Homework #10 *for practice*

(Note that all Griffiths Problems are from 4th Ed.)

1. On good graph paper, of some sort, please set up the usual x and ct-axes for observer $\mathcal{O}$, making them intersect at 90° at the origin, etc.

a. Now carefully draw the x' and ct' axes for an observer moving with $v = 0.6c$. Next, draw additional lines on this graph that correspond to the following:

$$x' = -2, x' = +2, ct' = -2, ct' = +2, ct' = +4 \ .$$

b. Now let a different observer to observed to travel from the event $x' = -2, ct' = -2$ to the event $x' = +2, ct' = +4$. Show the line that corresponds to her worldline on your graph, and measure the slope of that line, thereby determining its velocity as measured by $\mathcal{O}$.

2.

According to an observer $\mathcal{O}$, a particle of mass m is moving so that its total energy is $E = 2mc^2$. It then collides with an identical particle which is at rest, and they stick together.

a. What is the original velocity of the moving particle, and what is the final velocity of the composite, final particle?

b. What is the mass of the composite particle?

c. We define the center of mass reference frame for a system of particles as that frame in which the vector sum of all the 3-momenta of the particles is zero. As measured by $\mathcal{O}$, what is the velocity of the center of mass reference frame for the original two particle system?

3.

A pi meson is at rest, and after some time decays into a muon and a neutrino. Assuming that the mass of the neutrino is zero, determine the energy and velocity of the muon. You should not insert any actual numbers, but simply take the mass of the pi meson as m_π and that of the muon as m_μ.

Do recall that the relationship between energy, momentum, and mass of a particle is that given by the invariant of the 4-momentum, namely

$$\widetilde{p}^2 = -(mc)^2 \quad \Longleftrightarrow \quad E^2 = (cp)^2 + (mc^2)^2 \ ,$$

which is of course also true even when the mass, m, is zero.

4.

1 Consider the particle of Griffiths's Example 12.10, which is subjected to a constant force F in the x direction.

(a) *Draw* the trajectory [Eq. (12.62)] of the particle on a spacetime diagram for all times t, both positive and negative.

(b) *Find* the event $\mathcal{O}$ that is the same proper distance s from all events on the trajectory. What is s?

(c) Suppose the particle radiates pulses of light continuously in all directions. Do any of these pulses reach the event $\mathcal{O}$? Suppose a light pulse is emitted from event $\mathcal{O}$ in all directions. Does this light pulse ever reach the particle?

5.

) **The rocket and the barn.** Consider a rocket that is moving with velocity $v = 4c/5$ with respect to the lab. The tail of the rocket is at $x = 0$ at $t = 0$ (this is the origin $\mathcal{O}$ of the spacetime diagram), and the rocket has rest (proper) length $s_0 = 4\,\text{m}$.

Throughout this problem you should endeavor to make your spacetime diagram as accurate as possible.

(a) *Draw* a spacetime diagram that depicts both the lab and rocket coördinates and the light cone $x = ct$.

(b) *Label* the event $\mathcal{P}$ that corresponds to the front of the rocket at $t' = 0$. *Find* the coördinates of $\mathcal{P}$ in the lab and rocket frames, and label these coördinates on your diagram. *Verify* that these coördinates are consistent with the invariant interval between $\mathcal{O}$ and $\mathcal{P}$ and with the Lorentz transformation between the two frames. *Draw* and *label* the world lines of the front and tail of the rocket.

(c) The length of the rocket is measured at $t = 0$ in the lab frame. The origin $\mathcal{O}$ is the event corresponding to the measurement of the position of the tail of the rocket. *Label* the event $\mathcal{Q}$ that corresponds to the measurement of the position of the front of the rocket. *Find* the coördinates of $\mathcal{Q}$ in the lab and rocket frames, and label these coördinates on your diagram. *Verify* that these coördinates are consistent with the invariant interval between $\mathcal{O}$ and $\mathcal{Q}$ and with the Lorentz transformation between the two frames. What is the length of the rocket as measured in the lab frame?

A barn of proper length $s_0/\gamma = 2.4\,\text{m}$ (just large enough to contain the Lorentz-contracted rocket) is at rest in the lab frame. The left side of the barn is at $x = 0$ and the right side of the barn is at $x = s_0/\gamma$. The rocket passes through windows in the left and right sides of the barn.

(d) *Draw* and *label* the world lines of the two sides of the barn. The length of the barn is measured at $t' = 0$ in the rocket frame. The origin $\mathcal{O}$ is the event corresponding to the measurement of the position of the left side of the barn. *Label* the event $\mathcal{R}$ that corresponds to the measurement of the position of the right side of the barn. *Find* the coördinates of $\mathcal{R}$ in the lab and rocket frames, and label these coördinates on your diagram. *Verify* that these coördinates are consistent with the invariant interval between $\mathcal{O}$ and $\mathcal{R}$ and with the Lorentz transformation between the two frames. What is the length of the barn as measured in the rocket frame?

Suppose now that at $t = 0$ in the lab (just as the rocket reaches the right side of the barn), the window on the right side of the barn is closed. The resulting mayhem is hard to predict, but let us proceed with the following (very) unrealistic scenario: when the rocket contacts the closed window, a signal propagates toward the back of the rocket at the speed of light; when this signal informs a place in the rocket that the rocket has contacted a hard barrier, that place stops instantaneously in the lab frame.

(e) *Draw* the world line of the signal that propagates through the rocket. *Label* the event $\mathcal{S}$ at which the tail of the rocket gets the message that it must stop. What are the lab-frame coördinates of $\mathcal{S}$. What is the final rest length of the rocket?

6.

) Consider the particle of Griffiths's Example 12.10, which is subjected to a constant force F in the x direction.

(a) *Draw* the trajectory [Eq. (12.62)] of the particle on a spacetime diagram for all times t, both positive and negative.

(b) *Find* the event $\mathcal{O}$ that is the same proper distance s from all events on the trajectory. What is s?

(c) Suppose the particle radiates pulses of light continuously in all directions. Do any of these pulses reach the event $\mathcal{O}$? Suppose a light pulse is emitted from event $\mathcal{O}$ in all directions. Does this light pulse ever reach the particle?

7.

) Challenge problem. **Magnetism as a relativistic phenomenon.** Consider a wire made up of equal numbers of positive and negative charges. In the lab frame the positive charges have linear charge density $\lambda_+ = \lambda$ and move with velocity v along the wire, and the negative charges have charge density $\lambda_- = -\lambda$ and are at rest. A charge q, moving in the same direction and with the same velocity v as the positive charges in the wire, is located a distance d from the wire.

(a) *Find* the electric field, the magnetic field, and the force on q in the lab frame.

For the remainder of the problem we are interested in the lab (unprimed) frame and in the rest (primed) frame of q.

(b) Consider a length l of the wire at time $t = 0$ in the lab. This length contains positive charge $Q_+ = \lambda_+ l$ and negative charge $Q_- = \lambda_- l$. On a spacetime diagram depicting both the unprimed and primed frames, *draw* the world lines of the positive and negative charges in l, and *label* the lengths $l'_\pm$ occupied by the positive and negative charges in the primed frame. How are $l'_\pm$ related to l? *Relate* the charge densities $\lambda'_\pm$ of the positive and negative charges in the primed frame to λ, and *find* the total charge density λ'_{tot} in the primed frame. What is the current I' in the primed frame?

(c) *Find* the electric field, the magnetic field, and the force on q in the rest frame of q.

(d) *Check* that the force on q obeys the proper transformation rules in going from the lab frame to the rest frame of q.

(e) How do you reconcile the fact that the total charge on the wire is zero in the lab frame, yet appears to be negative in the primed frame?