Physics 480/581

Problem Session No. 11 Monday, 12 November, 2018

1. For the Kerr metric determine the area of the surface called the (outer) horizon, i.e., the surface with \( dt = 0 \) and

\[
r = r_+ = m + \sqrt{m^2 - a^2},
\]

which is where \( \Delta \) vanishes, so that \( g_{rr} \) is infinite.

2. Since the scalar product of a 4-velocity with a Killing vector is a constant, then \( u_\phi \), and \( u_t \) are constants. However, \( u^\mu = dx^\mu/d\tau \). How do these relate?

3. Given the standard orthonormal basis for 1-forms appropriate for the Kerr metric, find the corresponding reciprocal basis for tangent vectors.

\[
\begin{align*}
\varpi^r &= \sqrt{\frac{\Sigma}{\Delta}} \, dr, & \varpi^\theta &= \sqrt{\Sigma} \, d\theta, & \varpi^t &= \sqrt{\frac{\Sigma \Delta}{A}} \, dt, \\
\varpi^\phi &= \sqrt{\frac{A}{\Sigma}} \sin \theta \, d\varphi - \frac{2ma \sin \theta}{\sqrt{\Sigma A}} \, dt = \sqrt{\frac{A}{\Sigma}} \sin \theta \left( d\varphi - \omega \, dt \right),
\end{align*}
\]

where

\[
\begin{align*}
\Sigma &\equiv r^2 + (a \cos \theta)^2, \\
\Delta &\equiv r^2 + a^2 - 2mr, & \omega &\equiv \frac{2ma}{A} = -g_{rt}^{\phi t} = -\frac{g_{\phi t}}{g_{\phi \phi}}, \\
A &\equiv (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta = (r^2 + a^2) \Sigma + 2ma^2 r \sin^2 \theta = \Delta \Sigma + 2mr(r^2 + a^2).
\end{align*}
\]