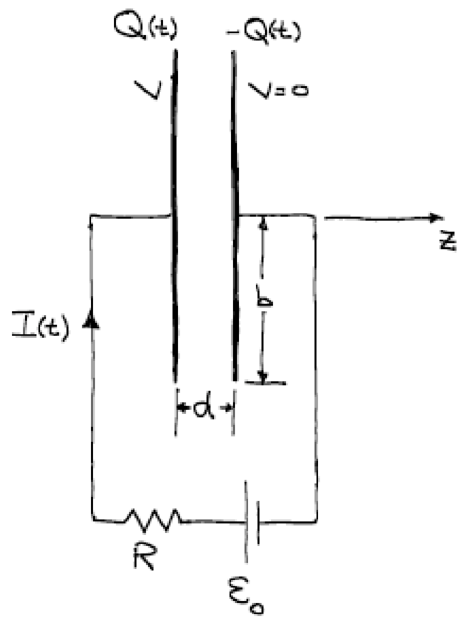
Homework #4 *due Tuesday 9/23/2013* ***in Class***
(see syllabus about late HWs)
(Note that all Griffiths Problems are from 4th Ed.)

1. A good problem that puts many of the concepts of Chap 7 and 8 together.

Consider a vacuum parallel-plate capacitor in which the two plates are disks of radius b, which are separated by a distance $d \ll b$, corresponding to a capacitance $C = \epsilon_0 \pi b^2/d$. A cross section through the capacitor is shown in the drawing below.



At time $t = 0$ a switch is thrown so that a battery, which generates emf $\mathcal{E}_0$, begins charging up the capacitor through a resistor. The wires that make up the charging circuit are far away from the region between the plates. The resistance R is much larger than the internal resistance of the plates, so at all times as the capacitor charges up, the charge on the plates is distributed uniformly. At time t there is a charge

$$Q(t) = C\mathcal{E}_0(1 - e^{-t/RC})$$

on the left plate and a charge $-Q(t)$ on the right plate, and there is a current

$$I(t) = dQ/dt = (\mathcal{E}_0/R)e^{-t/RC}$$

flowing in the wires.

(a) As the capacitor charges up, there is a surface current density $\mathbf{K} = K(s,t)\hat{\mathbf{e}}_s$ on the left plate and an equal, but opposite surface current on the right plate. These surface currents distribute the charge over the surface of the plates. Use the law of charge conservation to *find* the surface current $\mathbf{K}$. Write your answer in terms of $I(t)$.

(b) *Find* the displacement current density $\mathbf{J}_d$ between the capacitor plates. What is the total displacement current between the plates? Write your answers in terms of $I(t)$.

(c) *Find* the magnetic field $\mathbf{B}$ between the plates. Write your answer in terms of $I(t)$. (Hint: Use the Ampere-Maxwell law.)

(d) *Evaluate* the Poynting vector $\mathbf{S}$ between the plates. *Find* the electromagnetic power P that is transported *inward* through a tin can of radius s that is centered on the z axis and that spans the distance between the plates. What is the total electromagnetic energy U that is transported inward through the tin can as the capacitor charges up? Write your answer in terms of $\mathcal{E}_0$. *Interpret* your answer for the total electromagnetic energy transported through the tin can. Where does the energy go?

2)

A parallel-plate capacitor may be construed—if we ignore the fringing fields—as having a uniform electric field, $\vec{E} = E\hat{z}$, and a potential difference, $V = Ed$, where we suppose the distance between the plates is d, and we also go ahead and label their area as A. Place that capacitor in a uniform magnetic field, $\vec{B} = B\hat{x}$, again ignoring any fringing.

a. Find the total electromagnetic momentum in the space between the plates.

b. Now suppose that we connect a wire, with large resistance, between the plates, directly along the $\hat{z}$-axis, so that the capacitor will, slowly, discharge. As the current so generated finds itself in a magnetic field, it will have a force exerted on it. Eventually the current will of course cease; therefore, it is reasonable to ask for the total change in momentum of the wire as a result of this force. [Do recall that, since $dp/dt = F$, it follows that $\Delta p = \int dt\, F$.] Compare this quantity with the total electromagnetic momentum that, originally, resided in the space between the plates, as calculated in part (a).

3)

Take as a system a parallel-plate capacitor, where we treat the plates as infinite in extent, and make them normal to the $\hat{z}$-axis, placed symmetrically with respect to the origin, so that they are a total distance d apart. As well, choose the plate with positive charge density, $+\sigma$, to be the one at $z = +d/2$, relative to the origin.

a. Again, calculate all the components of the stress tensor, $\overleftrightarrow{T}$. Then, choose a closed area to use to determine the force per unit area of one plate on the other.

b. What is the momentum per unit area, per unit time, crossing some plane parallel to, and between, our two capacitor plates?

c. Now calculate the recoil effect on the top plate, as a result of the impact of this momentum, showing that it is the same as the force calculated in part (a). [This is of course a particular way of explaining why there should be such a force.]

4) A very long solenoid carries a current I. Coaxial with the solenoid is a large, circular ring of wire, with resistance R. When the current in the solenoid is gradually decreased, a current is induced in the ring. Take the solenoid to have n turns per unit length, and radius a, while the ring has radius $b >> a$.

a. Determine the current in the ring, as a function of dI/dt.

b. The current in the ring dissipates heat, by virtue of its resistance. That energy must have come from somewhere. Presumably it must have come from (magnetic) energy that once was contained within the solenoid. Confirm that this is so by determining the Poynting vector just exterior to the solenoid, and showing that an integration over the entire surface of the solenoid generates the correct power, above. Note that the change in magnetic flux is the source of the electric field needed for the existence of a non-zero Poynting vector, while the current that electric field generates within the large ring is the source of the magnetic field.

5) EXTRA CREDIT: Griffiths 8.3