Physics 160 Test 1
Solutions

1. \( \langle v \rangle = \frac{\Delta x}{\Delta t} \Rightarrow \text{time for entire trip} = \Delta t = \frac{\Delta x}{\langle v \rangle} \)
   
   or \( \Delta t = \frac{180 \text{ mi}}{40 \text{ mph}} = 4.5 \text{ h} \)

   Plot \( v \) vs. \( t \):  
   \[ v_1 = 30 \]
   \[ t_1 = 4.5 \text{ h} \]

   To find \( t_1 \), \( v_1 = \frac{\Delta x_1}{t_1} \) or \( t_1 = \frac{\Delta x_1}{v_1} = \frac{90 \text{ mi}}{30 \text{ mph}} = 3 \text{ h} \)

   The time traveled at speed \( v_2 \) is \( t_2 - t_1 = 1.5 \text{ h} \).

   The average speed for the entire trip can be written as:
   
   \[ \langle v \rangle = \frac{(v_1 \times t_1) + [v_2 \times (t_2 - t_1)]}{t_2} \]

   Solve this for \( v_2 \):
   
   \[ v_2 = \frac{\langle v \rangle \times t_2 - v_1 \times t_1}{(t_2 - t_1)} = \frac{(40 \text{ mph}) \times (4.5 \text{ h}) - (30 \text{ mph}) \times (3 \text{ h})}{4.5 \text{ h} - 3 \text{ h}} \]
   
   \[ = 60 \text{ mph} \ (E) \]
\[ v_x = \frac{dx}{dt} = \frac{d}{dt} (7t^2 - 7t - 6) = 14t - 7 \quad \text{set } 0 \]

so \( t = \frac{7}{14} = \frac{1}{2} \) sec when \( v_x = 0 \)

The y-acceleration is \( a_y = \frac{d^2 y}{dt^2} = \frac{d^2}{dt^2} (4t^3 - 3t^2 - 12t - 5) \)

Taking 1st deriv,
\[ a_y = \frac{d}{dt} (12t^2 - 6t - 12) = 24t - 6 \]

Evaluate this at \( t = \frac{1}{2} \) sec \( \implies a_y = 6 \frac{m}{s^2} \quad (A) \)

\[ \mathbf{v}_{p/g} \quad \mathbf{v}_{w/g} \quad \mathbf{v}_{p/w} = \mathbf{v}_{p/g} + \mathbf{v}_{w/g} \]

where \( \mathbf{v}_{w/g} = (120 \ \text{km/h}, \ \text{to the south}) \)

\[ |\mathbf{v}_{p/w}| = 200 \ \text{km/h} \quad \text{(direction not given)} \]

Since this is a right triangle, we can use the Pythagorean Theorem:
\[ (\mathbf{v}_{p/w})^2 = (\mathbf{v}_{p/g})^2 + (\mathbf{v}_{w/g})^2 \]

So \( \mathbf{v}_{p/g} = \sqrt{(\mathbf{v}_{p/w})^2 - (\mathbf{v}_{w/g})^2} = 160 \frac{\text{km}}{\text{h}} \quad (C) \)
4. \[ \vec{c} \times \vec{j} = (-3\hat{i} - 2\hat{j} - 3\hat{k}) \times \vec{j} \]
   \[= -3 (\vec{i} \times \vec{j}) - 2 (\vec{j} \times \vec{j}) - 3 (\vec{k} \times \vec{j}) \]
   \[= -3 \hat{k} + 0 - 3 (-\hat{i}) \]
   \[= 3\hat{i} - 3\hat{k} \quad \text{(A)} \]

5. First find the speed at which the tip of the blade is ejected. (It will be only an \textit{x}-component)

   \[v_0_x = \frac{2\pi R}{T} \]

   Now find the time to hit the ground using the \textit{y}-equations:

   \[y = -\frac{1}{2} gt^2 \quad \text{or} \quad t = \sqrt{-\frac{2y}{g}} \]

   where \(y = -h\), with \(h = \text{height of tip of blade} = 32\text{m}. \)

   \[t = \sqrt{\frac{2h}{g}} \]

   To find the range, use \textit{x}-motion:

   \[x = v_0_x \cdot t = \left(\frac{2\pi R}{T}\right) \sqrt{\frac{2h}{g}} = 160\text{m} \quad \text{(E)} \]
\[ \tan \phi = \frac{A_y}{A_x} = \frac{-4}{3.9} \]
\[ \phi = -46^\circ \]

Then \[ \theta = 360^\circ - 46^\circ = 314^\circ \] (A)

(10) From \[ a = \frac{v^2}{r} \Rightarrow a = \frac{(v_2)^2/r_2}{(v_1)^2/r_1} = \left(\frac{v_2}{v_1}\right)^2 \frac{r_1}{r_2} \]

or \[ a = \left(\frac{200 \text{ km/h}}{100 \text{ km/h}}\right)^2 \left(\frac{100 \text{ m}}{200 \text{ m}}\right) = 2 \] (C)

(11) \[ \vec{A} \cdot \vec{B} = (5\hat{i} - 2\hat{j} - 2\hat{k}) \cdot (-2\hat{i} - 7\hat{j} + 2\hat{k}) \]

\[ = -10 + 14 - 4 = 0 \] (A)

(12) This is 1-D motion. Choose y origin at top of building. Since the ball is in free fall, the y-motion is given by

\[ y = v_{0y} t - \frac{1}{2} g t^2 \]

The only unknown is time \( t \), since we choose \( y = -h \), \( h \) = height of building = 70 m

Solve this quadratic equation for \( t \):

\[ t = -\frac{v_{0y} \pm \sqrt{(v_{0y})^2 - 4 \left( -\frac{g}{2} \right) (-y)}}{2 \left( -\frac{g}{2} \right)} \]

with \( y = -h \)

We get \[ t = \begin{cases} -1.7 \text{ s} \\ +8.2 \text{ s} \end{cases} \] (A)

(The first root is time if ball were thrown up from the ground)