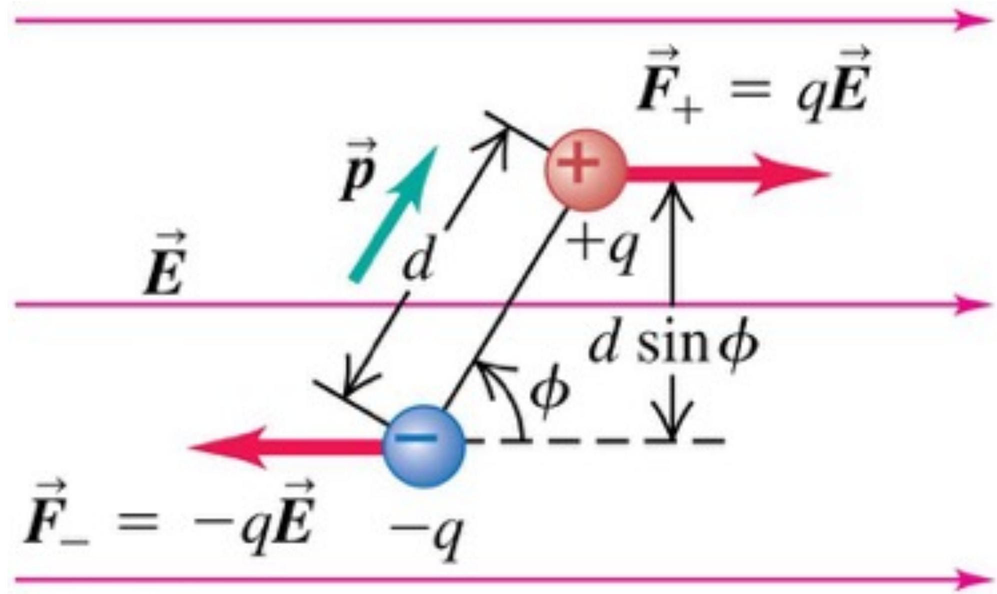
Lecture 11

PHYC 161 Fall 2016

Force and torque on a dipole

- When a dipole is placed in a uniform electric field, the net *force* is always zero, but there can be a net *torque* on the dipole.



Vector torque on an electric dipole: $\vec{\tau} = \vec{p} \times \vec{E}$ ($\vec{p}$: Electric dipole moment; $\vec{E}$: Electric field)

Potential energy for an electric dipole in an electric field: $U = -\vec{p} \cdot \vec{E}$ ($\vec{E}$: Electric field; $\vec{p}$: Electric dipole moment)

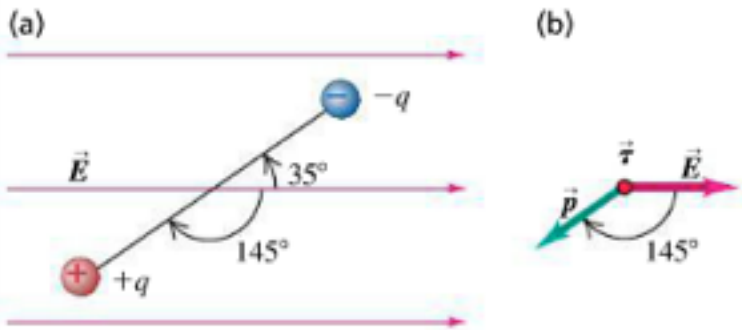
EXAMPLE 21.13 FORCE AND TORQUE ON AN ELECTRIC DIPOLE

Figure 21.32a shows an electric dipole in a uniform electric field of magnitude 5.0×10^5 N/C that is directed parallel to the plane of the figure. The charges are $\pm 1.6 \times 10^{-19}$ C; both lie in the plane and are separated by 0.125 nm $= 0.125 \times 10^{-9}$ m. Find (a) the net force exerted by the field on the dipole; (b) the magnitude and direction of the electric dipole moment; (c) the magnitude and direction of the torque; (d) the potential energy of the system in the position shown.

SOLUTION

IDENTIFY and SET UP: This problem uses the ideas of this section about an electric dipole placed in an electric field. We use the relationship $\vec{F} = q\vec{E}$ for each point charge to find the force on the dipole as a whole. Equation (21.14) gives the dipole moment, Eq. (21.16) gives the torque on the dipole, and Eq. (21.18) gives the potential energy of the system.

21.32 (a) An electric dipole. (b) Directions of the electric dipole moment, electric field, and torque ($\vec{\tau}$ points out of the page).



EXECUTE: (a) The field is uniform, so the forces on the two charges are equal and opposite. Hence the total force on the dipole is zero.

(b) The magnitude p of the electric dipole moment $\vec{p}$ is

$$p = qd = (1.6 \times 10^{-19}\ \text{C})(0.125 \times 10^{-9}\ \text{m})$$
$$= 2.0 \times 10^{-29}\ \text{C} \cdot \text{m}$$

The direction of $\vec{p}$ is from the negative to the positive charge, 145° clockwise from the electric-field direction (Fig. 21.32b).

(c) The magnitude of the torque is

$$\tau = pE\sin\phi = (2.0 \times 10^{-29}\ \text{C} \cdot \text{m})(5.0 \times 10^5\ \text{N/C})(\sin 145^\circ)$$
$$= 5.7 \times 10^{-24}\ \text{N} \cdot \text{m}$$

From the right-hand rule for vector products (see Section 1.10), the direction of the torque $\vec{\tau} = \vec{p} \times \vec{E}$ is out of the page. This corresponds to a counterclockwise torque that tends to align $\vec{p}$ with $\vec{E}$.

(d) The potential energy

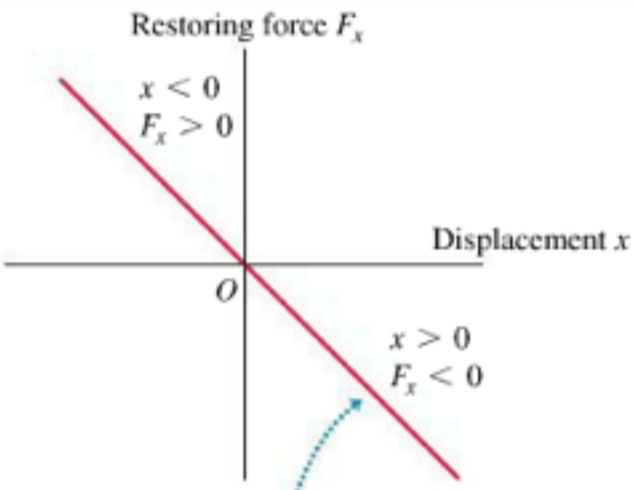
$$U = -pE\cos\phi$$
$$= -(2.0 \times 10^{-29}\ \text{C} \cdot \text{m})(5.0 \times 10^5\ \text{N/C})(\cos 145^\circ)$$
$$= 8.2 \times 10^{-24}\ \text{J}$$

EVALUATE: The charge magnitude, the distance between the charges, the dipole moment, and the potential energy are all very small, but are all typical of molecules.

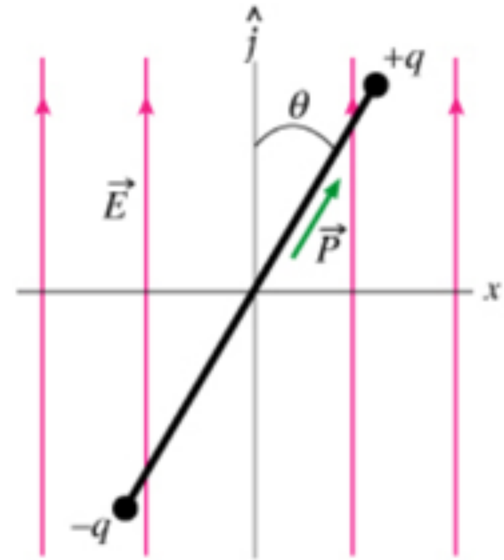
HW related…

Review Ch 14: Simple harmonic motion

Restoring force exerted by an **ideal spring**: $F_x = -kx$
- F_x: x-component of force
- x: Displacement
- k: Force constant of spring



The restoring force exerted by an idealized spring is directly proportional to the displacement (Hooke's law, $F_x = -kx$): the graph of F_x versus x is a straight line.

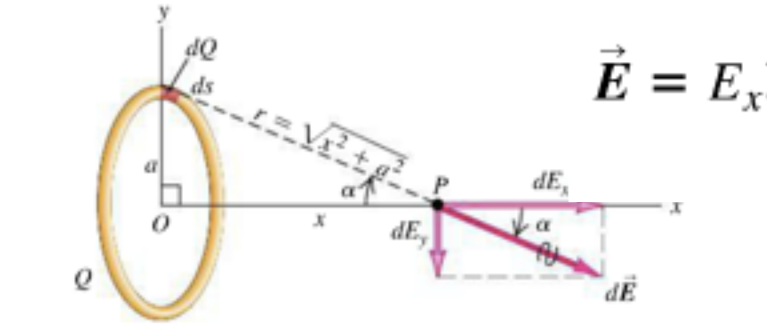


Angular frequency for simple harmonic motion: $\omega = \sqrt{\frac{k}{m}}$
- k: Force constant of restoring force
- m: Mass of object

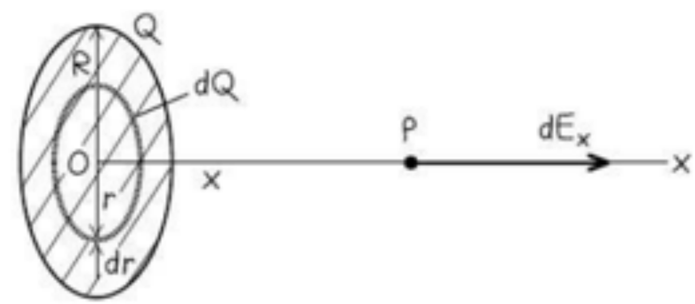
CAREFUL - not the same ω

$$K = \frac{1}{2} I \omega^2$$

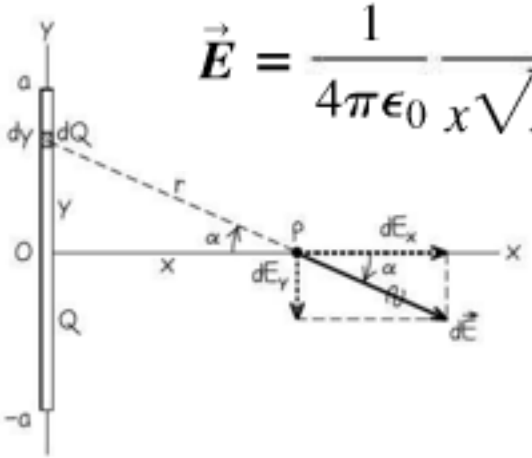
Electric Field of Various Charge Distributions



$$\vec{E} = E_x \hat{\imath} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + a^2)^{3/2}}\hat{\imath}$$



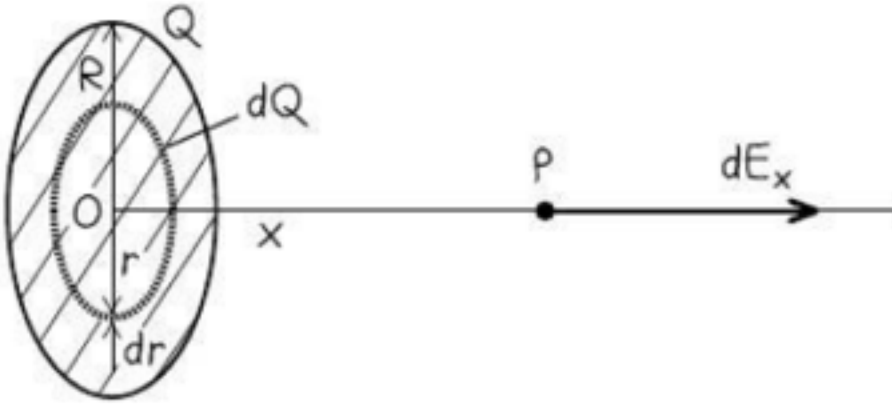
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2 + a^2}}\hat{\imath}$$



$$E_x = \frac{\sigma x}{2\epsilon_0}\left[-\frac{1}{\sqrt{x^2 + R^2}} + \frac{1}{x}\right]$$

$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R^2/x^2) + 1}}\right]$$

What is the electric field due to an infinite sheet?



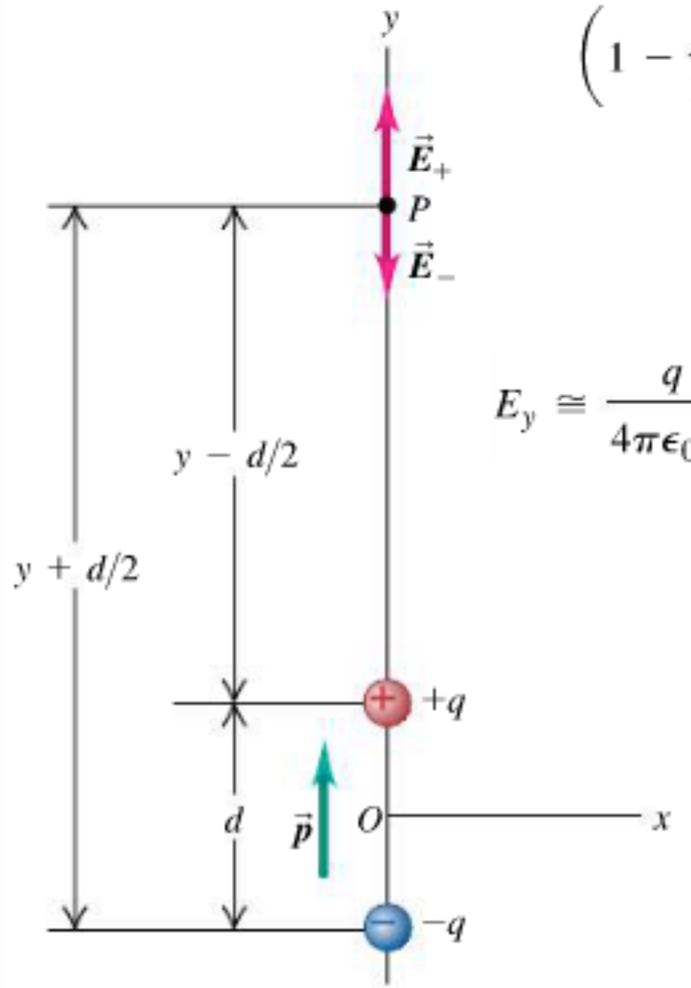
$$E_x = \frac{\sigma x}{2\epsilon_0}\left[-\frac{1}{\sqrt{x^2 + R^2}} + \frac{1}{x}\right]$$

$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R^2/x^2) + 1}}\right]$$

Given the expression above for a disk of charge, what is the field of an infinite sheet of charge?

HW related…

21.33 Finding the electric field of an electric dipole at a point on its axis.



$$\left(1 - \frac{d}{2y}\right)^{-2} \cong 1 + \frac{d}{y} \quad \text{and} \quad \left(1 + \frac{d}{2y}\right)^{-2} \cong 1 - \frac{d}{y}$$

$$E_y \cong \frac{q}{4\pi\epsilon_0 y^2}\left[1 + \frac{d}{y} - \left(1 - \frac{d}{y}\right)\right] = \frac{qd}{2\pi\epsilon_0 y^3} = \frac{p}{2\pi\epsilon_0 y^3}$$