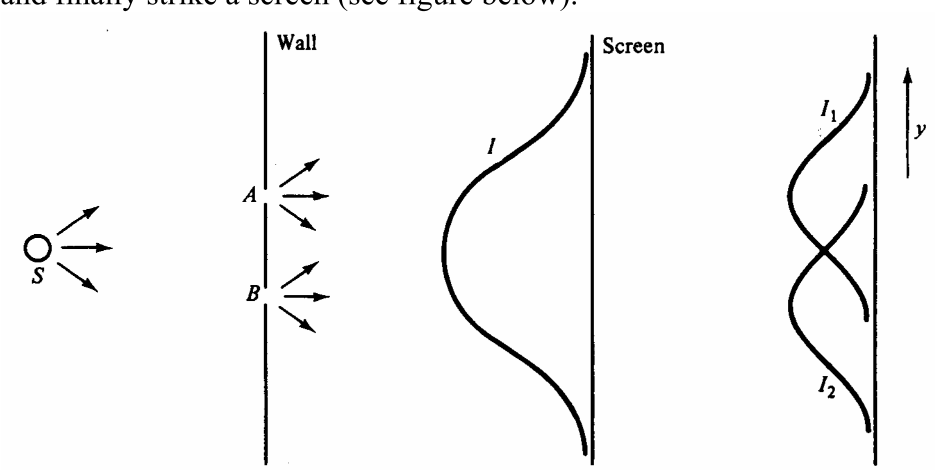
Born's Statistical Interpretation

We have seen that matter must be considered to have wave-like properties in order to explain experimental data, but what is the nature of these waves? What is "waving"? Born postulated that the wave function, Ψ, that describes a particle's behavior is related to the probability of finding the particle by:

$$\int_a^b |\Psi(x,t)|^2\, dx = \{\text{Probability of finding the particle at time t between a and b}\}$$

where $|\Psi|^2$ is the complex square, or $\Psi^* \Psi$ and Ψ^* is the complex conjugate of the (perhaps complex) wave function. This tells us that the wavefunction itself does not represent the probability, but a probability amplitude, and that the information contained in Ψ only represents the probability that one would measure a certain dynamical quantity, but cannot give pre-determined results in the same way that the deterministic classical mechanics did. Whereas classical mechanics was completely deterministic (if you know the initial conditions, you can say exactly where the particle will be at a later point in time), quantum mechanics only tells you statistical information about what the possible measurements will be. This interpretation, although since born out by much experimentation, caused much debate in the history of quantum mechanics, and continues to prick at our intuition.

Let's use the double slit experiment to demonstrate this difference. Consider a source of particles far away from a wall with two slits in it that allow the particles to pass through and finally strike a screen (see figure below).



Classical particles will follow either one of two trajectories, and pass through either one slit or the other. If they pass through the upper slit A, there will be some distribution of hits on the screen I_1. You would get this distribution, for instance, if you blocked slit B and measured where the particles hit on the screen. Likewise, if you blocked slit A, you would get the intensity pattern I_2. When you allow particles to pass through both slits, the intensities just add, $I = I_1 + I_2$ and you get the pattern in the middle of the figure. This is our intuition when it comes to classical trajectories.

Born's interpretation of the wavefunction tells us that for quantum mechanical particles (waves), the probability of finding a particle is the complex square of the total amplitude, so that $I = |\Psi|^2 = |\Psi_1 + \Psi_2|^2$. If we use the complex form of the free particle wavefunctions,

$$\Psi_1 = |\Psi_1| e^{i\alpha_1}$$

$$\Psi_2 = |\Psi_2| e^{i\alpha_2}$$

where α is the phase of the waves (given by the distance to the screen from the two slits at a particular point on the screen). Then, if we cover slit B, we get the intensity on the screen as

$$I_1 = |\Psi_1| e^{i\alpha_1} |\Psi_1| e^{-i\alpha_1} = |\Psi_1|^2$$

and likewise

$$I_2 = |\Psi_2| e^{i\alpha_2} |\Psi_2| e^{-i\alpha_2} = |\Psi_2|^2.$$

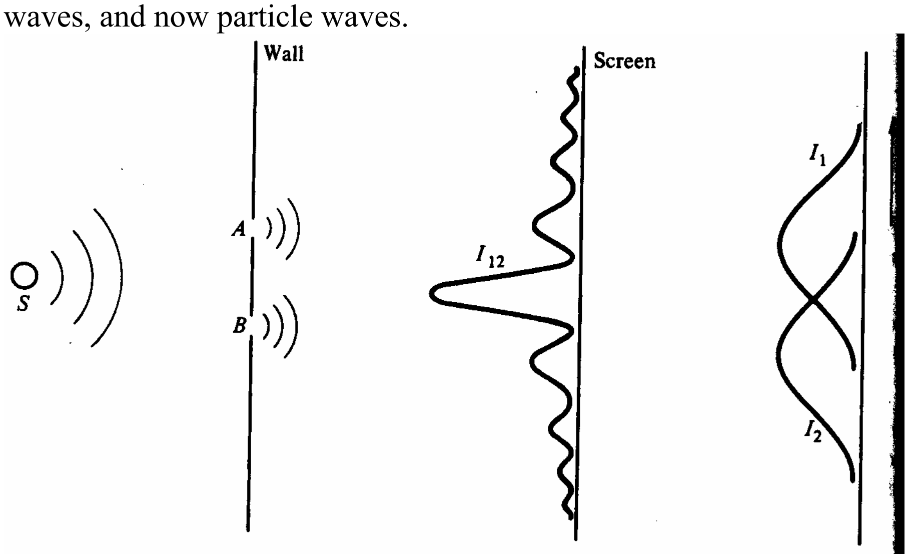
With both slits uncovered however, we have

$$I = |\Psi|^2 = |\Psi_1 + \Psi_2|^2 = (\Psi_1 + \Psi_2)^*(\Psi_1 + \Psi_2)$$

$$= |\Psi_1|^2 + |\Psi_2|^2 + |\Psi_1 \Psi_2| \left[e^{i(\alpha_1 - \alpha_2)} + e^{-i(\alpha_1 - \alpha_2)} \right]$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\alpha_1 - \alpha_2)$$

The difference in this total intensity as see in the figure below, is the presence of the interference term. This is seen in all wave mechanics, i.e., water waves, electromagnetic waves, and now particle waves.



Homework: Liboff 2.21.