

Lecture 13

PHYC 161 Fall 2016

Gauss's law

- Carl Friedrich Gauss helped develop several branches of mathematics, including differential geometry, real analysis, and number theory.
- The “bell curve” of statistics is one of his inventions.
- Gauss also made state-of-the-art investigations of the earth's magnetism and calculated the orbit of the first asteroid to be discovered.
- While completely equivalent to Coulomb's law, **Gauss's law** provides a different way to express the relationship between electric charge and electric field.

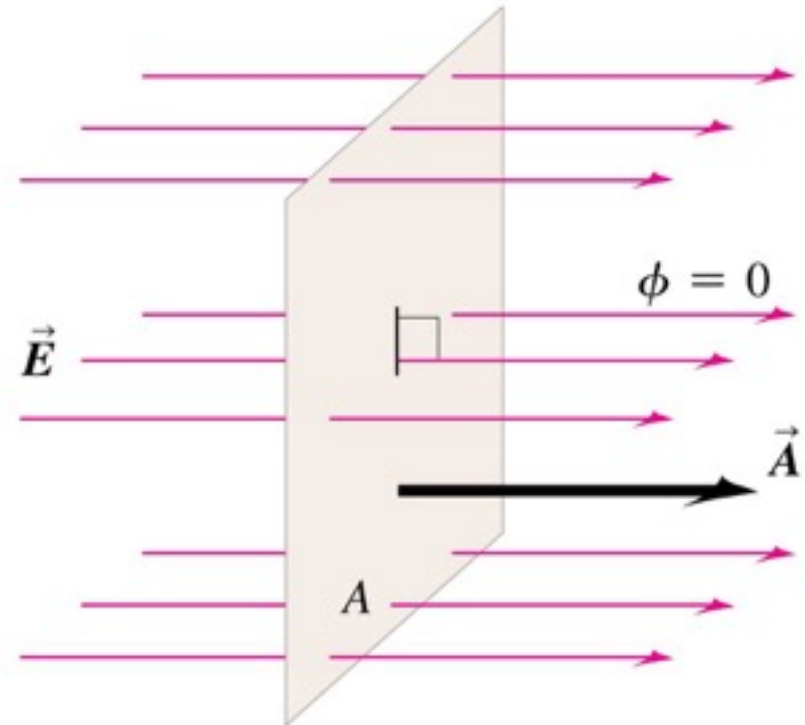


Calculating electric flux

- Consider a flat area perpendicular to a uniform electric field.
- Increasing the area means that more electric field lines pass through the area, increasing the flux.
- A stronger field means more closely spaced lines, and therefore more flux.

Surface is face-on to electric field:

- \vec{E} and \vec{A} are parallel (the angle between \vec{E} and \vec{A} is $\phi = 0$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA$.

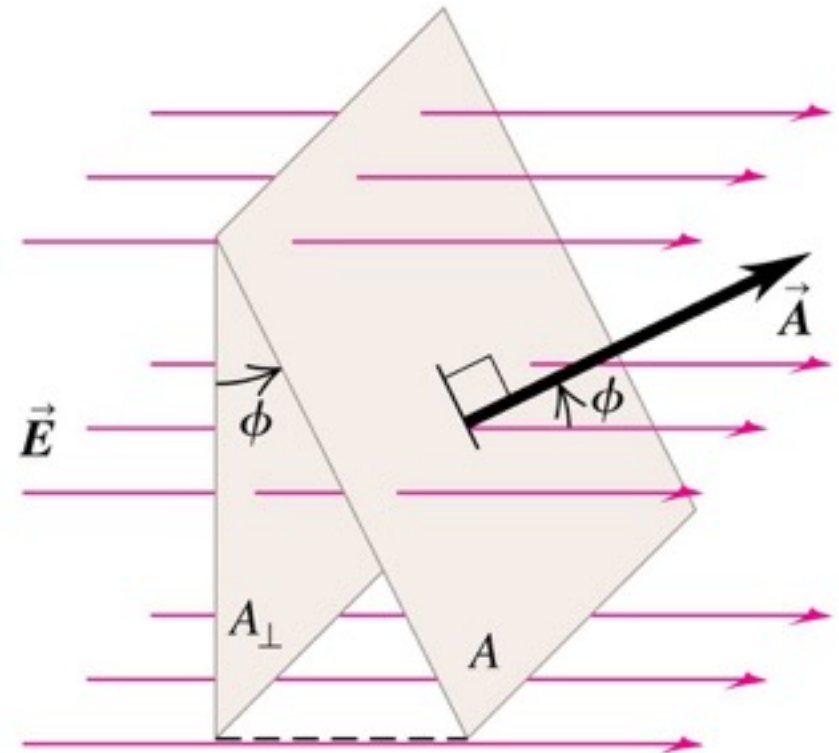


Calculating electric flux

- If the area is not perpendicular to the field, then fewer field lines pass through it.
- In this case the area that counts is the silhouette area that we see when looking in the direction of the field.

Surface is tilted from a face-on orientation by an angle ϕ :

- The angle between \vec{E} and \vec{A} is ϕ .
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi$.



Flux of a nonuniform electric field

- In general, the flux through a surface must be computed using a **surface integral** over the area:

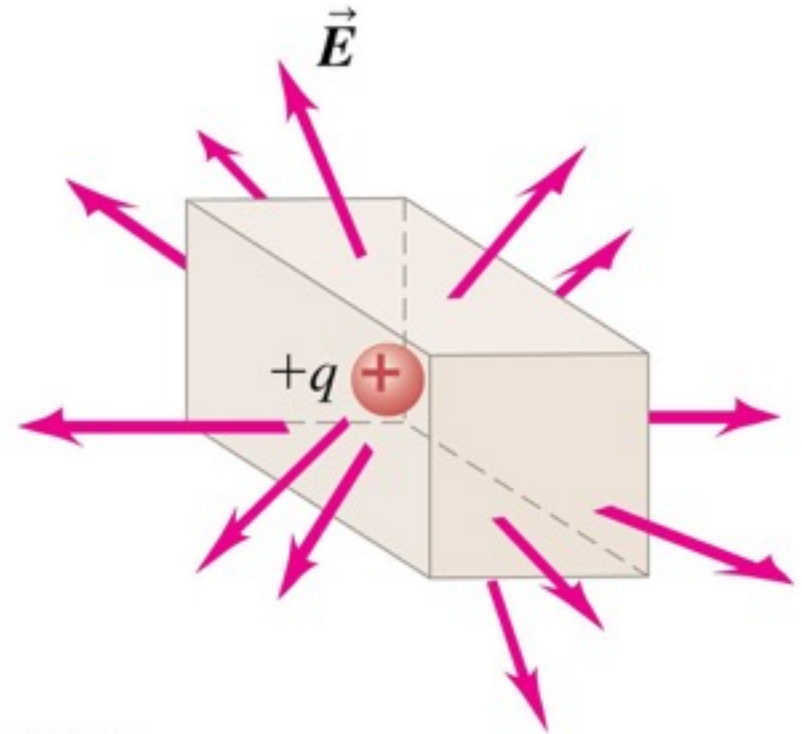
The diagram illustrates the calculation of electric flux through a surface. It features the equation $\Phi_E = \int E \cos \phi \, dA = \int E_{\perp} \, dA = \int \vec{E} \cdot d\vec{A}$ with several labels and arrows explaining its components:

- Electric flux through a surface**: Points to the symbol Φ_E .
- Magnitude of electric field \vec{E}** : Points to the E in the first integral.
- Angle between \vec{E} and normal to surface**: Points to the ϕ in the first integral.
- Element of surface area**: Points to the dA in the first and second integrals.
- Component of \vec{E} perpendicular to surface**: Points to the E_{\perp} in the second integral.
- Vector element of surface area**: Points to the $d\vec{A}$ in the third integral.

- The SI unit for electric flux is $1 \text{ N} \cdot \text{m}^2/\text{C}$.

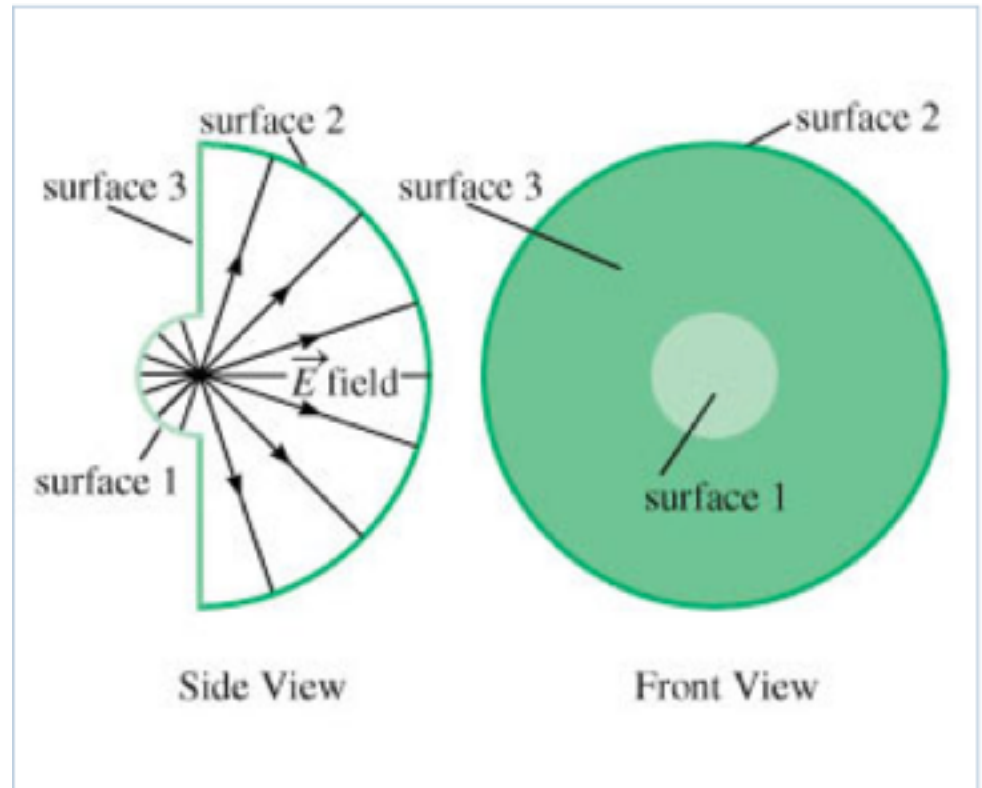
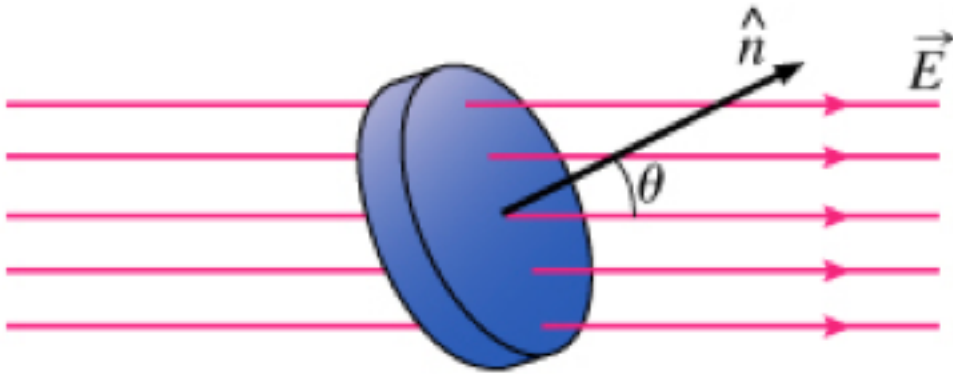
Q22.1

The figure shows a Gaussian surface with rectangular sides and positive point charge $+q$ at its center. If all the dimensions of the Gaussian surface double, but charge $+q$ remains at its center, the electric flux through the surface will



- A. increase by a factor of 4.
- B. increase by a factor of 2.
- C. remain the same.
- D. decrease by a factor of $1/2$.
- E. decrease by a factor of $1/4$.

On your HW:



General form of Gauss's law

- Let Q_{encl} be the total charge enclosed by a surface.
- Gauss's law states that **the total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ϵ_0 :**

Gauss's law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Electric flux through a closed surface of area A = surface integral of \vec{E}

Total charge enclosed by surface

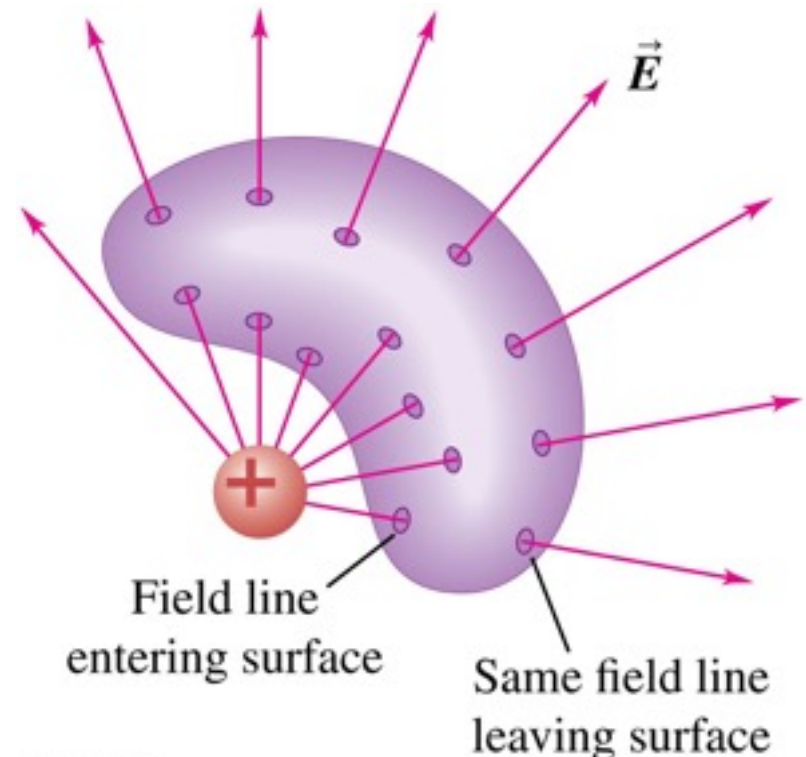
Electric constant

Gauss's law in a vacuum

- For a closed surface enclosing no charge:

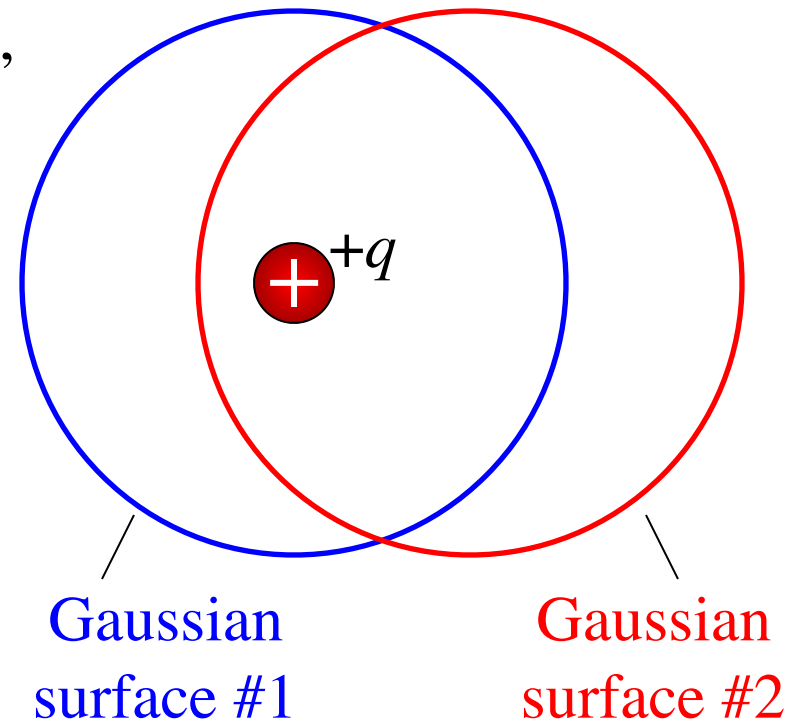
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = 0$$

- The figure shows a point charge *outside* a closed surface that encloses no charge.
- If an electric field line from the external charge enters the surface at one point, it must leave at another.



Q22.2

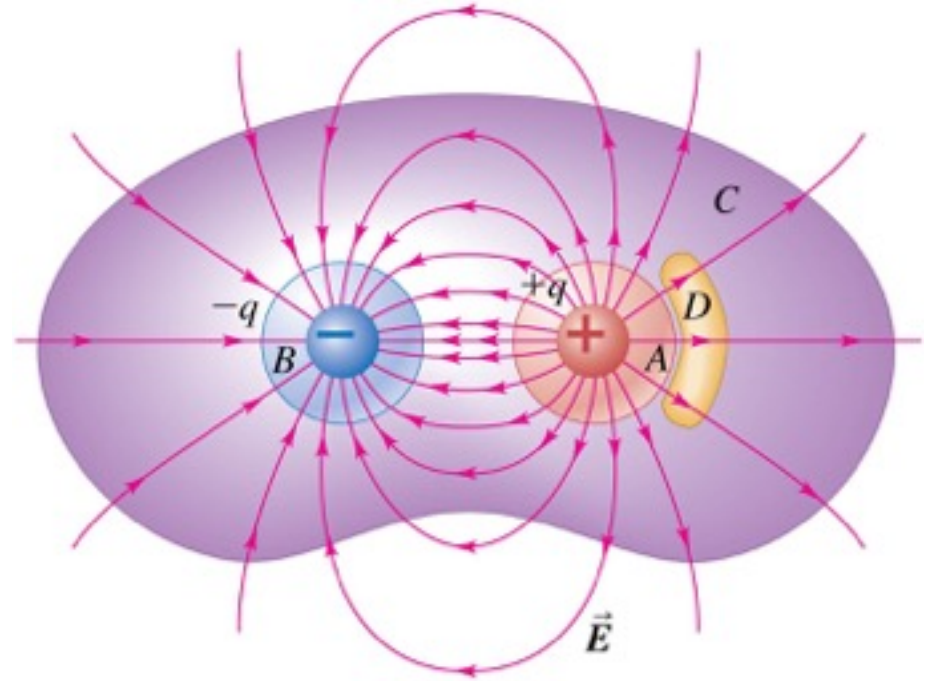
Spherical Gaussian surface #1 has point charge $+q$ at its center. Spherical Gaussian surface #2, of the same size, also encloses the charge but is not centered on it. There are no other charges inside either Gaussian surface. Compared to the electric flux through surface #1, the flux through surface #2 is



- A. greater.
- B. the same.
- C. less, but not zero.
- D. zero.
- E. Not enough information is given to decide.

Q22.3

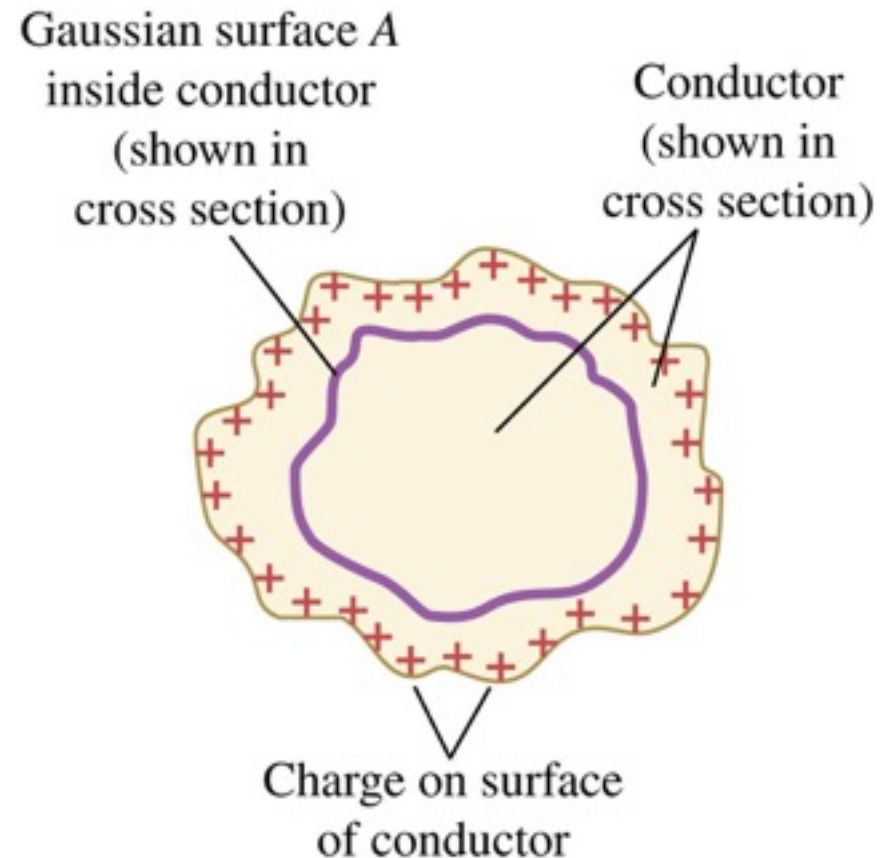
Two point charges, $+q$ (in red) and $-q$ (in blue), are arranged as shown. Through which closed surface(s) is/are the net electric flux equal to zero?



- A. surface A
- B. surface B
- C. surface C
- D. surface D
- E. both surface C and surface D

Applications of Gauss's law

- Suppose we construct a Gaussian surface inside a conductor.
- Because $\vec{E} = 0$ everywhere on this surface, Gauss's law requires that the net charge inside the surface is zero.
- Under electrostatic conditions (charges not in motion), any excess charge on a solid conductor resides entirely on the conductor's surface.



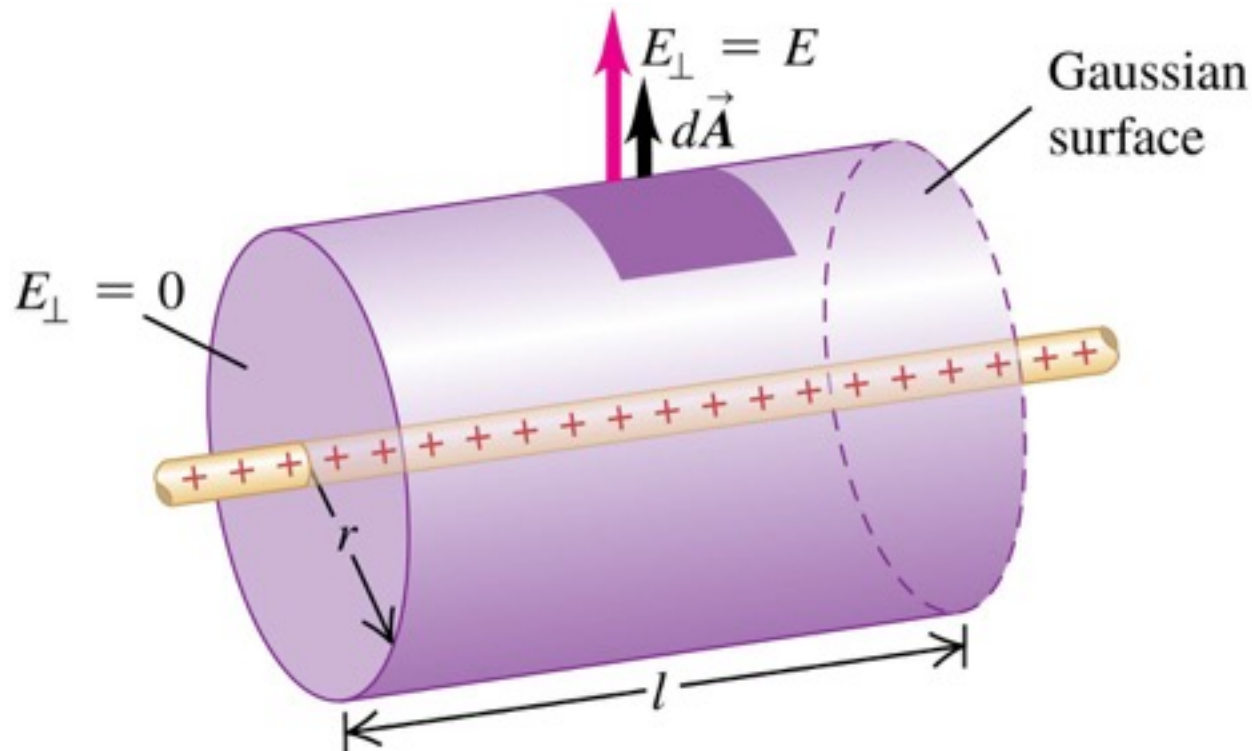
Q22.5

There is a negative surface charge density in a certain region on the surface of a solid conductor. Just beneath the surface of this region, the electric field

- A. points outward, toward the surface of the conductor.
- B. points inward, away from the surface of the conductor.
- C. points parallel to the surface.
- D. is zero.
- E. Not enough information is given to decide.

Field of a uniform line charge

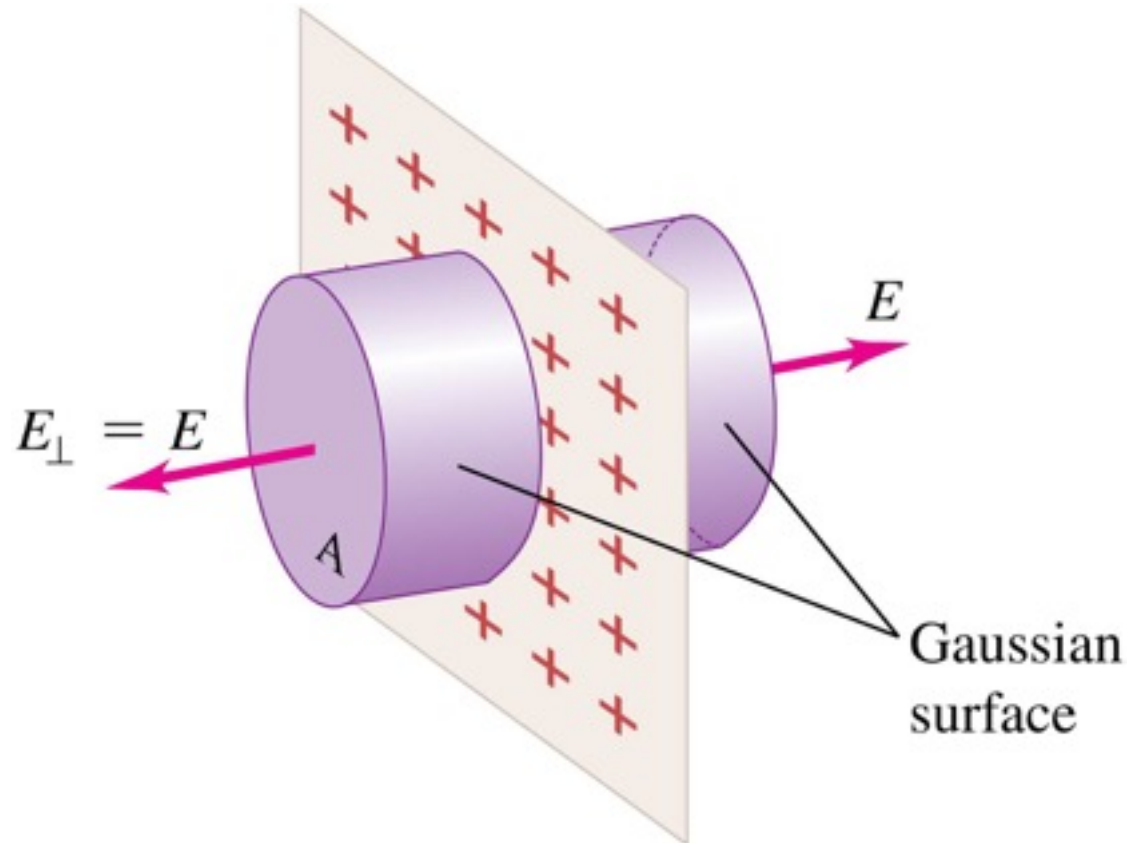
- Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is λ (assumed positive).
- Using Gauss's law, we can find the electric field:
$$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$$



Field of an infinite plane sheet of charge

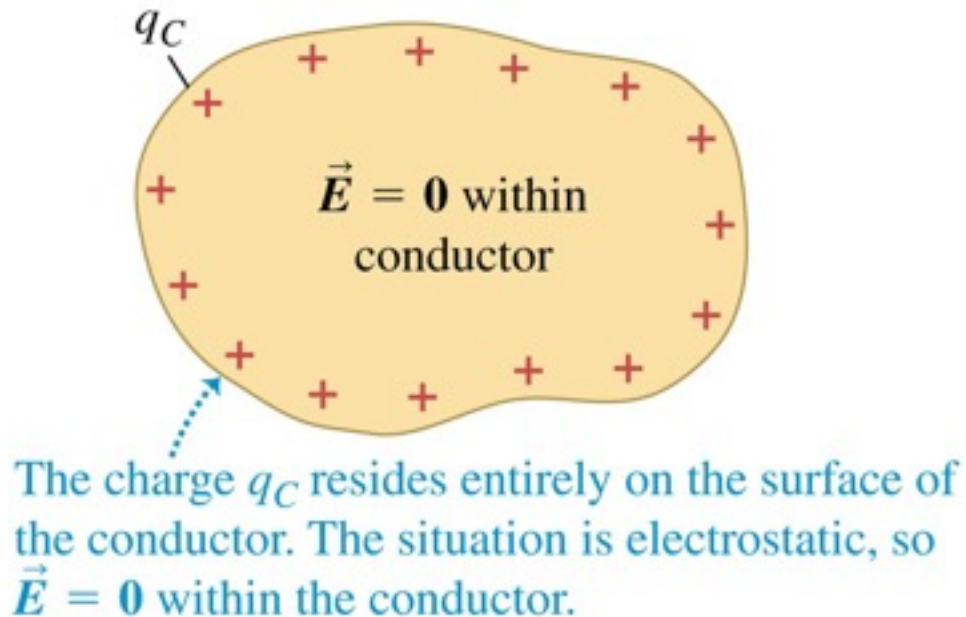
- Gauss's law can be used to find the electric field caused by a thin, flat, infinite sheet with a uniform positive surface charge density σ .

$$E = \frac{\sigma}{2\epsilon_0}$$



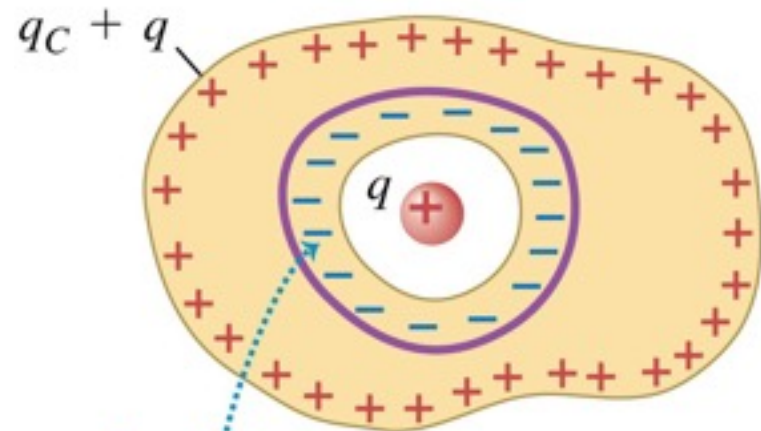
Charges on conductors

- Consider a solid conductor with a hollow cavity inside.
- If there is no charge within the cavity, we can use a Gaussian surface such as A to show that the net charge on the surface of the cavity must be zero, because $\vec{E} = 0$ everywhere on the Gaussian surface.



Charges on conductors

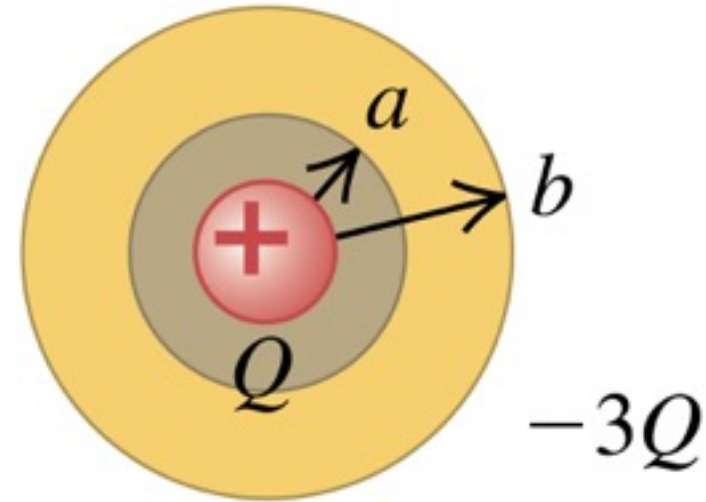
- Suppose we place a small body with a charge q inside a cavity within a conductor. The conductor is uncharged and is insulated from the charge q .
- According to Gauss's law the total there must be a charge $-q$ distributed on the surface of the cavity, drawn there by the charge q inside the cavity.
- The total charge on the conductor must remain zero, so a charge $+q$ must appear on its outer surface.



For \vec{E} to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$.

Q22.4

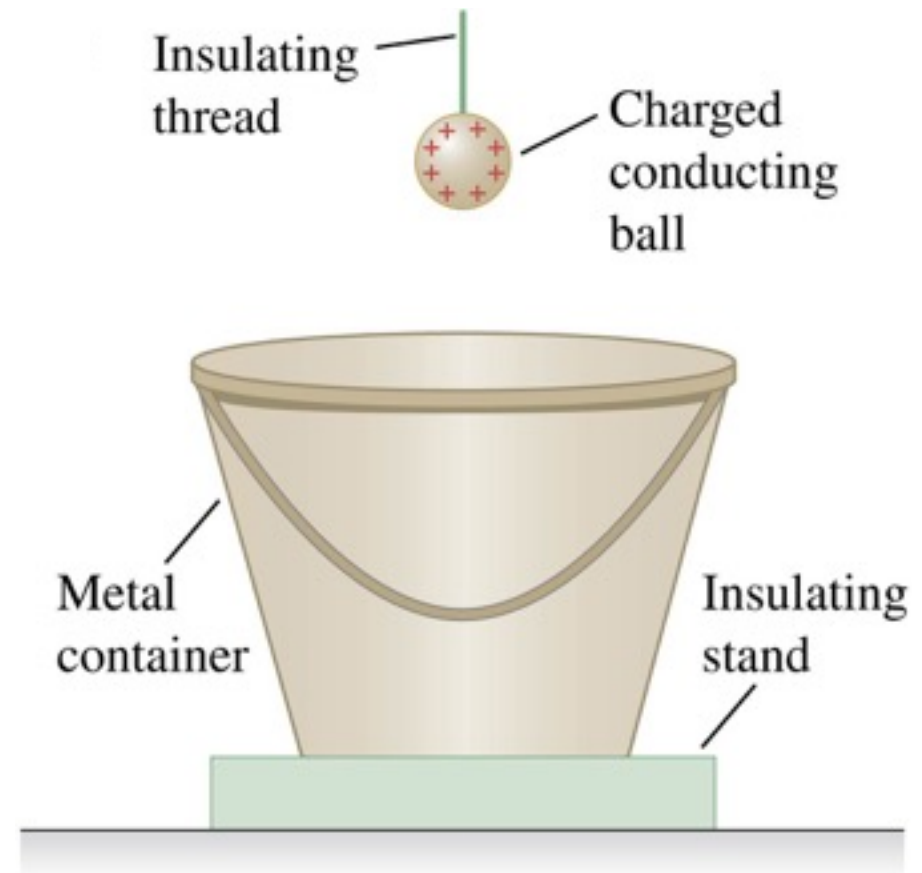
A conducting spherical shell with inner radius a and outer radius b has a positive point charge Q located at its center. The total charge on the shell is $-3Q$, and it is insulated from its surroundings. In the region $a < r < b$,



- A. the electric field points radially outward.
- B. the electric field points radially inward.
- C. the electric field points radially outward in parts of the region and radially inward in other parts of the region.
- D. the electric field is zero.
- E. Not enough information is given to decide.

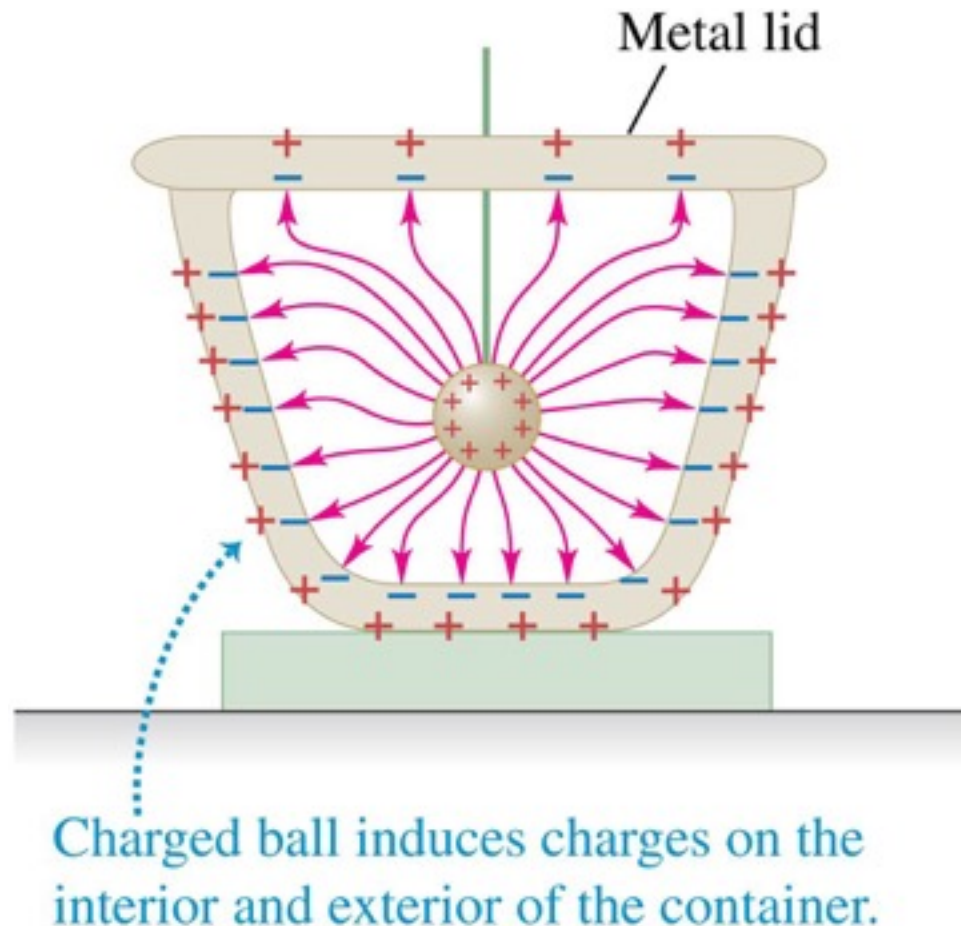
Faraday's icepail experiment: Slide 1 of 3

- We now consider Faraday's historic **icepail experiment**.
- We mount a conducting container on an insulating stand.
- The container is initially uncharged.
- Then we hang a charged metal ball from an insulating thread, and lower it into the container.



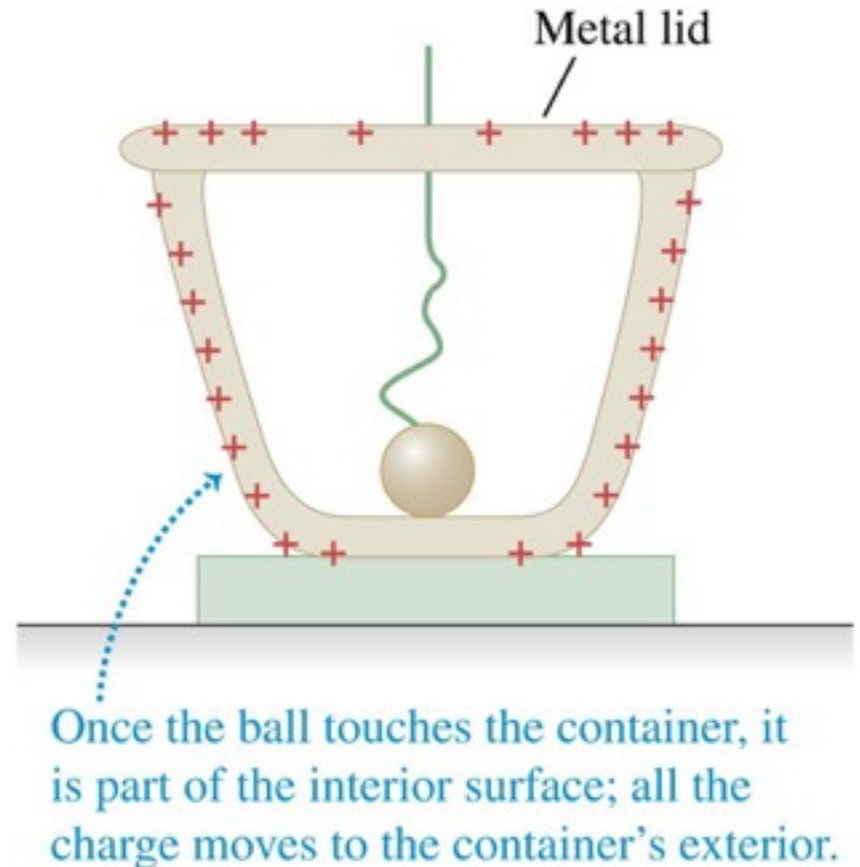
Faraday's icepail experiment: Slide 2 of 3

- We lower the ball into the container, and put the lid on.
- Charges are induced on the walls of the container, as shown.



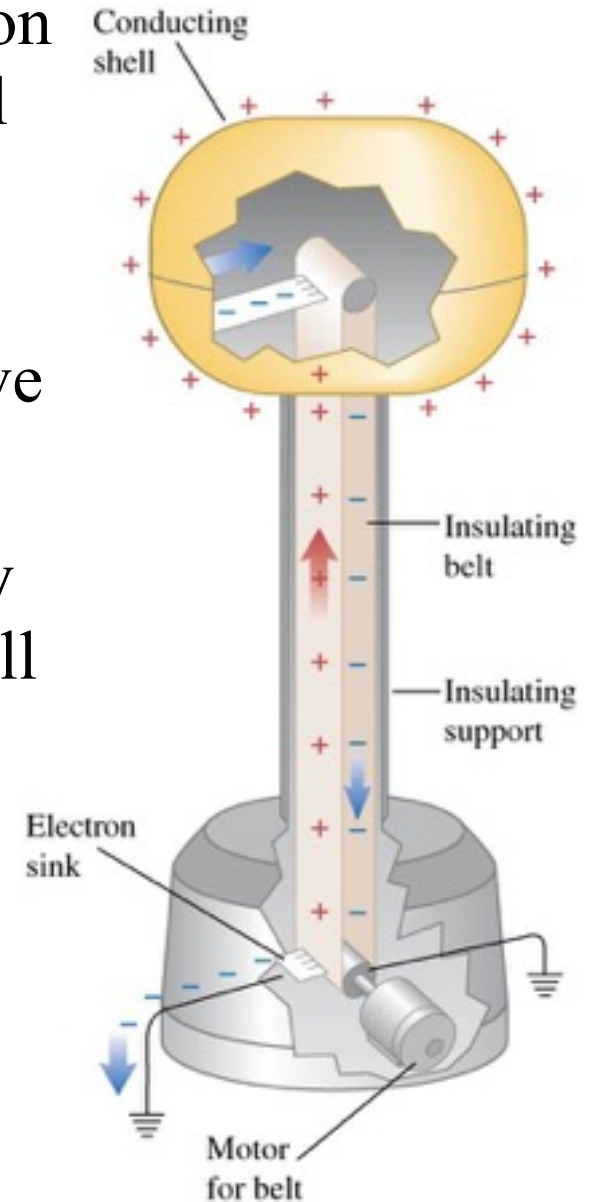
Faraday's icepail experiment: Slide 3 of 3

- We now let the ball touch the inner wall.
- The surface of the ball becomes part of the cavity surface, thus, according to Gauss's law, the ball must lose all its charge.
- Finally, we pull the ball out; we find that it has indeed lost all its charge.



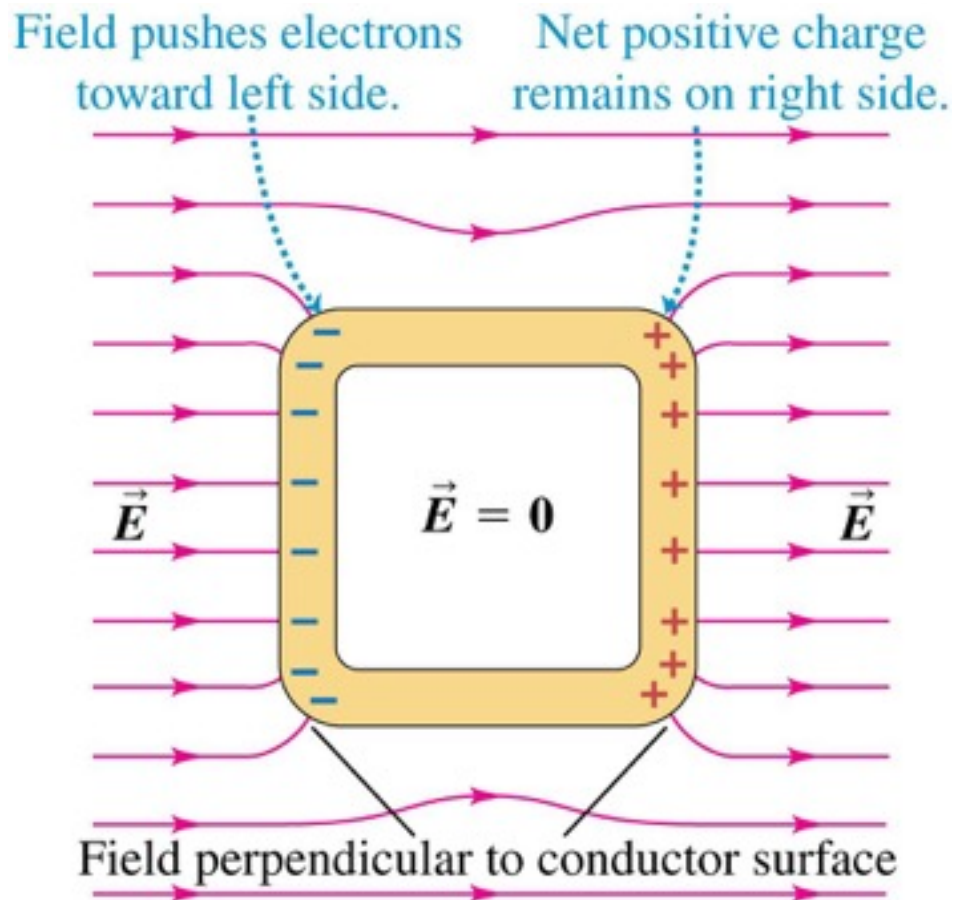
The Van de Graaff generator

- The **Van de Graaff generator** operates on the same principle as in Faraday's icepail experiment.
- The electron sink at the bottom draws electrons from the belt, giving it a positive charge.
- At the top the belt attracts electrons away from the conducting shell, giving the shell a positive charge.



Electrostatic shielding

- A conducting box is immersed in a uniform electric field.
- The field of the induced charges on the box combines with the uniform field to give *zero* total field inside the box.



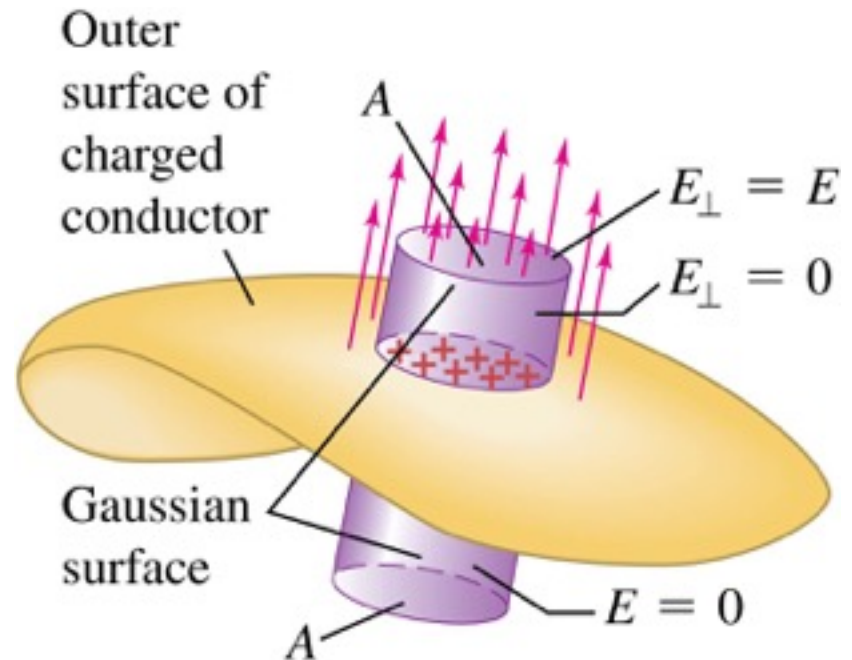
Electrostatic shielding

- Suppose we have an object that we want to protect from electric fields.
- We surround the object with a conducting box, called a Faraday cage.
- Little to no electric field can penetrate inside the box.
- The person in the photograph is protected from the powerful electric discharge.



Field at the surface of a conductor

- Gauss's law can be used to show that the direction of the electric field at the surface of any conductor is always perpendicular to the surface.
- The magnitude of the electric field just outside a charged conductor is proportional to the surface charge density σ .



Electric field at surface of a conductor, \vec{E} perpendicular to surface $\rightarrow E_{\perp} = \frac{\sigma}{\epsilon_0}$

σ Surface charge density
 ϵ_0 Electric constant