Lecture 35
(de Broglie & Matter Waves)

Physics 262-01 Spring 2019

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Clicker Quiz

- For a certain metal, the work function is 4eV. If light at frequency 1.9×10^{15}Hz strikes the metal, what is the maximum kinetic energy of the ejected electrons? $h = 4.135667662(25) \times 10^{-15}$ eV*s

A) 1eV

B) 2eV

C) 3eV

D) 4eV

E) They aren't ejected

Symmetry

- I’ve talked about symmetry in physics a lot…
  - The symmetry of the Maxwell’s equations in E and B (in vacuum, at least).
  - The symmetry of space and time in special relativity.
  - Many others
- But the initial development of Quantum Mechanics is a beautiful example of how one can use symmetry to theorize something that has yet to be observed.
- This was done by Louise-Victor de Broglie in his PhD thesis of 1924…

Particle Waves

- De Broglie hypothesized that since light waves behaved like particles, perhaps particles should also behave like waves.
- Since, for light: $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{c}\frac{c}{\lambda} \Rightarrow \quad \lambda = \frac{h}{p} \quad \Rightarrow p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k$
- Then, the same should also hold true for matter (or particles).
- Hence, for a particle: $\lambda = \frac{h}{p} = \frac{h}{\frac{mv}{\sqrt{1-v^2/c^2}}} = \frac{h}{\gamma mv}$
- Which, of course, reduces in the limit of low v to:

$$\lambda = \frac{h}{mv}$$

Particle Waves

- Similarly, the frequency of a photon is given by its energy:

$$E = hf \Rightarrow$$

$$f = \frac{E}{h}$$

- So, for a particle:

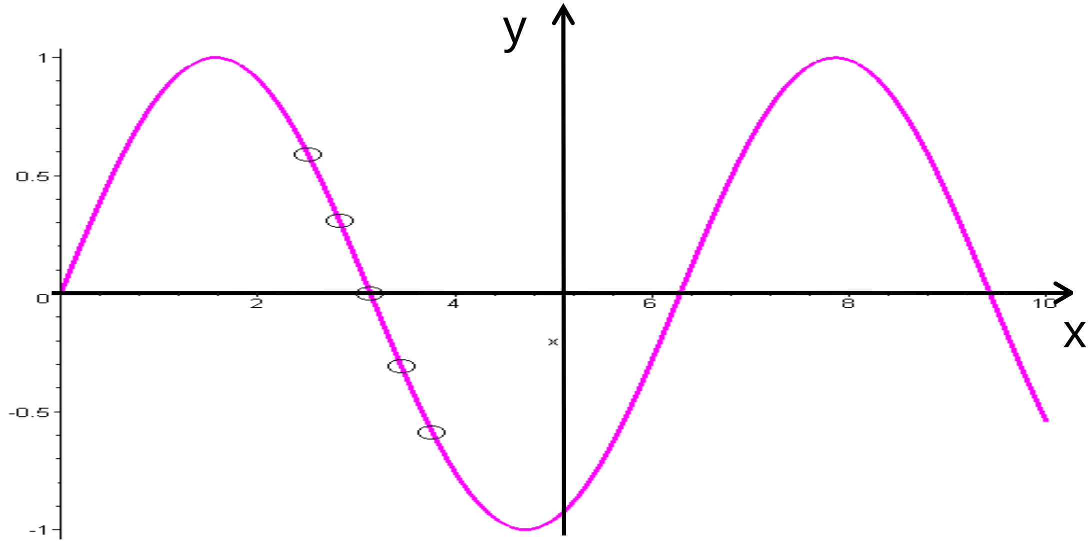
$$f = \frac{E}{h}$$

- And, again, at low v, this reduces to:

$$f = \frac{\frac{1}{2}mv^2}{h} = \frac{p^2}{2mh}$$

Periodic Wave Description

- Remember:



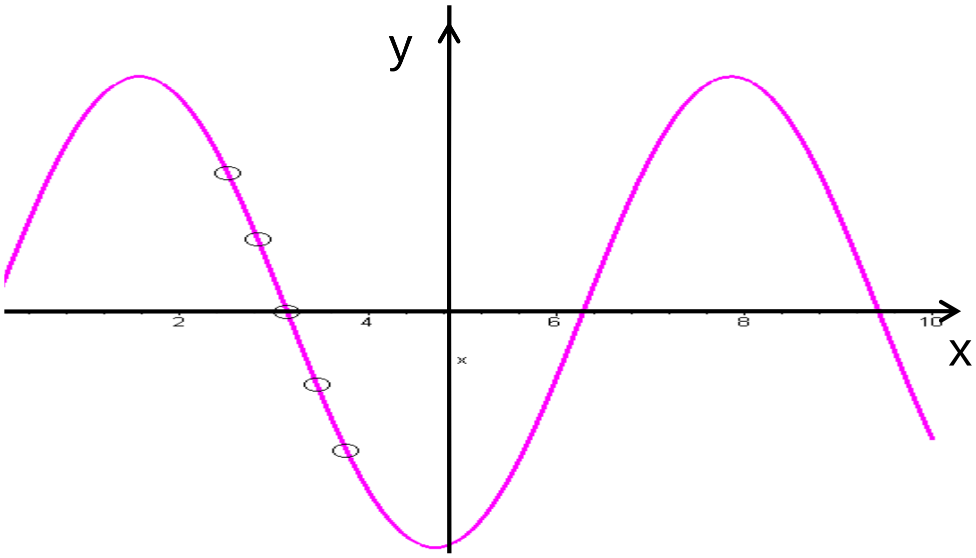
$$y(x,t) = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}, \quad \omega = \frac{2\pi}{T}$$

Wave travelling to the right

Periodic Wave Description

- How fast is the wave moving?



$$y(x,t) = 0 = A\cos(kx - \omega t) \Rightarrow$$

$$kx - \omega t = \frac{\pi}{2} \Rightarrow$$

$$x = \frac{\pi}{2k} + \frac{\omega}{k} t \Rightarrow$$

$$v = \frac{dx}{dt} = 0 + \frac{\omega}{k} = \frac{\omega}{k} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = f\lambda$$

Wave Velocity

- Let's look at this in relation to what we (should) know about wave motion.
- Since we know that the velocity of a wave is given by the angular frequency ω and wave number k, let's calculate these:

$$f = \frac{p^2}{2mh} = \frac{h^2}{\lambda^2}\frac{1}{2mh} = \frac{1}{\lambda^2}\frac{h}{2m} \Rightarrow$$

$$\omega = 2\pi f = \frac{2\pi}{\lambda^2}\frac{h}{2m} = \frac{(2\pi)^2}{\lambda^2}\frac{\frac{h}{2\pi}}{2m} = \frac{\hbar k^2}{2m}$$

- And we take our normal way of finding the velocity of a wave:

$$v = \frac{\omega}{k} = \frac{\frac{\hbar k^2}{2m}}{k} = \frac{\hbar k}{2m} = \frac{p}{2m} = \frac{1}{2}v$$

- Crap.

Group and Phase Velocity

- But ω/k is not the wave velocity, it's the phase velocity:

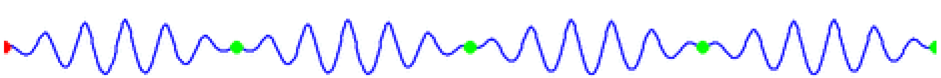
$$v_{\mathrm{p}} = \frac{\omega}{k}$$

- If we want to know how fast an isolated pulse is going we have to look at the group velocity: $v_g \equiv \frac{\partial \omega}{\partial k}$

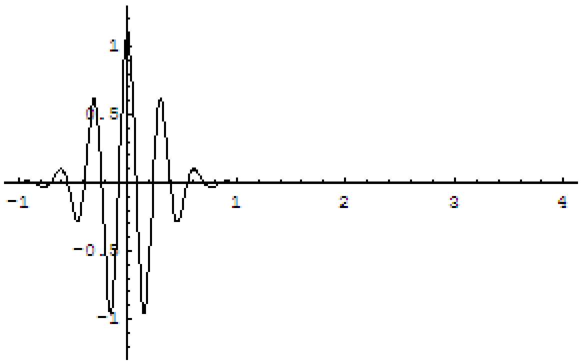
$$\omega = \frac{\hbar k^2}{2m} \Rightarrow$$

$$\frac{\partial \omega}{\partial k} = \frac{2\hbar k}{2m} = \frac{\hbar k}{m} = \frac{p}{m} = v$$

- The phase velocity doesn't represent the motion of the particle, but the difference between the phase and group velocities do become important when we think about dispersion (spreading of the wave packet) later.



"Wave group" by Kraaiennest - Own work. Licensed under GFDL via Commons - https://commons.wikimedia.org/wiki/File:Wave_group.gif#/media/File:Wave_group.gif



$$\lambda = \frac{h}{p} = \frac{h}{\gamma m v} = \frac{h}{mv}\sqrt{1 - \frac{v^2}{c^2}}$$

$$f = \frac{E}{h} = \frac{\gamma m c^2}{h} = \frac{mc^2}{h} / \sqrt{1 - \frac{v^2}{c^2}}$$

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\partial (E/\hbar)}{\partial (p/\hbar)} = \frac{\partial E}{\partial p}$$

Relativistic

$$v_g = \frac{\partial E}{\partial p}$$

$$= \frac{\partial}{\partial p}\left(\sqrt{p^2 c^2 + m_0^2 c^4}\right)$$

$$= \frac{pc^2}{\sqrt{p^2 c^2 + m_0^2 c^4}}$$

$$= \frac{pc^2}{E}$$

Non relativistic

$$v_g = \frac{\partial E}{\partial p} = \frac{\partial}{\partial p}\left(\frac{1}{2}\frac{p^2}{m}\right) = \frac{p}{m} = v$$

Example

An electron is accelerated through a potential difference of 10V. What is the wavelength of this electron?

Do we need to use special relativity?

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Do we need to use special relativity?

$$KE\left(10eV\right) = E - mc^2\left(511{,}000eV\right) \Rightarrow$$
$$E \sim mc^2$$

$$p = \frac{1}{c}\sqrt{E^2 - m^2c^4}$$
$$v = \frac{pc^2}{E} = c\sqrt{1 - \frac{m^2c^4}{E^2}}$$

No, v=0.006c

A good rule of thumb is that if the kinetic energy is much less than the rest energy, you can use classical mechanics. Otherwise, need special relativity.

Example

An electron is accelerated through a potential difference of 10V. What is the wavelength of this electron?

$$KE = \frac{p^2}{2m} \qquad \lambda = \frac{h}{p}$$

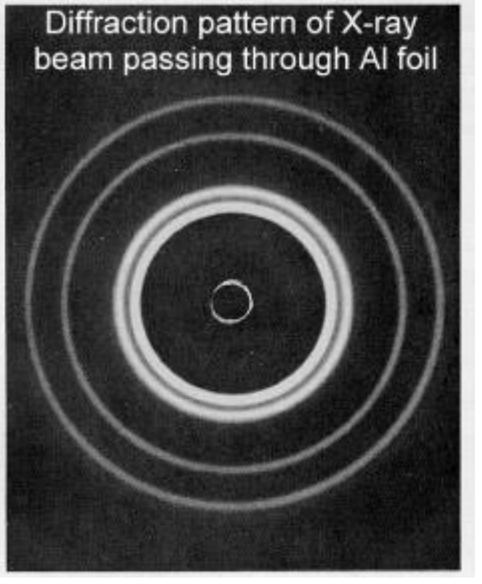
$$KE = \frac{p^2}{2m} \Rightarrow$$

$$p = \sqrt{2m(KE)} = \sqrt{2\left(511\times10^3 eV/c^2\right)\left(10eV\right)} = 3197\, eV/c \Rightarrow$$

$$\lambda = \frac{h}{p} = \frac{4.14\times10^{-15} eV\cdot s}{3197eV}\left(3.0\times10^8\, m/s\right) = 0.39nm$$

Electron Diffraction

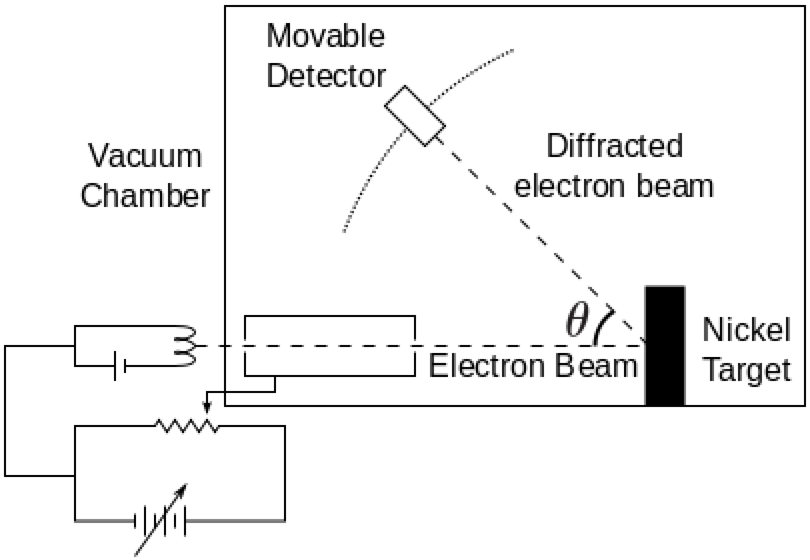
- So, if electrons travel as waves, we should be able to see electron diffraction, right?



Davisson-Germer Experiment

- From 1921 to 1925 physicists Clinton Davisson and Lester Germer (at Bell Labs) were trying to understand electron scattering off of metals.
- By accidentally overheating the metal, they formed regions where the metal became crystalline in nature.
- Because of the crystal spacing, the electrons scattered in a similar manner to a two-slit experiment (although repeated in 2D).

$$n\lambda = 2d\sin\left(90^\circ - \frac{\theta}{2}\right)$$



"Davisson-Germer experiment" by Roshan - Own work. Licensed under CC BY-SA 3.0 via Commons - https://commons.wikimedia.org/wiki/File:Davisson-Germer_experiment.svg#/media/File:Davisson-Germer_experiment.svg

for $n = 1$, $\theta = 50°$, and for the spacing of the crystalline planes of nickel ($d = 0.091$ nm) obtained from previous X-ray scattering experiments on crystalline nickel.[2] According to the de Broglie relation, electrons with kinetic energy of 54 eV have a wavelength of 0.167 nm. The experimental outcome was 0.165 nm via Bragg's law, which closely matched the predictions. **Davisson and Germer's accidental discovery of the diffraction of electrons was the first direct evidence confirming de Broglie's hypothesis that particles can have wave properties as well**. Davisson's attention to detail, his resources for conducting basic research, the expertise of colleagues, and luck all contributed to the experimental success.

…Wikipedia

Example

What is the de Broglie wavelength of a typical major league baseball fast pitch (m_{ball} = 0.145kg, v_{ball} = 150km/hr?

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What is the de Broglie wavelength of a typical major league baseball fast pitch (m_{ball} = 0.145kg, v_{ball} = 150km/hr)?

$$p = mv = 0.145kg \cdot 1.5\times10^{5}\ m/hr \cdot \frac{1}{3600}\ hr/s = 6.04\,kg\cdot m/s \Rightarrow$$

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\ J\cdot s}{6.04\,kg\cdot m/s} = 1.1\times10^{-34}\,m$$