Lecture 33
(Emission and Interaction of Photons)

Physics 262-01 Spring 2019
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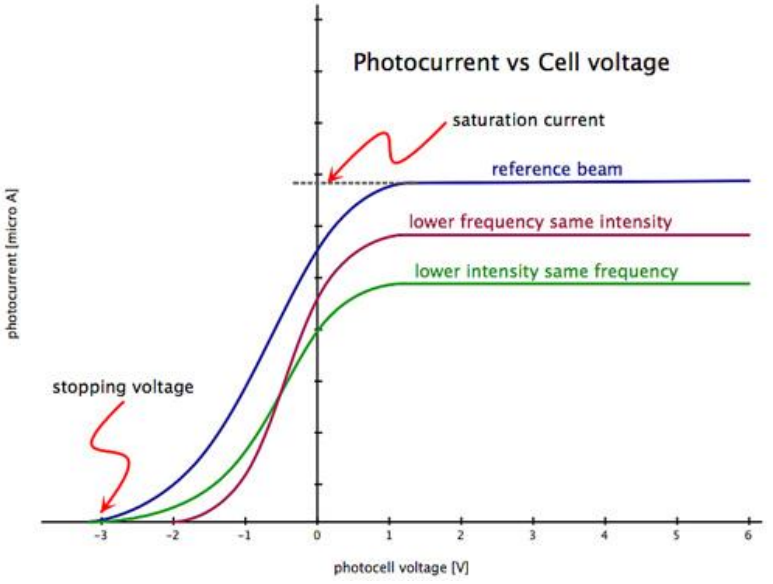
Simulation

- https://phet.colorado.edu/en/simulation/photoelectric

Photoelectric Effect Results

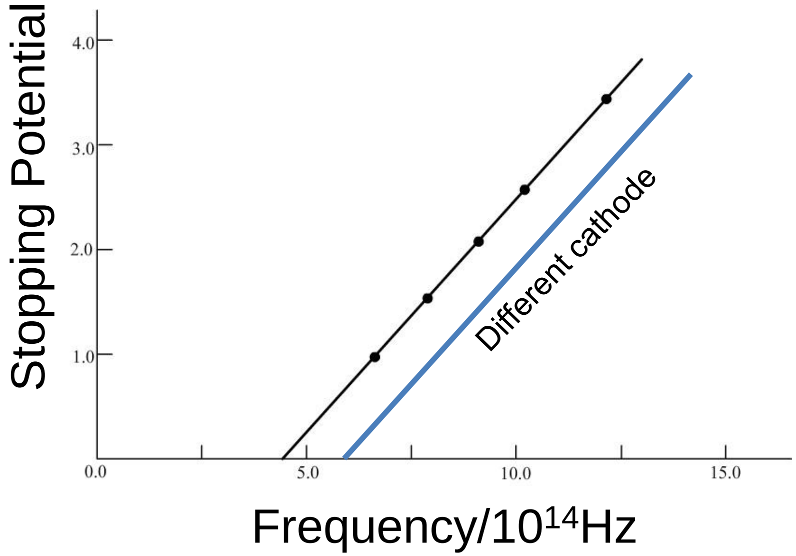
- Now, if we look at the current as a function of the voltage between the cathode and anode (labeled photocell voltage below) we see (blue line):
  - Above a certain positive voltage, and for a fixed intensity, the current remains constant.
  - All electrons that make it out of the cathode are accelerated and make it to the anode and are measured as a current.
  - As the voltage is lowered, some electrons don't make it to the anode – the current is reduced.
  - Even with negative voltage, there is some current, until at some point the current goes to zero.
- If we lower the intensity (green line):
  - The saturation current is lowered, but the point at which the current becomes zero remains the same.

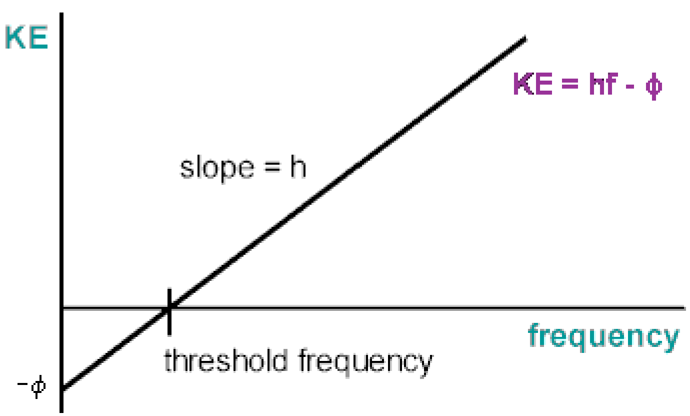
- It turns out that **the stopping voltage** stays the same regardless of intensity for a particular cathode material, but is different for different cathode materials.
- The stopping potential for a particular cathode material also changes depending on the frequency of light shining on it (red line):
  - Lower frequencies have stopping potentials closer to zero.
- Below some frequency for a certain cathode material, no current is seen regardless of intensity or voltage.
- Regardless of intensity, there is no time delay between turning on the light and measuring a current



Understanding the Results

- The stopping potential as a function of the frequency (for a particular cathode) turns out to be linear.
- Different cathodes have the same slope, ***h***, but cross the horizontal axis at a different point.
- If we think of the stopping potential as the maximum kinetic energy which an emitted electron has after leaving the surface,
- then the linear function has the form: $KE = hf - \phi$
- Where ϕ is called the work function and is only dependent on the cathode material – it is the amount of energy needed to just get an electron out of the surface.





Classical Wave Expectations

- What was the classical physics expectations for this experiment?
  1. The energy carried by an electromagnetic wave is dependent on its intensity (~amplitude of electric field squared), not on the frequency, so there should be current at all frequencies with sufficient magnitude, and the current should depend on the intensity, not on the frequency.
  2. Since the minimum energy to knock out an electron (the work function ϕ) could be built up over time, at low intensities one should see a time delay between turning on the light and seeing the current.
  3. The stopping potential should increase with increasing intensity, and should be independent of frequency.
- Needless to say, the experimental results were astonishing at the time!

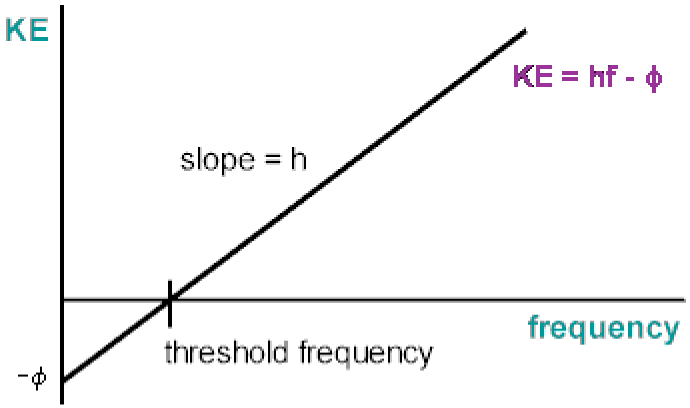
Einstein’s Explanation

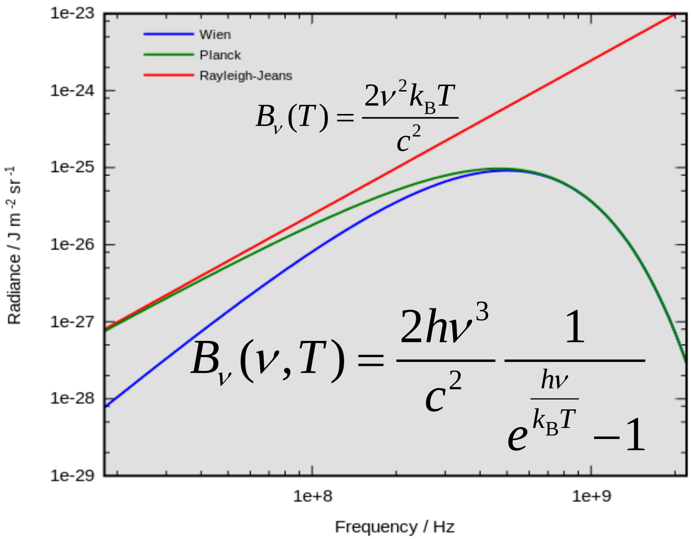
- Einstein proposed a solution to the problem that seems easy to see in hindsight, but was not well accepted at the time.
- He proposed that light itself was quantized – it could only be found in discreet (as opposed to continuous) packages of energy, proportional to the frequency – ***nhf***.
- So a single photon (n=1) would have energy

$$E_{photon} = hf = \frac{hc}{\lambda}, \quad h = 6.62606896(33)\times 10^{-34}\, J\cdot s$$

Relation to Blackbody Radiation

- Notice that the constant h (known as Planck's constant) is popping up everywhere we look.





$$E_{photon} = hf = \frac{hc}{\lambda}, \quad h = 6.62606896(33)\times 10^{-34}\, J\cdot s$$

Photons

- Just to give you a better idea of scale, for a photon of visible light with λ = 600nm:

$$f = \frac{c}{\lambda} = \frac{3\times10^{8}\, m/s}{600\times10^{-9}\, m} = 5\times10^{14}\, Hz$$

$$E_{photon} = hf = \left(6.626\times10^{-34}\, J\cdot s\right)\cdot\left(5\times10^{14}\, s^{-1}\right) = 3\times10^{-19}\, J$$

- Now, as we deal with these very small energies, we tend to use different units, in particular the unit of energy which is tied to the energy of an electron accelerated through 1V of potential difference:

$$1eV = 1.602\times10^{-19}\, J$$

$$so$$

$$E_{\text{visible}} = \frac{3\times10^{-19}\, J}{1.602\times10^{-19}\ J/eV} = 1.87eV \sim 2eV$$

A laser pointer with a power output of 5.00 mW emits red light with λ = 650nm.

a) How many photons does the laser pointer emit each second?
b) What is the momentum of each photon?
c) What is the reactive force felt by holding a laser pointer (turned on).

$$E_{photon} = hf = \frac{hc}{\lambda}, \quad h = 6.62606896(33)\times 10^{-34}\, J\cdot s$$

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a)

$$P = 5.00\times10^{-3}\ J/s$$

$$E_{photon} = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ J\cdot s\right)\cdot\left(3\times10^{8}\ m/s\right)}{650\times10^{-9}\,m} = 3.06\times10^{-19}\ J$$

$$\frac{\text{\#photons}}{s} = \left(\frac{\text{total energy}}{s}\right)\left(\frac{\text{photon}}{\text{Energy}}\right)$$

$$= \frac{5.00\times10^{-3}\ J/s}{3.06\times10^{-19}\ J} = 1.63\times10^{16}\ \frac{\text{photons}}{s}$$

A laser pointer with a power output of 5.00 mW emits red light with λ = 650nm.

a) How many photons does the laser pointer emit each second?

b) What is the momentum of each photon?

c) What is the reactive force felt by holding a laser pointer (turned on).

b) $$p = \frac{E}{c} = \frac{3.06\times10^{-19}\,J}{3\times10^{8}\,m/s} = 1.02\times10^{-27}\,kg\cdot m/s$$

c) $$F = \frac{dp}{dt} = \left(\frac{\#\text{photons}}{s}\right)(\text{photon momentum})$$

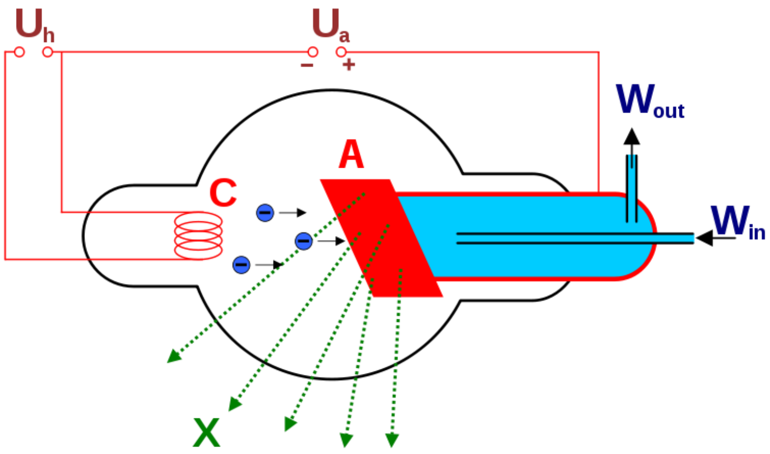
$$= \left(1.63\times10^{16}\,\frac{\text{photons}}{s}\right)\cdot\left(1.02\times10^{-27}\,kg\cdot m/s\right)$$

$$= 1.66\times10^{-11}\,N$$

X-ray Photons

- So, light is absorbed in quanta, but what about emission?
- In the late 1800s, Röntgen first observed high energy electromagnetic emission (x-rays) using a Crooke’s tube.

"Crookes tube xray experiment" by William J. Morton - Downloaded 2007-12-23 from William J. Morton and Edwin W. Hammer (1896) The X-ray, or Photography of the Invisible and its value in Surgery, American Technical Book Co., New York, fig. 54 on Google Books. Licensed under Public Domain via Commons - https://commons.wikimedia.org/wiki/File:Crookes_tube_xray_experiment.jpg#/media/File:Crookes_tube_xray_experiment.jpg



"WaterCooledXrayTube" by Roentgen-Roehre.svg: Hmilchderivative work: Coolth (talk) - Roentgen-Roehre.svg. Licensed under Public Domain via Commons - https://commons.wikimedia.org/wiki/File:WaterCooledXrayTube.svg#/media/File:WaterCooledXrayTube.svg

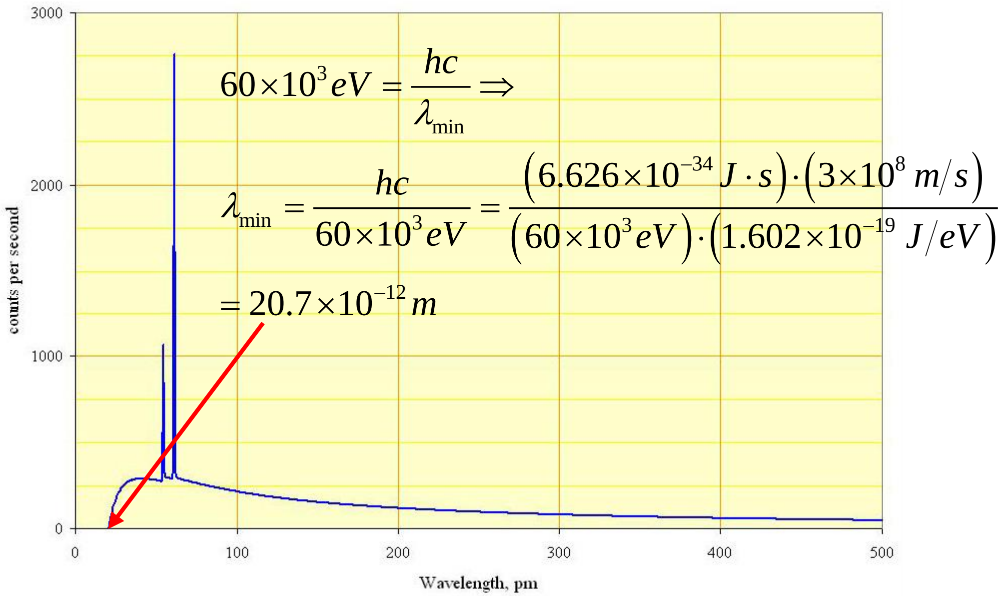
X-ray Photons

- X-rays pass through low-Z materials (like water), but get scattered from high-Z materials (like calcium), so they can be used to images bones…

X-ray Photons

- The wave model prediction for the oscillation of the electrons inside the anode struck by an electron was that all frequencies would be excited (the book makes the analogy of the sound made when a gong is hit), hence, all wavelengths of light should be emitted.
- But that's not what was seen. There is a cutoff in the wavelength, and hence frequency that corresponds to:

$$\frac{hc}{\lambda_{\min}} = hf_{\max} = eV_A$$



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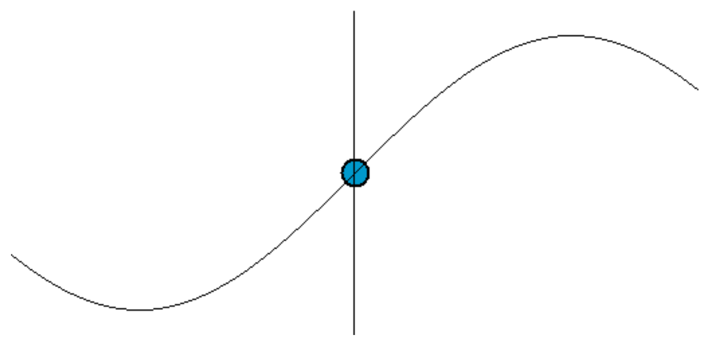
Compton Scattering

- In the early 1900s, there was a flurry of experiments meant to test both the photon theory of light and the newly discovered special relativity of spacetime.
- Perhaps none were so convincing as the x-ray scattering experiment of Arthur H. Compton in 1923.
- The idea behind the experiment was this:
  - What should be the wavelength and angular distribution of light scattered off of an electron?
- Let’s first consider the answer based on classical physics…

Classical Light Scattering

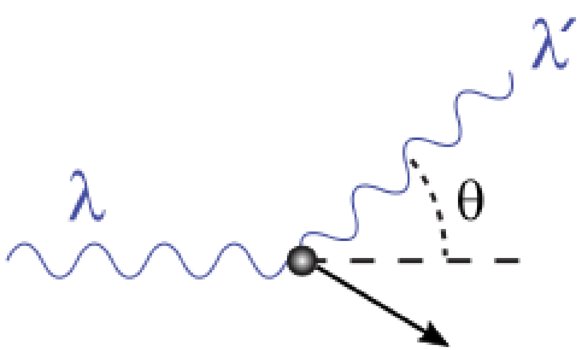
- Let’s say we have light hitting an electron.
- The light has a fixed frequency.
- What will the electron do?



- It just acts like an antenna, oscillating at the same frequency as incoming light, and hence re-radiating light of the same frequency with a dipole radiation distribution.

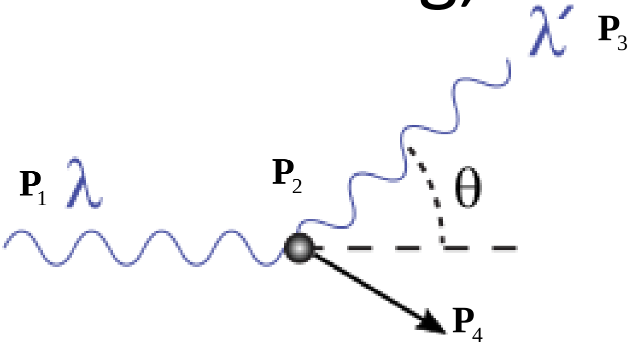
Photon Scattering

- Alright, now let's use some of the physics we have recently learned, and ask how would a photon scatter from an electron?
- We have to use special relativity if the energy of the incoming photon is large compared to the rest mass of the electron (511 keV).
- We will assume that the electron is unbound (but will come back to that later).



Photon Scattering

- We know that four-momentum must be conserved in the scattering,



Total Four-Momentum Before = Total Four-Momentum After

$$\mathbf{P} = \left[\frac{m}{\sqrt{1-v^2}}, \frac{mv_x}{\sqrt{1-v^2}}, \frac{mv_y}{\sqrt{1-v^2}}, \frac{mv_z}{\sqrt{1-v^2}}\right] = \left[E, P_x, P_y, P_z\right]$$

$$\begin{bmatrix} E_1 \\ P_{1x} \\ P_{1y} \\ P_{1z} \end{bmatrix} + \begin{bmatrix} E_2 \\ P_{2x} \\ P_{2y} \\ P_{2z} \end{bmatrix} = \begin{bmatrix} E_3 \\ P_{3x} \\ P_{3y} \\ P_{3z} \end{bmatrix} + \begin{bmatrix} E_4 \\ P_{4x} \\ P_{4y} \\ P_{4z} \end{bmatrix}$$

Photon Scattering

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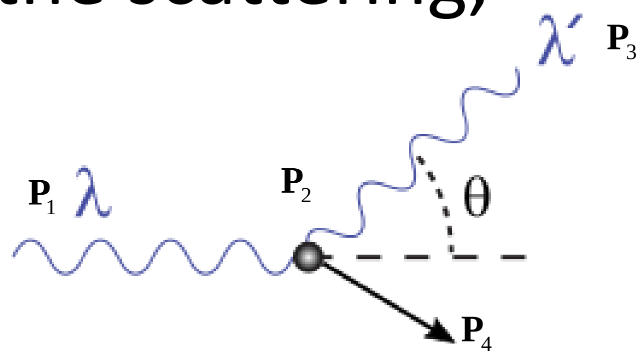


Total Four-Momentum Before = Total Four-Momentum After

$$E_{phot} = hf$$
$$c = f\lambda \Rightarrow$$
$$E_{phot} = \frac{hc}{\lambda}$$
$$E^2 = p^2c^2 + \cancel{m^2}c^4 \Rightarrow E = pc$$
$$p = \frac{E}{c} \Rightarrow$$
$$p = \frac{h}{\lambda}$$

$$E_1 = \frac{hc}{\lambda} \quad P_{1x} = \frac{h}{\lambda} \quad P_{1y} = 0 \quad P_{1z} = 0$$

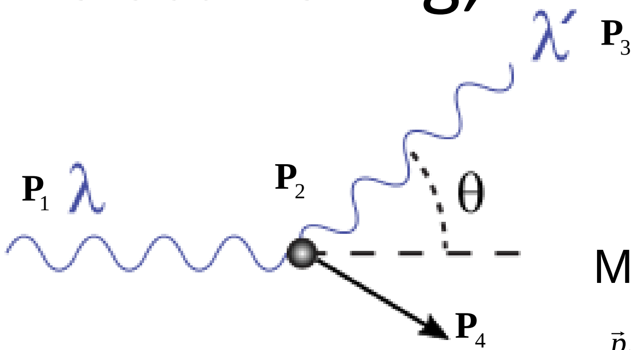
$$E_2 = m_e c^2 \quad P_{2x} = 0 \quad P_{2y} = 0 \quad P_{2z} = 0$$

$$E_3 = \frac{hc}{\lambda'} \quad P_{3x} = p_3 \cos\theta \quad P_{3y} = p_3 \sin\theta \quad P_{3z} = 0$$

$$E_4 = ? \quad P_{4x} = ? \quad P_{4y} = ? \quad P_{4z} = 0$$

Photon Scattering

- We know that four-momentum must be conserved in the scattering,



Energy Conservation

$$E_1 + E_2 = E_3 + E_4 \Rightarrow$$
$$p_1c + m_ec^2 = p_3c + E_4 \Rightarrow$$
$$p_1c - p_3c + m_ec^2 = E_4$$
$$but,$$
$$E_4^2 = p_4^2c^2 + m_e^2c^4$$
$$so,$$
$$\left(p_1c - p_3c + m_ec^2\right)^2 = p_4^2c^2 + m_e^2c^4 \Rightarrow$$
$$p_1^2c^2 + p_3^2c^2 + m_e^2c^4 - 2p_1p_3c^2 + 2p_1m_ec^3 - 2p_3m_ec^3 = p_4^2c^2 + m_e^2c^4$$
$$p_1^2 + p_3^2 - 2p_1p_3 + 2p_1m_ec - 2p_3m_ec = p_4^2$$

Momentum Conservation

$$\vec{p}_1 + \cancel{\vec{p}_2}^{\,0} = \vec{p}_3 + \vec{p}_4 \Rightarrow$$
$$\vec{p}_4 = \vec{p}_1 - \vec{p}_3 \Rightarrow$$
$$p_4^2 = \vec{p}_4 \cdot \vec{p}_4 = \left(\vec{p}_1 - \vec{p}_3\right) \cdot \left(\vec{p}_1 - \vec{p}_3\right)$$
$$= p_1^2 + p_3^2 - 2p_1p_3\cos\theta$$

Photon Scattering

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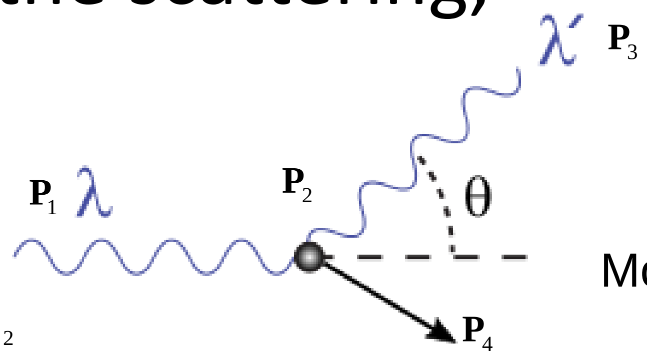

Energy Conservation

$$p_1^2 + p_3^2 - 2p_1p_3 + 2p_1m_ec - 2p_3m_ec = p_4^2$$

Momentum Conservation

$$p_4^2 = p_1^2 + p_3^2 - 2p_1p_3\cos\theta$$

Substitution

$$p_1^2 + p_3^2 - 2p_1p_3 + 2p_1m_ec - 2p_3m_ec = p_1^2 + p_3^2 - 2p_1p_3\cos\theta \Rightarrow$$

$$p_1m_ec - p_3m_ec = p_1p_3\left(1-\cos\theta\right) \Rightarrow$$

$$\frac{1}{p_3} - \frac{1}{p_1} = \frac{1}{m_ec}\left(1-\cos\theta\right) \Rightarrow$$

$$\lambda' - \lambda = \frac{h}{m_ec}\left(1-\cos\theta\right)$$

Compton Scattering Formula

$$\frac{h}{m_ec} = 2.426\times10^{-12}\,m$$

Compton Scattering Results

- We know that four-momentum must be conserved in the scattering,

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Compton Scattering Formula