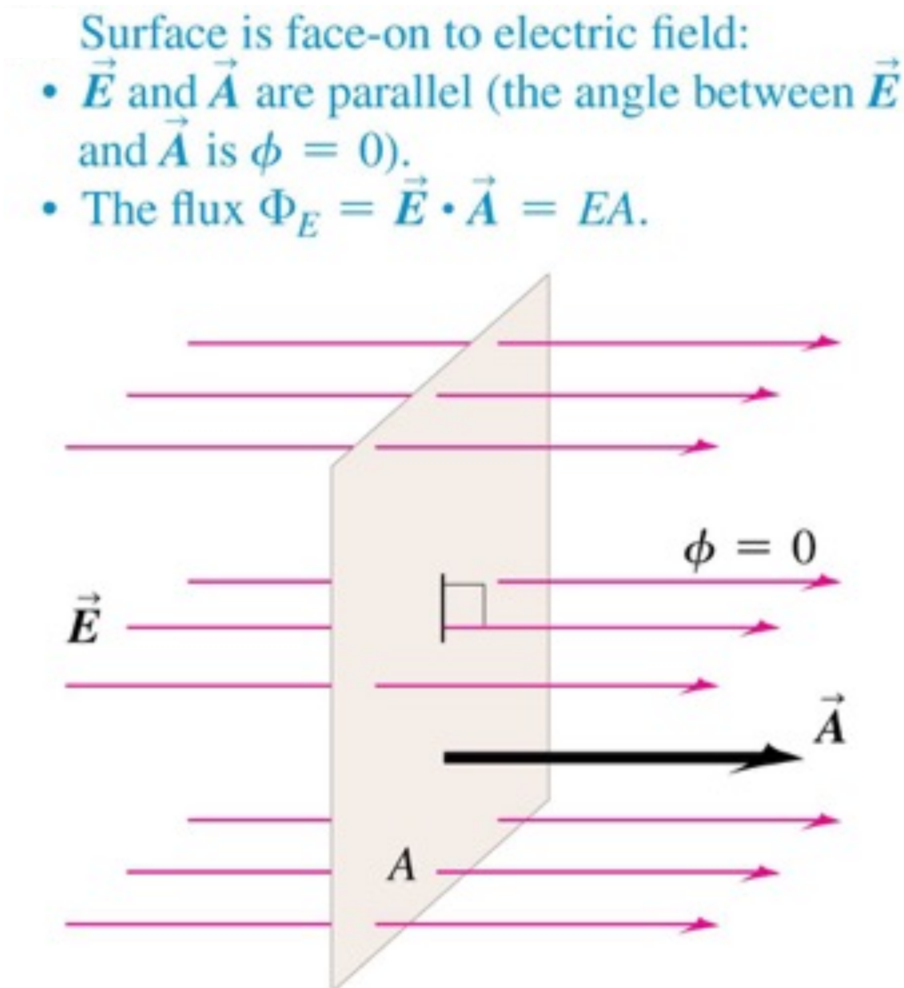
PHYC 161 Fall 2016

Gauss's law

- Carl Friedrich Gauss helped develop several branches of mathematics, including differential geometry, real analysis, and number theory.
- The "bell curve" of statistics is one of his inventions.
- Gauss also made state-of-the-art investigations of the earth's magnetism and calculated the orbit of the first asteroid to be discovered.
- While completely equivalent to Coulomb's law, **Gauss's law** provides a different way to express the relationship between electric charge and electric field.

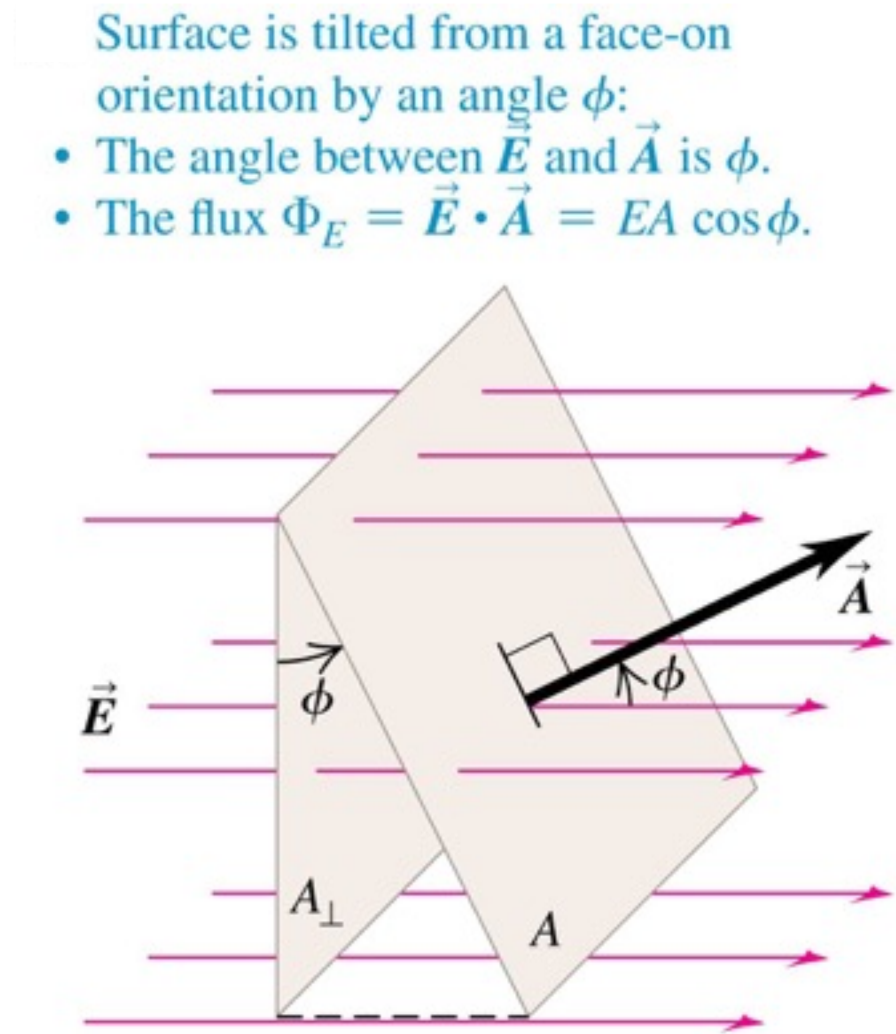
Calculating electric flux

- Consider a flat area perpendicular to a uniform electric field.
- Increasing the area means that more electric field lines pass through the area, increasing the flux.
- A stronger field means more closely spaced lines, and therefore more flux.



Calculating electric flux

- If the area is not perpendicular to the field, then fewer field lines pass through it.
- In this case the area that counts is the silhouette area that we see when looking in the direction of the field.



Flux of a nonuniform electric field

- In general, the flux through a surface must be computed using a **surface integral** over the area:

$$\Phi_E = \int E\cos\phi\, dA = \int E_\perp\, dA = \int \vec{E} \cdot d\vec{A}$$

Electric flux through a surface (Φ_E)

Magnitude of electric field $\vec{E}$ (E)

Angle between $\vec{E}$ and normal to surface (ϕ)

Element of surface area (dA)

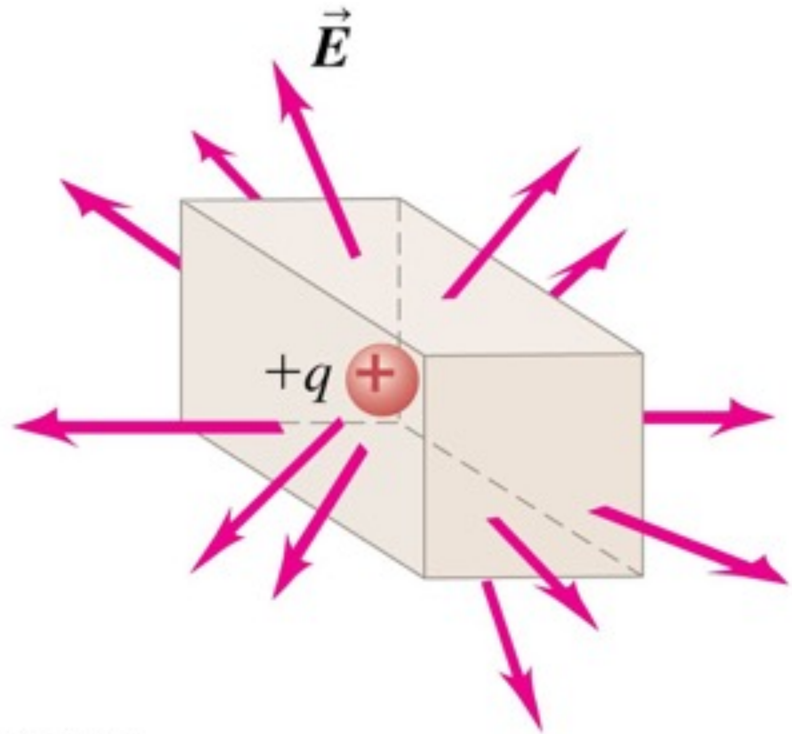
Component of $\vec{E}$ perpendicular to surface ($E_\perp$)

Vector element of surface area ($d\vec{A}$)

- The SI unit for electric flux is $1\ \text{N} \cdot \text{m}^2/\text{C}$.

Q22.1

The figure shows a Gaussian surface with rectangular sides and positive point charge $+q$ at its center. If all the dimensions of the Gaussian surface double, but charge $+q$ remains at its center, the electric flux through the surface will



A. increase by a factor of 4.

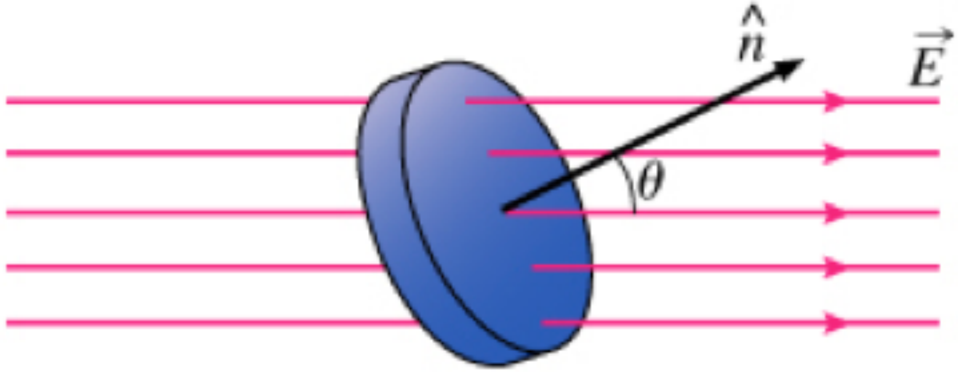
B. increase by a factor of 2.

C. remain the same.

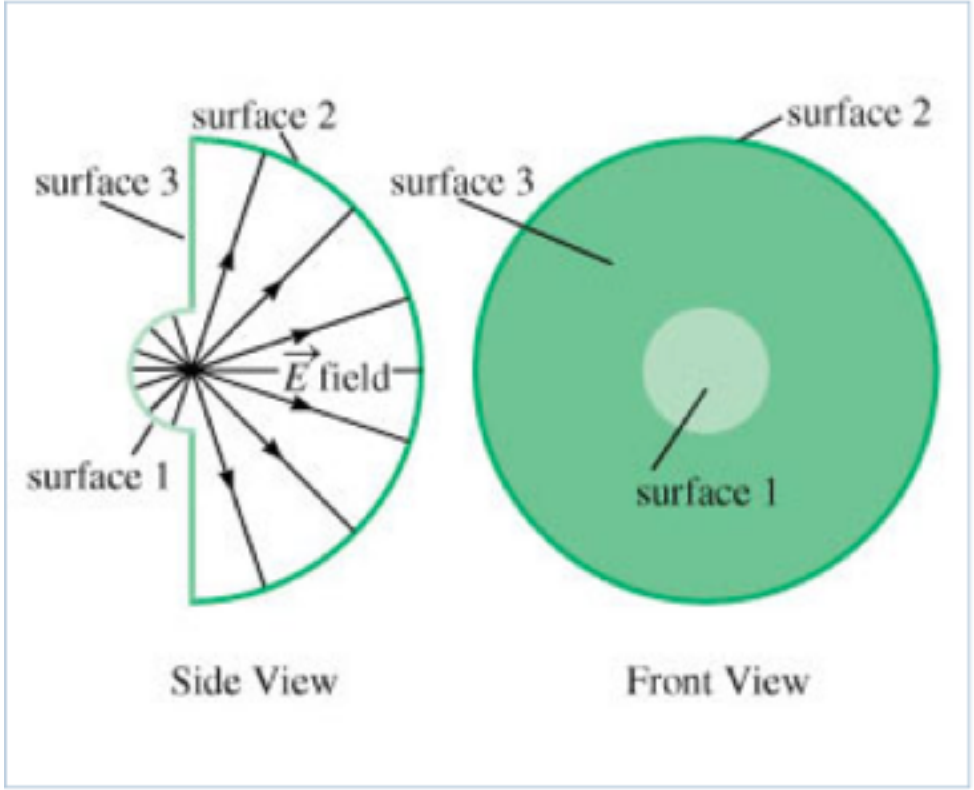
D. decrease by a factor of 1/2.

E. decrease by a factor of 1/4.

On your HW:



$\hat{n}$
$\vec{E}$
θ



surface 2
surface 3
$\vec{E}$ field
surface 1
surface 2
surface 3
surface 1
Side View
Front View

General form of Gauss's law

- Let Q_{encl} be the total charge enclosed by a surface.
- Gauss's law states that **the total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by** ε_0**:**

Gauss's law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

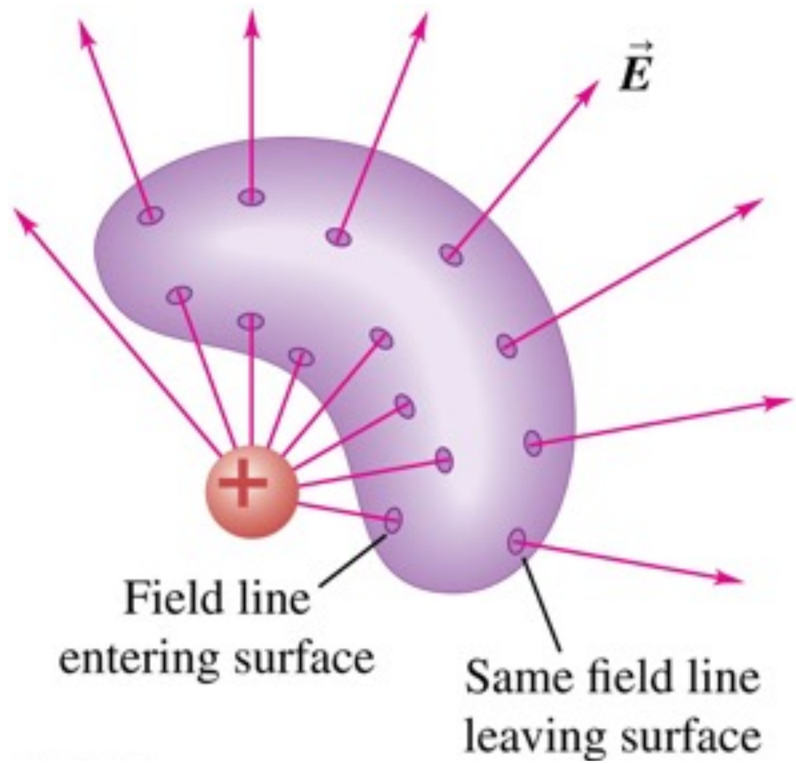
- Q_{encl}: Total charge enclosed by surface
- $\Phi_E = \oint \vec{E} \cdot d\vec{A}$: Electric flux through a closed surface of area A = surface integral of $\vec{E}$
- ϵ_0: Electric constant

Gauss's law in a vacuum

- For a closed surface enclosing no charge:

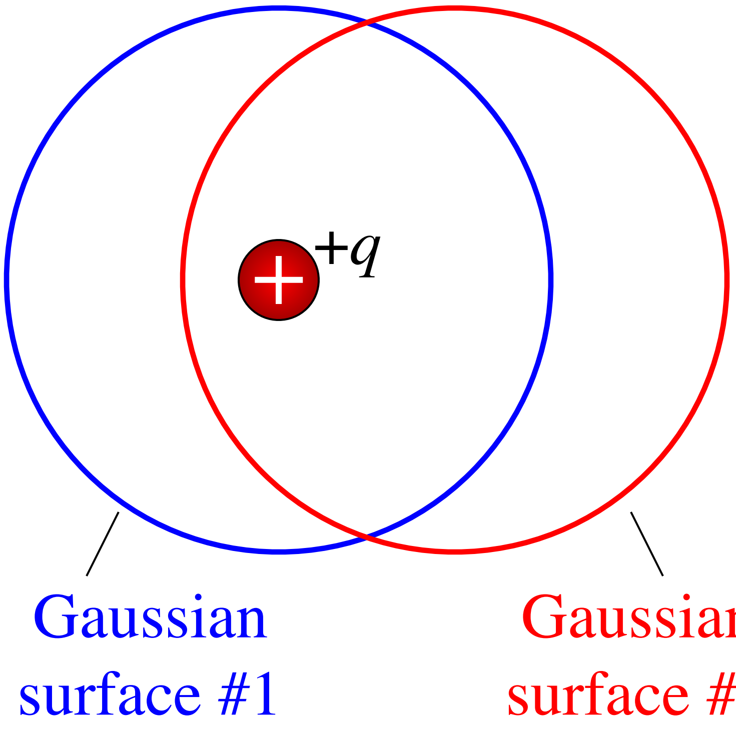
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = 0$$

- The figure shows a point charge *outside* a closed surface that encloses no charge.
- If an electric field line from the external charge enters the surface at one point, it must leave at another.



Q22.2

Spherical Gaussian surface #1 has point charge $+q$ at its center. Spherical Gaussian surface #2, of the same size, also encloses the charge but is not centered on it. There are no other charges inside either Gaussian surface. Compared to the electric flux through surface #1, the flux through surface #2 is



A. greater.

B. the same.

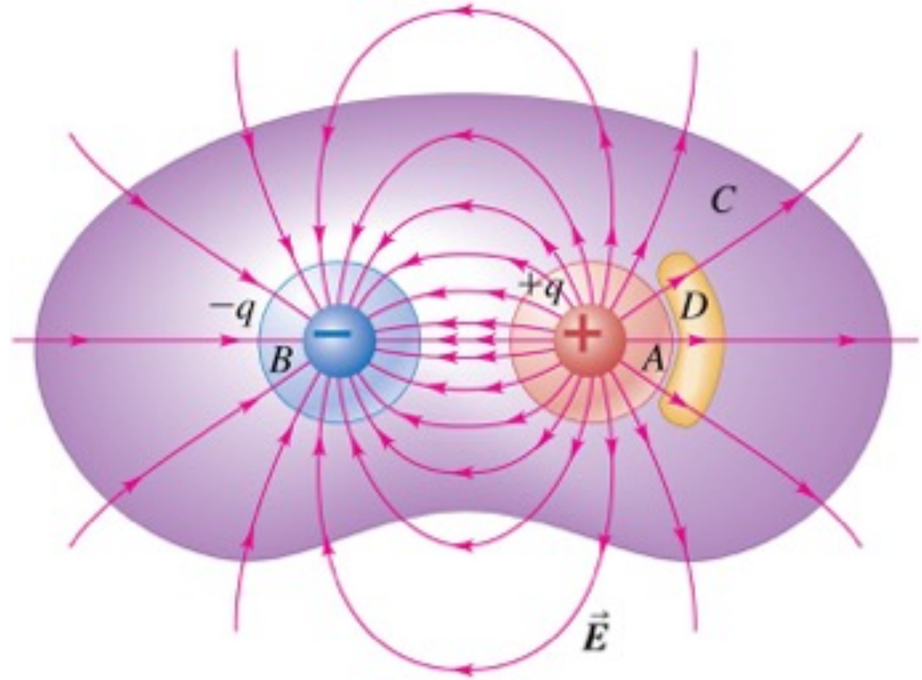
C. less, but not zero.

D. zero.

E. Not enough information is given to decide.

Q22.3

Two point charges, $+q$ (in red) and $-q$ (in blue), are arranged as shown. Through which closed surface(s) is/are the net electric flux equal to zero?



A. surface A

B. surface B

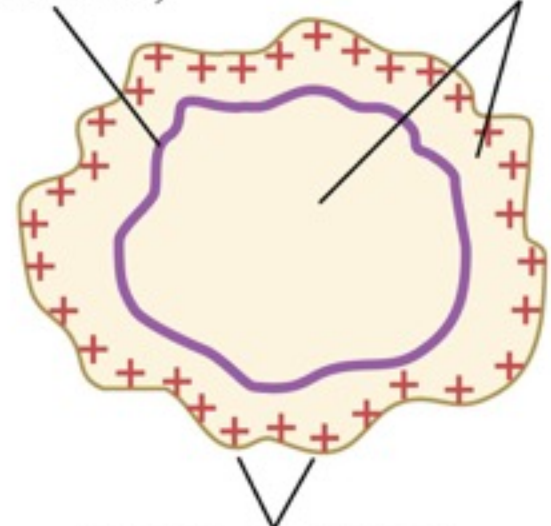
C. surface C

D. surface D

E. both surface C and surface D

Applications of Gauss's law

- Suppose we construct a Gaussian surface inside a conductor.
- Because $\vec{E} = 0$ everywhere on this surface, Gauss's law requires that the net charge inside the surface is zero.
- Under electrostatic conditions (charges not in motion), any excess charge on a solid conductor resides entirely on the conductor's surface.



Q22.5

There is a negative surface charge density in a certain region on the surface of a solid conductor. Just beneath the surface of this region, the electric field

A. points outward, toward the surface of the conductor.

B. points inward, away from the surface of the conductor.

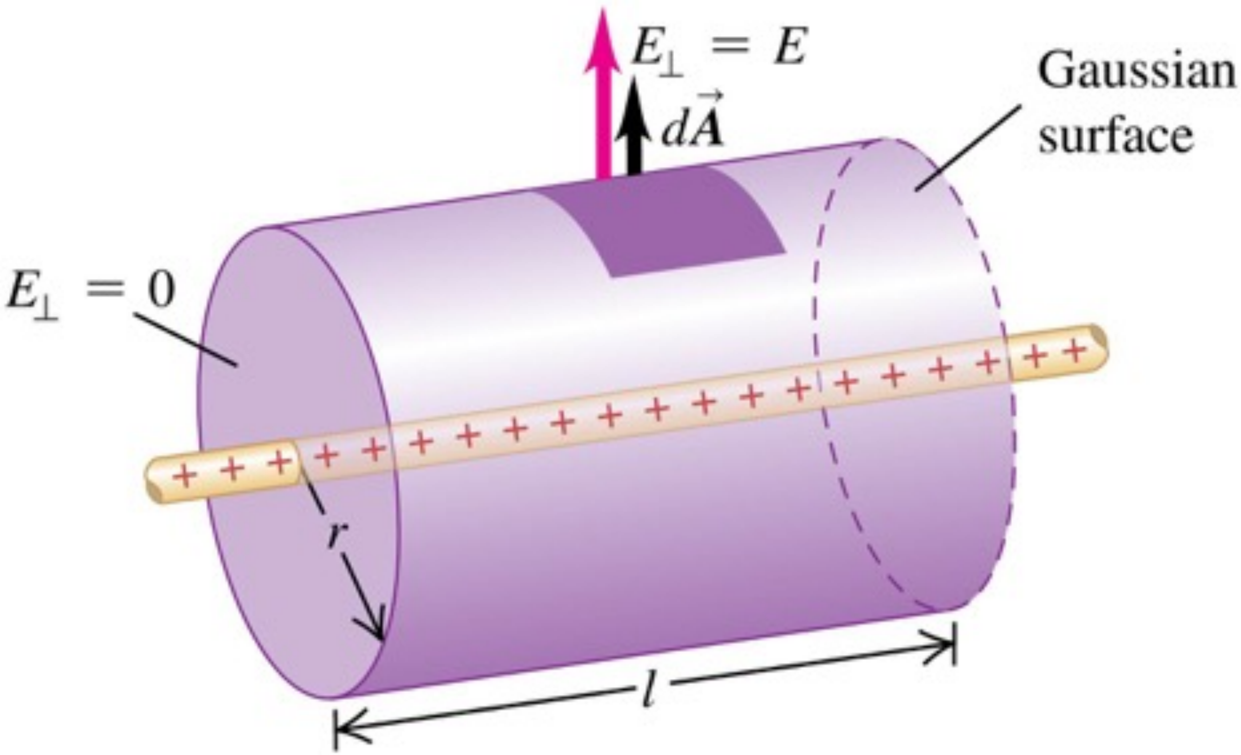
C. points parallel to the surface.

D. is zero.

E. Not enough information is given to decide.

Field of a uniform line charge

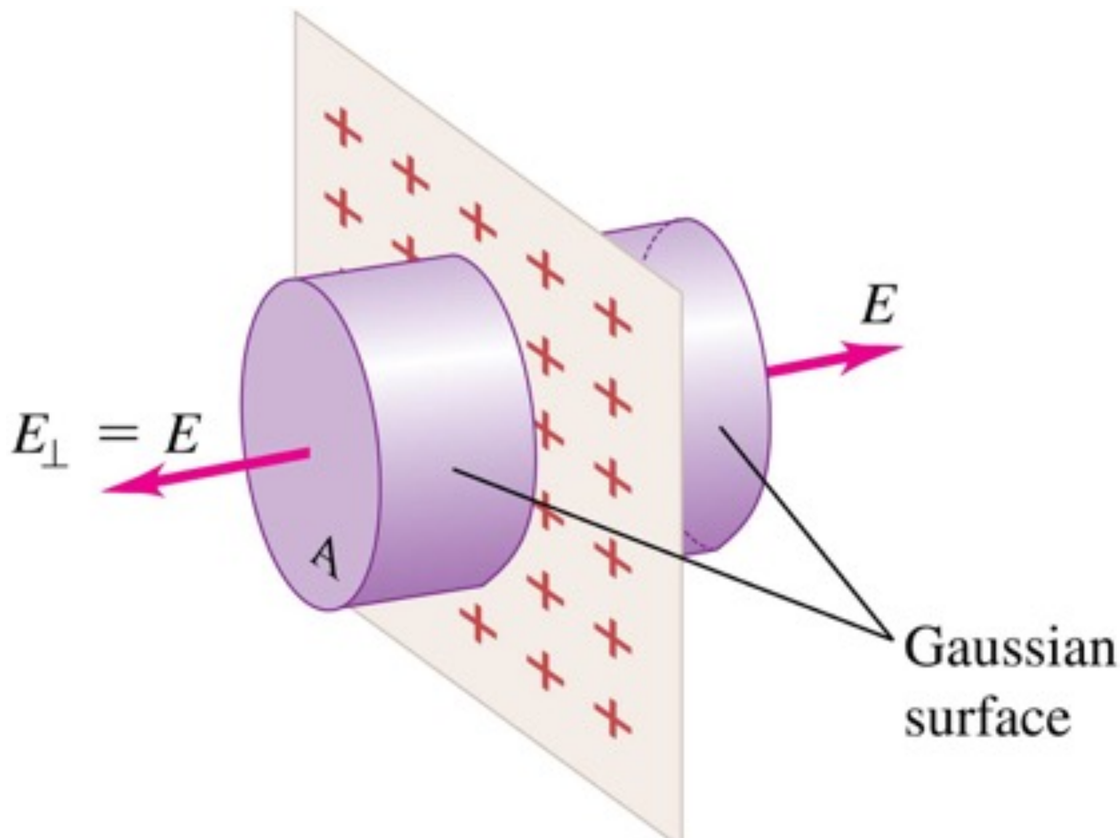
- Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is λ (assumed positive).
- Using Gauss's law, we can find the electric field: $E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$



Field of an infinite plane sheet of charge

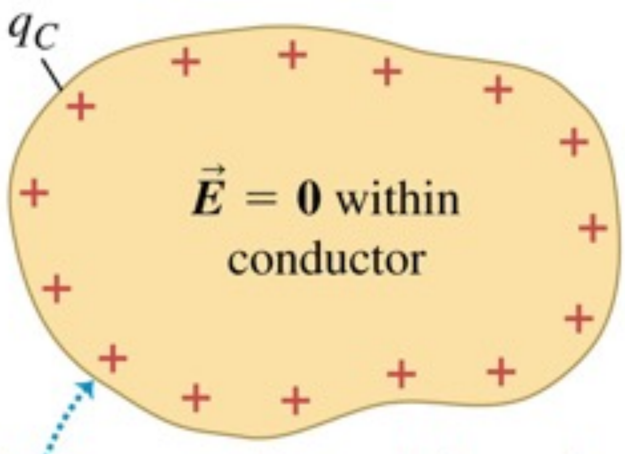
- Gauss's law can be used to find the electric field caused by a thin, flat, infinite sheet with a uniform positive surface charge density σ.

$$E = \frac{\sigma}{2\epsilon_0}$$



Charges on conductors

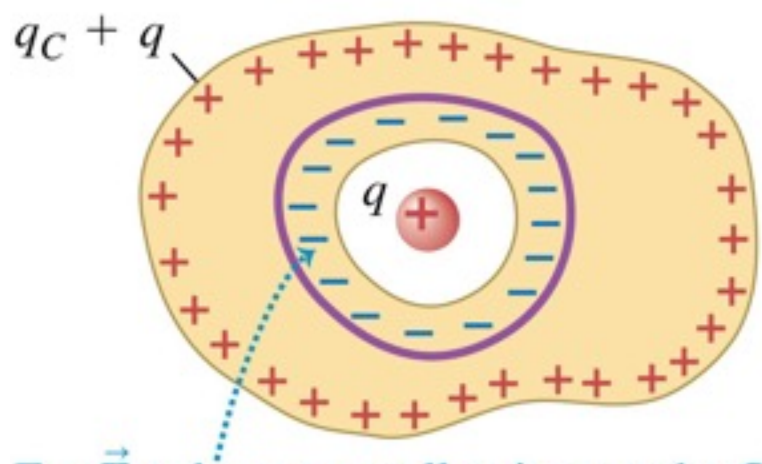
- Consider a solid conductor with a hollow cavity inside.
- If there is no charge within the cavity, we can use a Gaussian surface such as *A* to show that the net charge on the surface of the cavity must be zero, because $\vec{E} = 0$ everywhere on the Gaussian surface.



The charge q_C resides entirely on the surface of the conductor. The situation is electrostatic, so $\vec{E} = \mathbf{0}$ within the conductor.

Charges on conductors

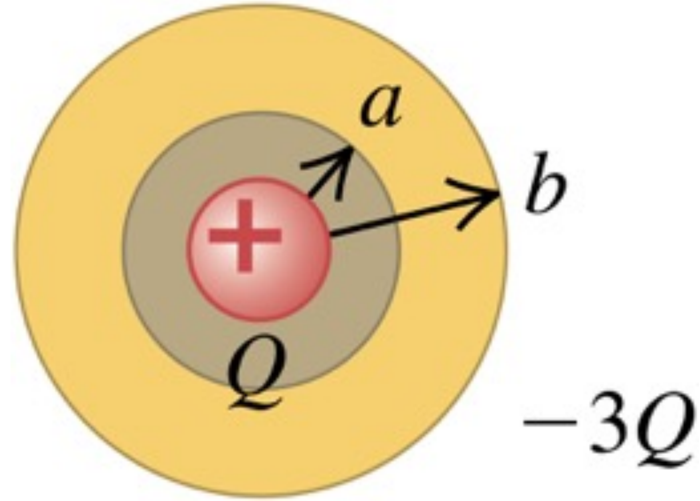
- Suppose we place a small body with a charge q inside a cavity within a conductor. The conductor is uncharged and is insulated from the charge q.
- According to Gauss's law the total there must be a charge $-q$ distributed on the surface of the cavity, drawn there by the charge q inside the cavity.
- The total charge on the conductor must remain zero, so a charge $+q$ must appear on its outer surface.



For $\vec{E}$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$.

Q22.4

A conducting spherical shell with inner radius a and outer radius b has a positive point charge Q located at its center. The total charge on the shell is $-3Q$, and it is insulated from its surroundings. In the region $a < r < b$,



A. the electric field points radially outward.

B. the electric field points radially inward.

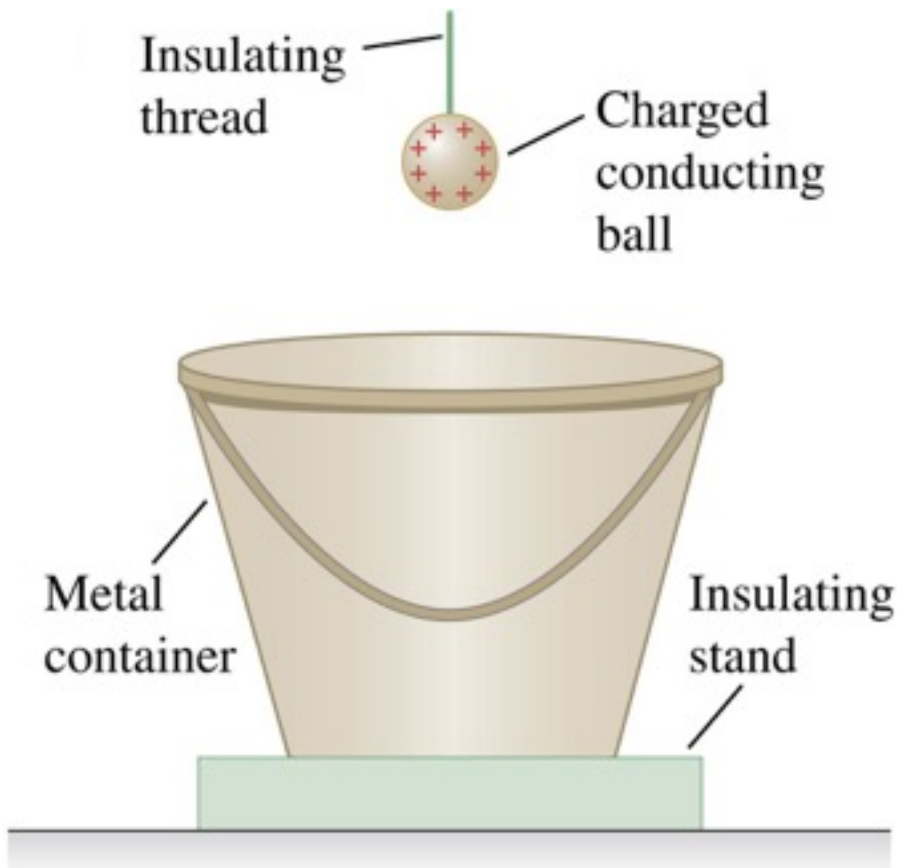
C. the electric field points radially outward in parts of the region and radially inward in other parts of the region.

D. the electric field is zero.

E. Not enough information is given to decide.

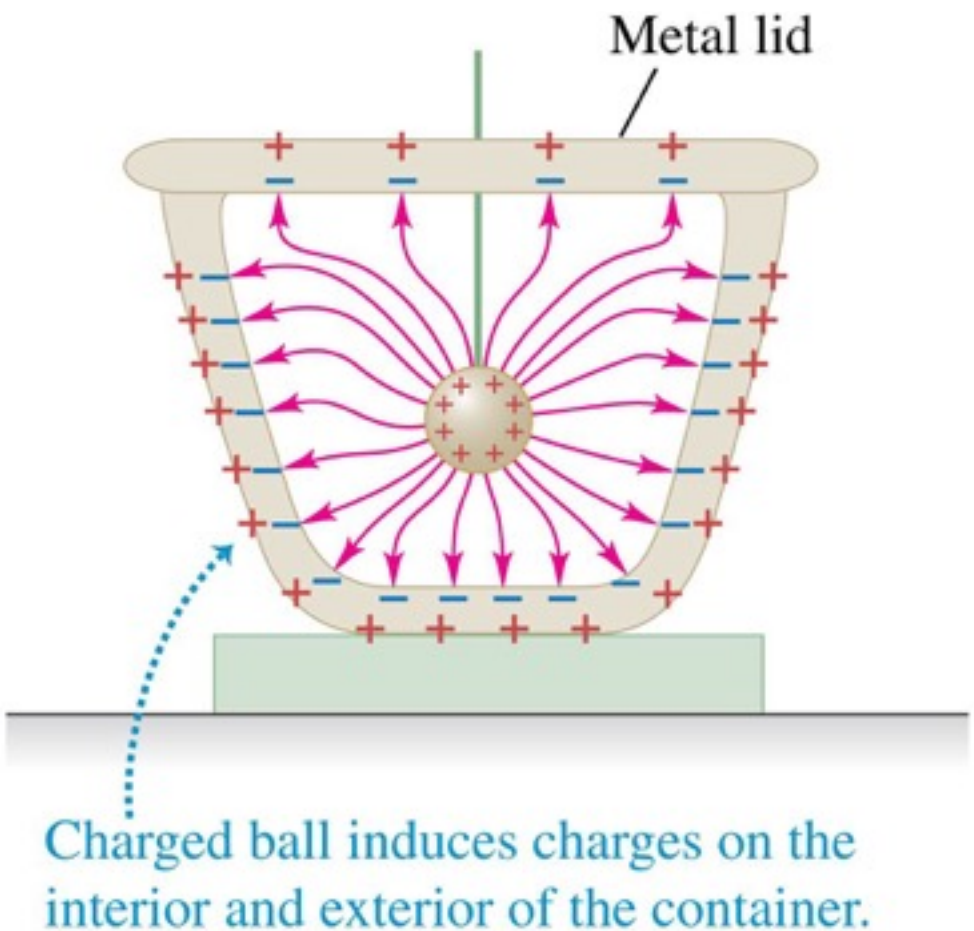
Faraday’s icepail experiment: Slide 1 of 3

- We now consider Faraday’s historic **icepail experiment**.
- We mount a conducting container on an insulating stand.
- The container is initially uncharged.
- Then we hang a charged metal ball from an insulating thread, and lower it into the container.



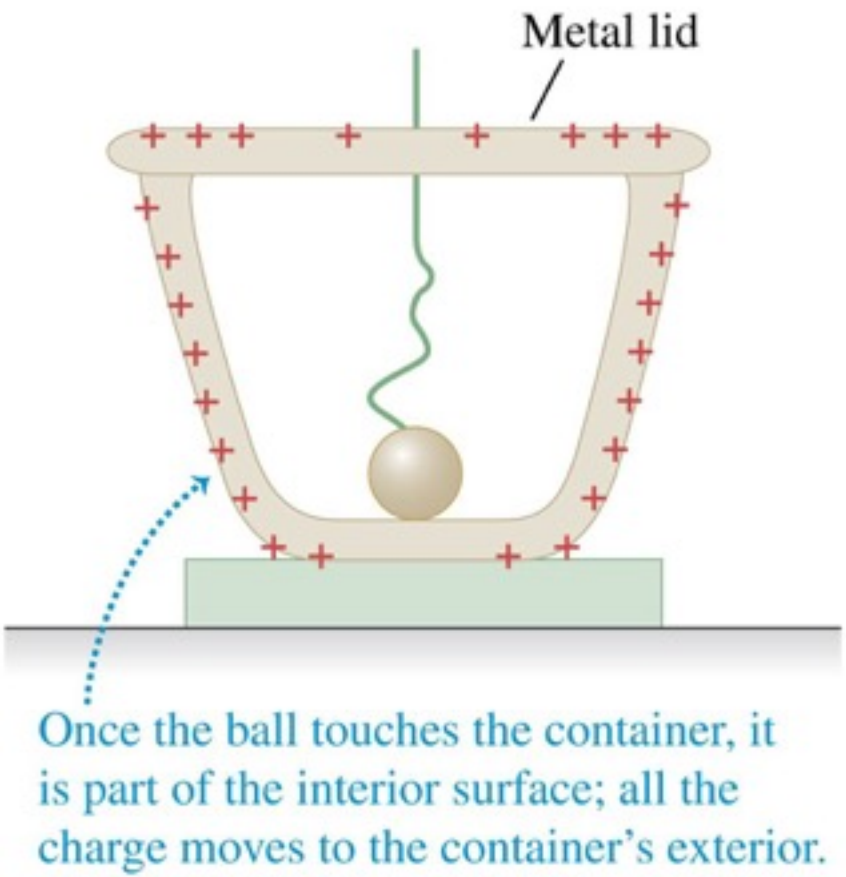
Faraday's icepail experiment: Slide 2 of 3

- We lower the ball into the container, and put the lid on.
- Charges are induced on the walls of the container, as shown.



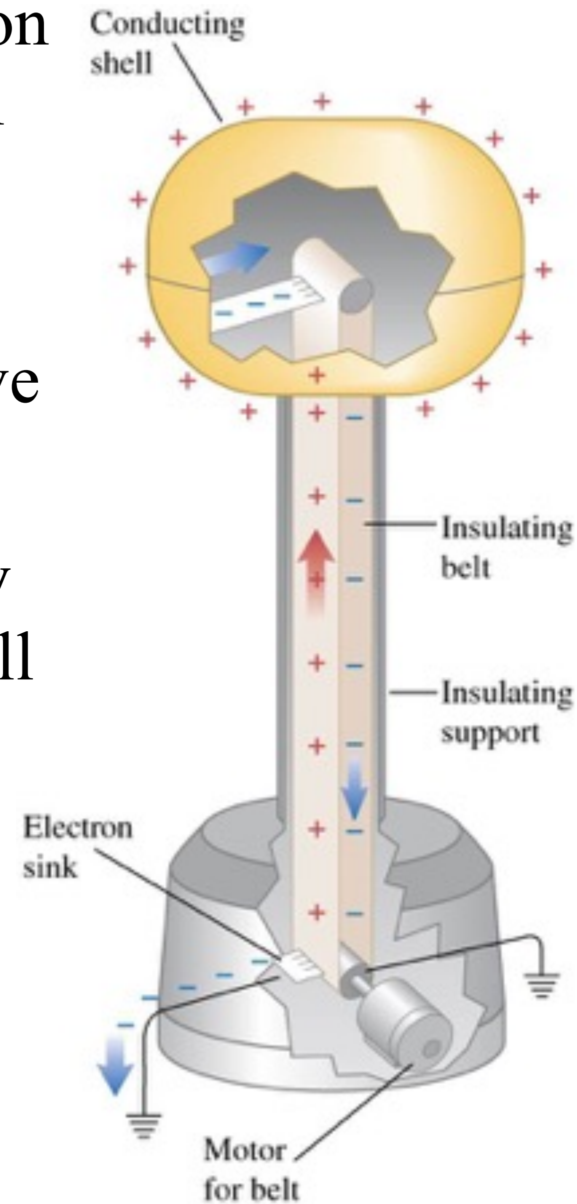
Faraday's icepail experiment: Slide 3 of 3

- We now let the ball touch the inner wall.
- The surface of the ball becomes part of the cavity surface, thus, according to Gauss's law, the ball must lose all its charge.
- Finally, we pull the ball out; we find that it has indeed lost all its charge.



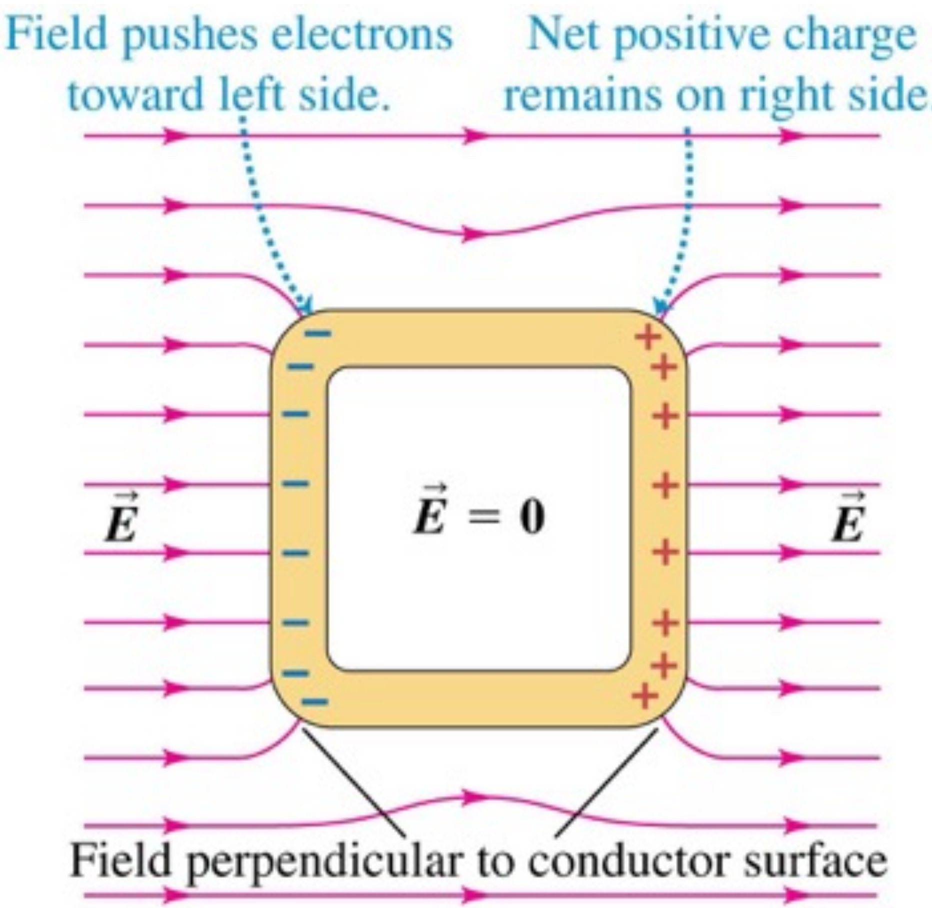
The Van de Graaff generator

- The **Van de Graaff generator** operates on the same principle as in Faraday's icepail experiment.
- The electron sink at the bottom draws electrons from the belt, giving it a positive charge.
- At the top the belt attracts electrons away from the conducting shell, giving the shell a positive charge.



Electrostatic shielding

- A conducting box is immersed in a uniform electric field.
- The field of the induced charges on the box combines with the uniform field to give *zero* total field inside the box.

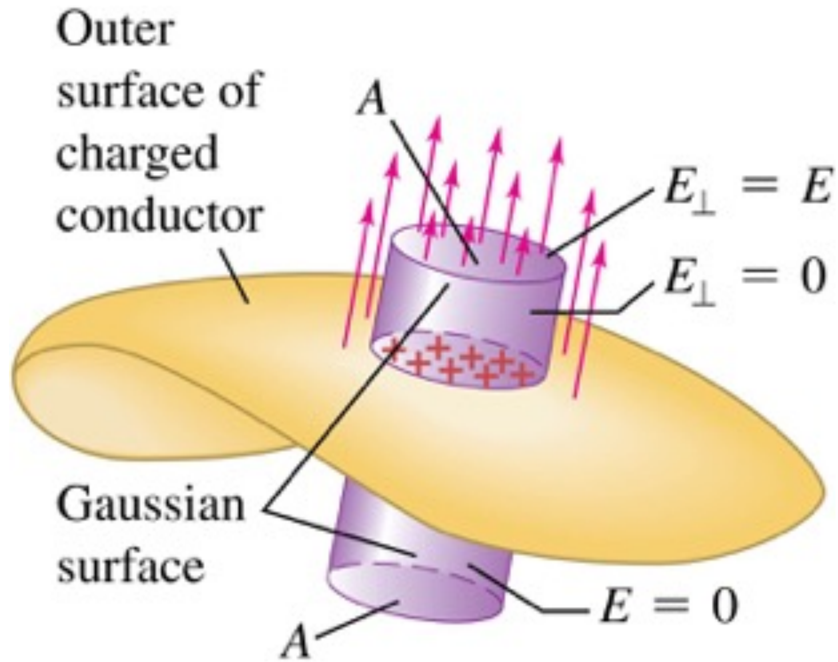


Electrostatic shielding

- Suppose we have an object that we want to protect from electric fields.
- We surround the object with a conducting box, called a Faraday cage.
- Little to no electric field can penetrate inside the box.
- The person in the photograph is protected from the powerful electric discharge.

Field at the surface of a conductor

- Gauss's law can be used to show that the direction of the electric field at the surface of any conductor is always perpendicular to the surface.
- The magnitude of the electric field just outside a charged conductor is proportional to the surface charge density σ.



Electric field at surface of a conductor, $\vec{E}$ perpendicular to surface

$$E_\perp = \frac{\sigma}{\epsilon_0}$$

σ: Surface charge density

ϵ_0: Electric constant