Homework #1 *Due Friday 8/29/2014 by 5PM in my mailbox*
(Note that all Griffiths Problems are from 4th Ed.)

1) Some mathematical review of material that will be useful this semester.

a) Consider a physical quantity Q (this could be charge, energy, momentum, etc.). If $\rho(\mathbf{r},t)$ is the local density of Q per unit volume, $\mathbf{J}(\mathbf{r},t)$ is the local flux density of Q (i.e. rate at which Q flows through a unit area), and $R(\mathbf{r},t)$ is the local rate at which Q is produced or destroyed per unit volume, show that the general "continuity equation"

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = R ,$$

is a local conservation law for Q. That is, show that the total amount of "Q" inside of a closed volume changes because Q flows through the volume, or because it is created/destroyed inside the volume.

b) Express each of the following complex numbers in both real/imag and polar forms

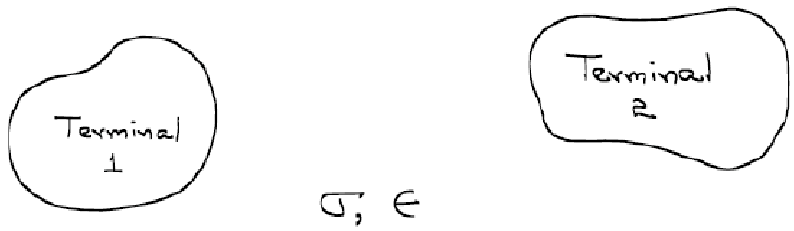
(i) $(-1-i)^2$ (ii) $\dfrac{1}{\omega^2 - \omega_0^2 - i\omega\Gamma}$

(iii) $\sqrt{-i}$ (iv) $\dfrac{e^{-i\omega t}}{\omega^2 - \omega_0^2 - i\omega\Gamma}$

Here ω, ω_0, Γ are real numbers.

2)
Consider a small ring-shaped coil of N turns and area A. It is connected to an external circuit by a twisted pair of leads. The resistance of the circuit, including the coil itself, is R.
(a) Suppose the magnetic flux through the coil is somehow changed from an initial steady value Φ_i to a final steady value Φ_f. *Find* the total charge Q which passes through the circuit as a result, and *show* that it is independent of the *rate* of change of flux.
A coil like this, called a "flip coil", is often used to measure the field strength in a magnet. Suppose the coil is placed with its plane perpendicular to a field **B**, which is uniform across the coil. *Find* the charge Q in terms of $B = |\mathbf{B}|$, N, A, and R when the coil is flipped through
(b) 90°
and
(c) 180°.

3)



(a) Two metallic terminals are embedded in a conducting medium that has conductivity σ and permittivity ϵ, as shown in the drawing above. Show that the resistance R of the medium and the capacitance C between the terminals are related by

$$RC = \frac{\epsilon}{\sigma} \,.$$

(b) Discuss how you can be sure that your analysis in part (a) is consistent with charge conservation. Would things be different if ϵ or σ depended on position?

(c) Show explicitly that the general Joule heating law,

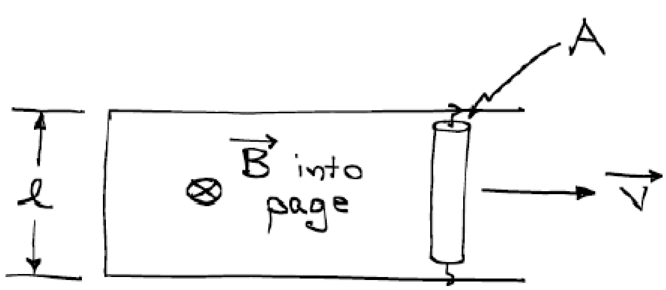
$$P = \int d\tau \, \mathbf{E} \cdot \mathbf{J} = \sigma \int d\tau \, E^2 \,,$$

reduces to $P = I^2 R$.

(d) Suppose you connect a battery between the two terminals and charge them up to a potential difference V_0. If you then disconnect the battery, the charge will leak off. Show that $V(t) = V_0 e^{-t/\tau}$, and find the "time constant" τ in terms of ϵ and σ. Evaluate τ for a good conductor like copper. Do you think this answer for τ makes sense?

4)

A cylindrical rod of resistive material slides frictionlessly on two parallel conducting rails. The rod has length l, cross-sectional area A, mass density ρ_m, and electrical conductivity σ. A uniform magnetic field **B**, pointing into the page, fills the entire region. The rod moves to the right with velocity v.



(a) What current flows through the circuit? In what direction does the current flow? Assume that all the resistance in the circuit is contained in the rod.

(b) What is the magnetic force (magnitude and direction) on the rod?

(c) Suppose the rod is given initial velocity v_0 and is then allowed to slide freely. Show that the velocity of the rod as a function of time is

$$v(t) = v_0 e^{-\Gamma t} , \quad \text{where} \quad \Gamma = \frac{B^2 \sigma}{\rho_m} .$$

(d) The initial kinetic energy was $mv_0^2/2$. At $t = \infty$ the rod has lost all its energy.

Into what sink does the energy go? Prove that energy is conserved by showing the total energy that goes into the sink is $mv_0^2/2$.