HW#7, due in class on 28th October 2014

30% of this HW is extra credit, but try some parts of each problem!

1.

A plane-polarized EM wave of angular frequency ω is incident normally on the flat surface of a nonpermeable medium of conductivity σ and permittivity ϵ. Show that for a very good conductor, the power reflection coefficient is approximately

$$R \approx 1 - 2\frac{\omega}{c}\delta,$$

where δ is the skin depth.

2. Energy in Absorbing Dielectrics

Consider a neutral plasma (gas of electrons and positive ions), with electron density N_e. At high frequencies, because the ions are very heavy, we can consider them to be essentially fixed and any current due solely to the light electrons. The total charge density can be set to zero for an electrically neutral gas.

(a) Use Maxwell's Equations to derive the wave equation for the electric field,

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \mu_0 \frac{\partial \mathbf{J}}{\partial t}.$$

(b) For a monochromatic wave, $\tilde{\mathbf{E}} = \mathrm{Re}(\tilde{\mathbf{E}} e^{-i\omega t})$, and ignoring any collision between electrons show that

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} = \frac{\omega_p^2}{c^2}\tilde{\mathbf{E}}, \text{ where } \omega_p = \sqrt{N_e e^2/\varepsilon_0 m_e} \text{ is the “plasma frequency”}$$

(c) Derive the dispersion relation $k = \sqrt{\omega^2 - \omega_p^2}/c$. Sketch the graph $\omega(k)$.

(d) Give the real electric field when $\omega<\omega_p$.

(e) What is the reflection coefficient R of a wave traveling from vacuum into a plasma at normal incidence? What is the value of R when $\omega<\omega_p$.

3. Silver as a conductor:

(a) Given the empirical DC conductivity of silver as σ=6.17×10^7/(ohm m) and your knowledge of the periodic table, estimate the plasma frequency and collision rate of electrons for silver at standard temperature and pressure.

(b) Give an approximate expression for the real and imaginary parts of the index of refraction at a typical microwave, visible, and x-ray frequency (take ε=1, μ=1). What is the skin depth of silver at a radio frequency?

c) What is the skin depth of silver at radio frequencies?

4. Waves in Plasmas:

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(a) Use Maxwell's Equations to derive the wave equation for the electric field,

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \mu_0 \frac{\partial \mathbf{J}}{\partial t}.$$

(b) For a monochromatic wave, $\tilde{\mathbf{E}} = \mathrm{Re}(\tilde{\mathbf{E}} e^{-i\omega t})$, and ignoring any collision between electrons show that

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\tilde{\mathbf{E}} = \frac{\omega_p^2}{c^2}\tilde{\mathbf{E}}, \text{ where } \omega_p = \sqrt{N_e e^2/\varepsilon_0 m_e} \text{ is the “plasma frequency”}$$

(c) Derive the dispersion relation $k = \sqrt{\omega^2 - \omega_p^2}/c$. Sketch the graph $\omega(k)$.

5)

Anomalous dispersion in dielectrics. The group velocity in an absorptive dispersive medium is defined as:

$v_g = \frac{1}{\frac{dk_R(\omega)}{d\omega}}$, where $k_R(\omega)$ is the real part of the wave number.

(a) Show that $v_g(\omega) = \frac{c}{n_R(\omega) + \omega(dn_R/d\omega)}$,

(b) For a single resonance in the Lorentz model, we found,

$$n_R(\omega) = 1 + \frac{\omega_p^2(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \omega^2\Gamma^2}, \text{ or near resonance, } n_R(\omega) \approx 1 + A\left(\frac{-\Delta}{\Delta^2 + \Gamma^2/4}\right)$$

$$\text{where } \Delta = \omega - \omega_0 \text{ and } A = \frac{\omega_p^2}{2\omega_0}.$$

Sketch the phase velocity as a function of Δ, comment.

(c) Show that in the region of anomalous dispersion $dn_R/d\omega<0$ even the group velocity can be greater than c!! What is the condition on the index of refraction? (Give a general analytic expression. You need not evaluate it explicitly for the case under consideration).

The physical meaning of this is still under controversy. Rest assured though, this does not mean that any signal can be sent faster than the speed of light