

Lecture 3

(Coupled Harmonic Oscillators)

Physics 262-01 Spring 2019

Douglas Fields

Tutor Table

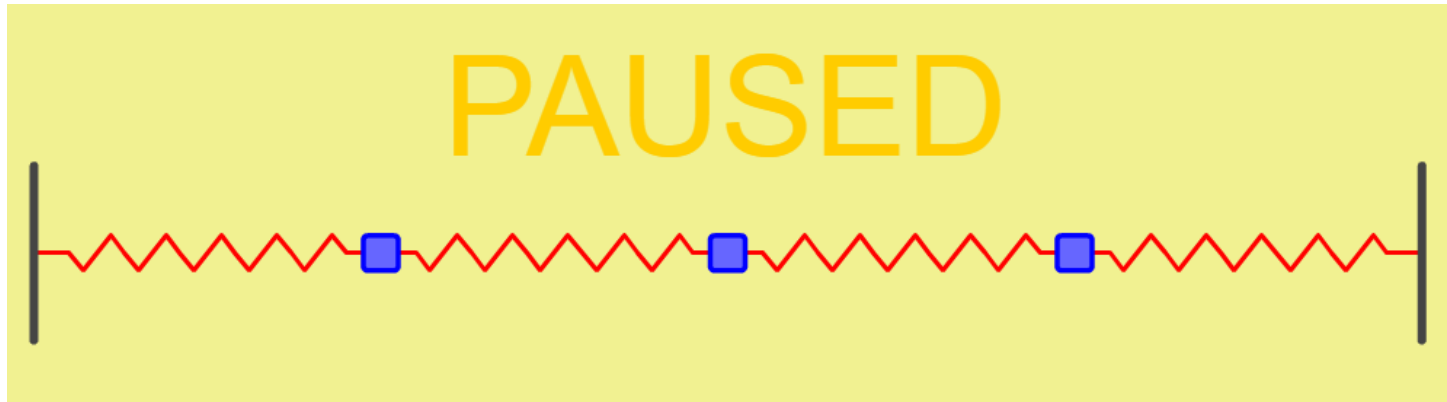
<http://physics.unm.edu/pandaweb/undergraduate/tutoring.php>

Spring 2019 Physics Tutoring Table - Weekly Schedule
Regener 111

	Monday	Tuesday	Wednesday	Thursday	Friday
8AM-9AM					Buchemmavari
9AM-10AM		Wickramasinghe			Buchemmavari
10AM-11AM		Wickramasinghe			Buchemmavari
11AM-12PM				Dhanasingham	
12PM-1PM		Seth	Morrison	Medina	Zhao
1PM-2PM	Thurtell	Slezak	Richards	Babaa	
2PM-3PM	Demme	Dunagin	Patra	Babaa	
3PM-4PM	Wickramasinghe		Sauer	Babaa	Seth
4PM-5PM	Omanakuttan	Ward	Bassett		Seth
5PM-6PM	Yu			Aryal	

Reading Quiz

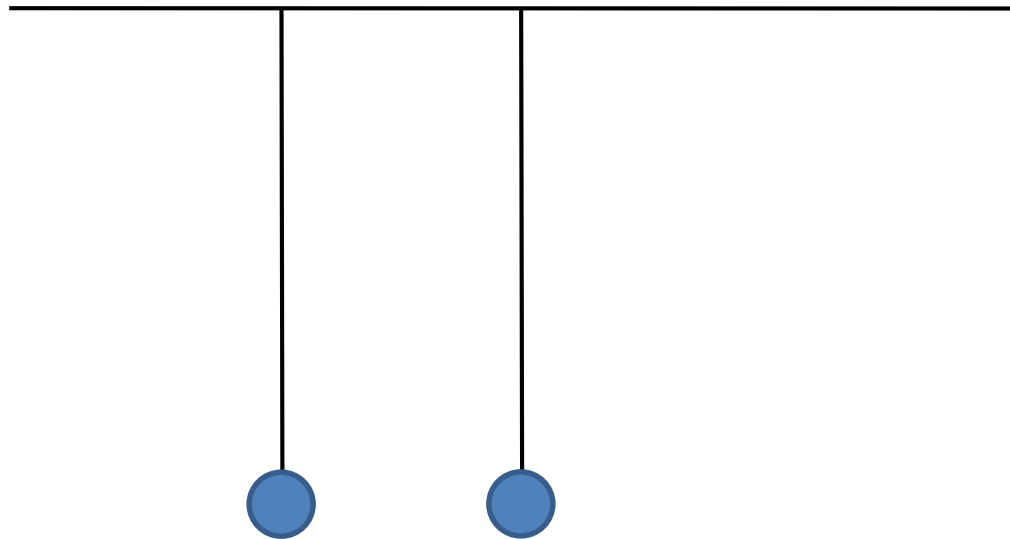
- With only three masses coupled together, how many normal modes would there be?



- A) 0
- B) 1
- C) 2
- D) 3
- E) 4

Coupled Harmonic Oscillators

- Let's go back to our discussion of harmonic oscillators, specifically pendulums.
- When two SHO are put into contact some way, such that energy from one can be transferred to the other, we have coupled harmonic oscillators.



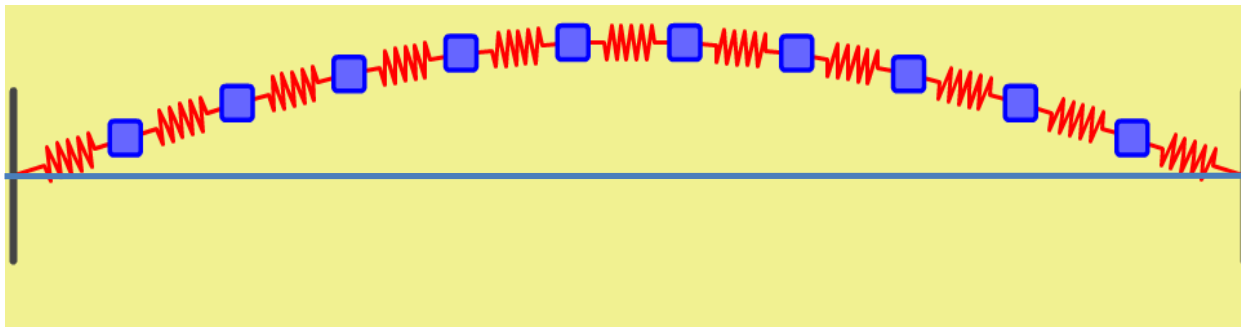
- https://phet.colorado.edu/sims/normal-modes/normal-modes_en.html

Normal modes?

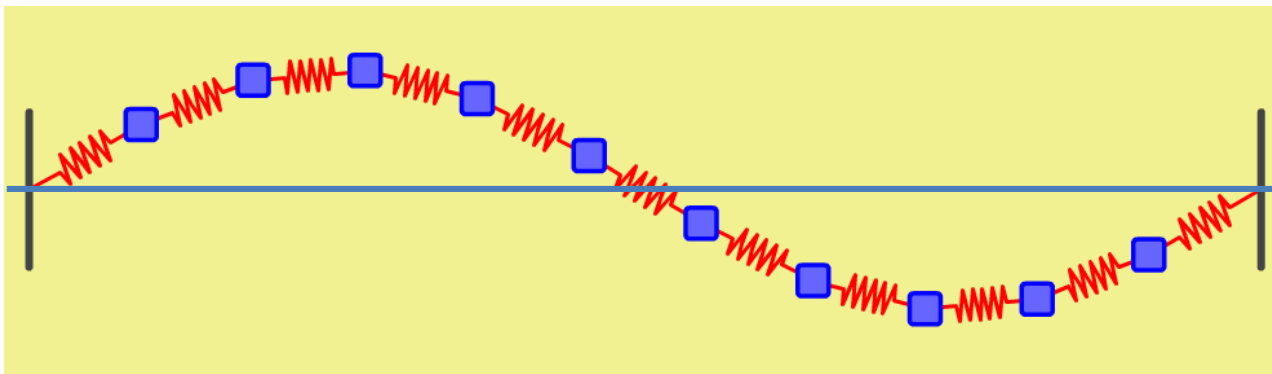
- **Notice** that:
 - For a particular normal mode, every mass has the same frequency of oscillation.
 - There is the same total number of normal modes as there are masses.
 - Normal modes alternate between being symmetric (with respect to the center) and anti-symmetric.
 - Each normal mode has a certain number of nodes and antinodes:
 - Nodes are points where the amplitude of oscillations is zero.
 - Antinodes are points where the amplitude of oscillations is maximal.

Normal modes?

- Notice that, for a given normal mode, the amplitude of oscillation for a particular mass does not change over time, but are different from mass to mass:



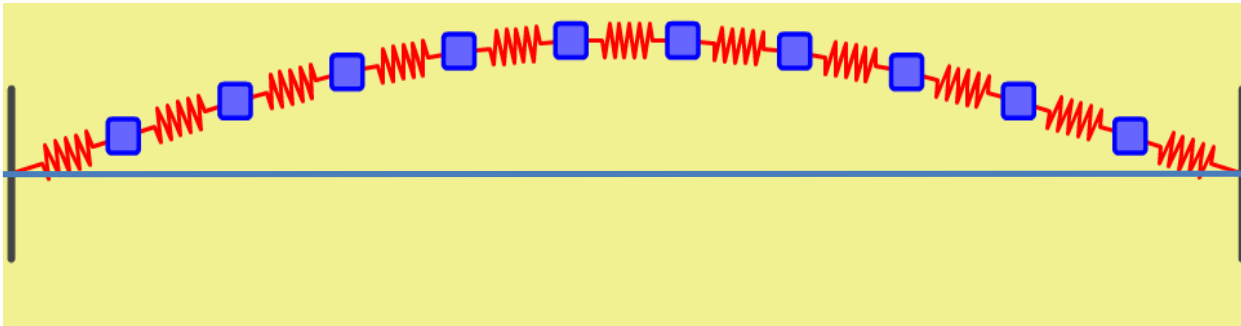
1st Normal mode; $N = 1$



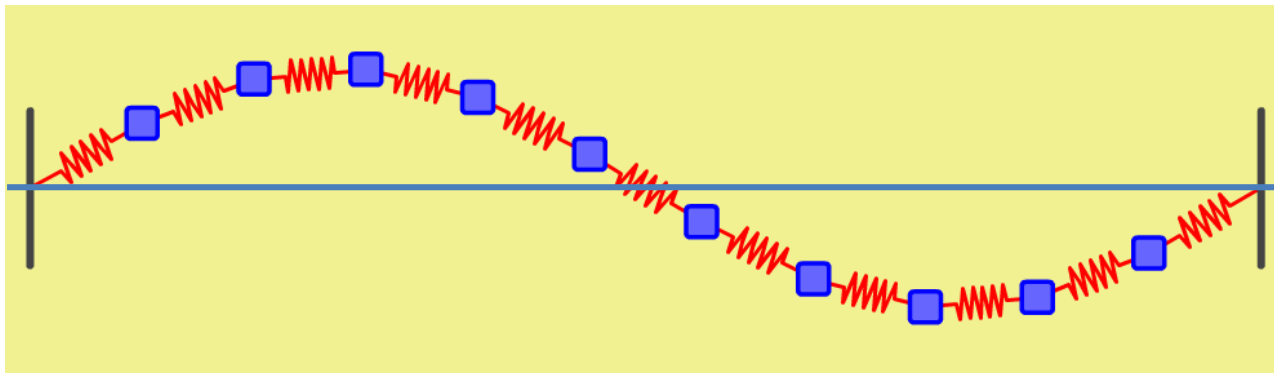
2nd Normal mode; $N = 2$

Normal modes?

- To try to make a mathematical description of the normal modes, we notice that the amplitudes vary as a sine function of the position:



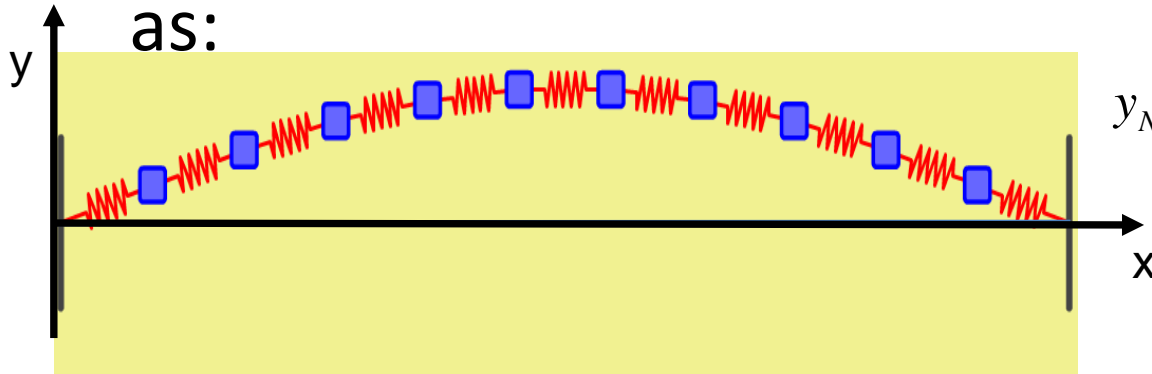
$$A_N(x_i) \propto \sin(k_N x_i) \\ = A \sin(k_N x_i)$$



Read: the amplitude of the i^{th} mass in the N^{th} normal mode is proportional to the sin of the x -position of the i^{th} mass times a constant which depends on the mode number

Time Dependence

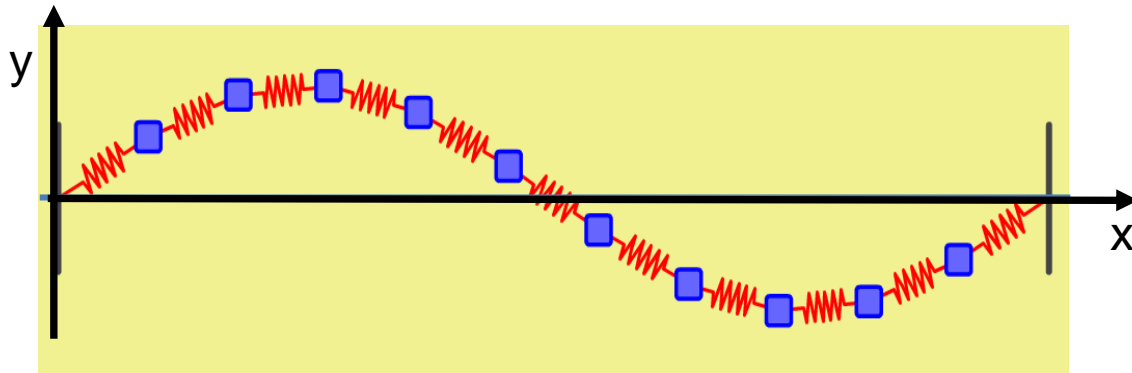
- Since, for the normal modes, all of the masses oscillate with the same frequency, their displacement from equilibrium can be described as:



$$y_N(x_i, t) = A_N \cos(\omega_N t)$$

$$= \underbrace{A \sin(k_N x_i)}_{\text{spatial part}} \cos(\omega_N t)$$

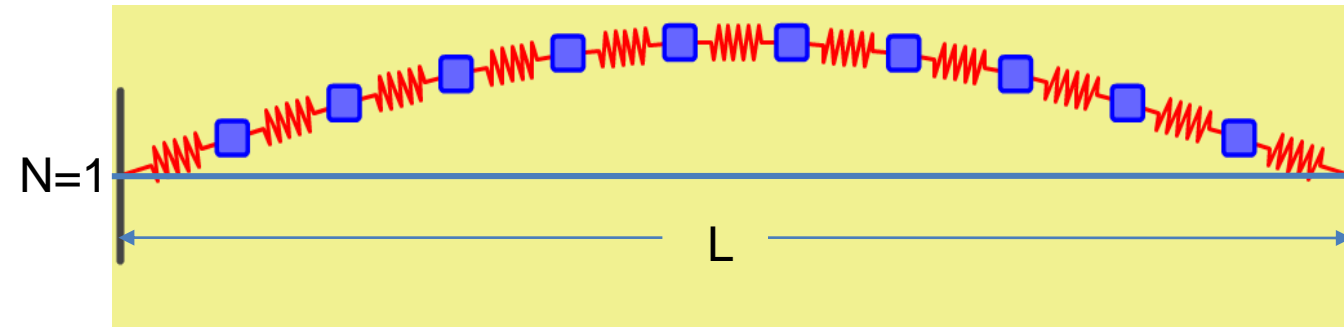
A blue arrow points from the A_N term in the first equation to the A term in the second equation.



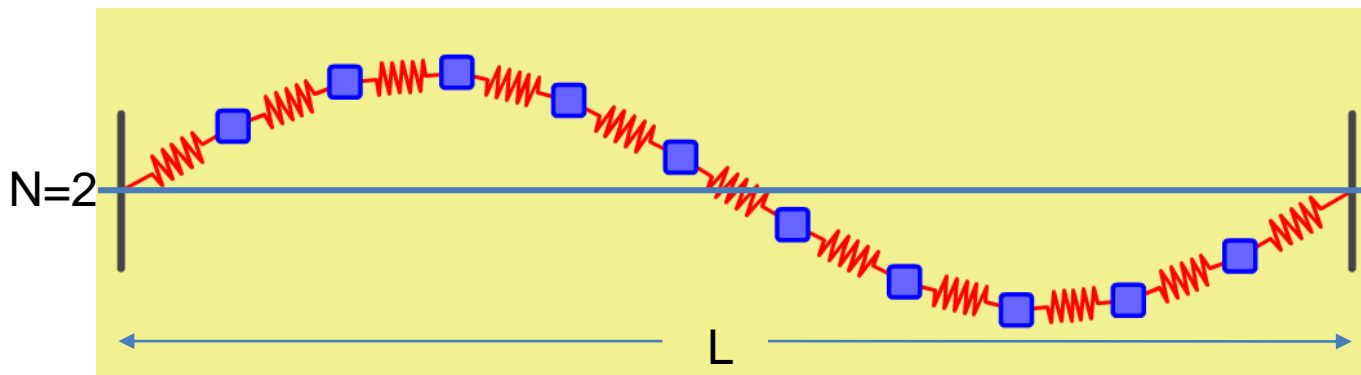
Questions

- What units do the k_N have?
- What units do the ω_N have?
- What is the value of k_2 ?

$$y_N(x_i, t) = A \sin(k_N x_i) \cos(\omega_N t)$$



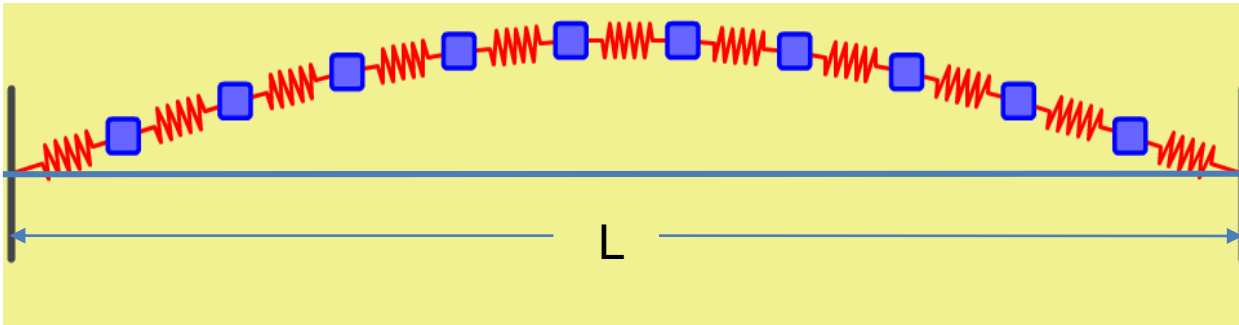
$$A_1(x_i) \propto \sin(k_1 x_i)$$



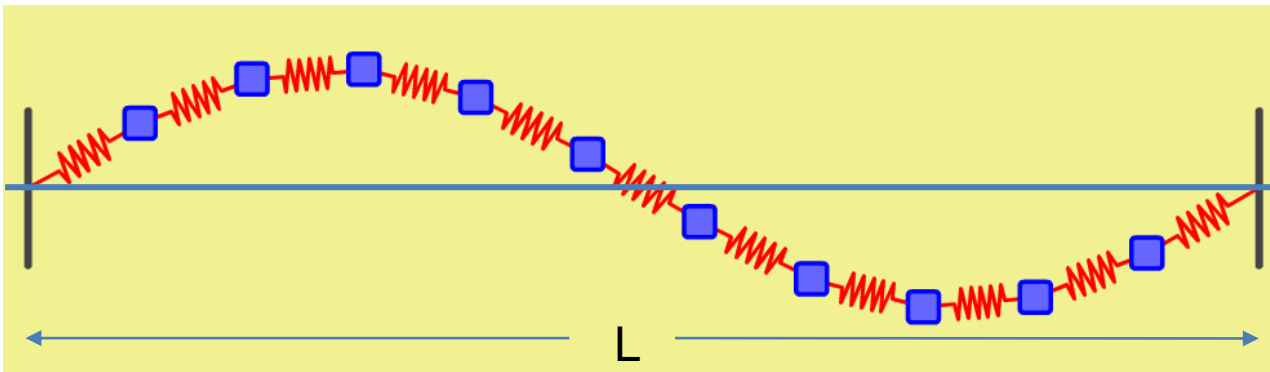
$$A_2(x_i) \propto \sin(k_2 x_i)$$

Answers

- The argument of a trigonometric function must be unitless, so k_N must have units of m^{-1} .
- For $N=2$, after the distance L , the sine function repeats, and since sine repeats every 2π , k_2 must be $2\pi/L$.
- k_1 would then be $2\pi/2L$.



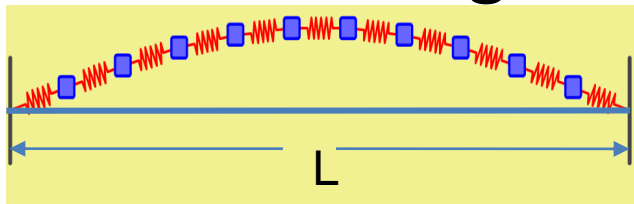
$$A_1(x_i) \propto \sin(k_1 x_i)$$
$$A_1(x_i) \propto \sin\left(\frac{\pi}{L} x_i\right)$$



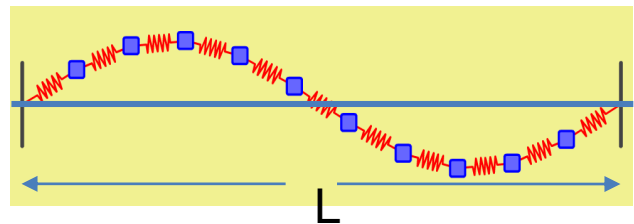
$$A_2(x_i) \propto \sin(k_2 x_i)$$
$$A_2(x_i) \propto \sin\left(\frac{2\pi}{L} x_i\right)$$

General k

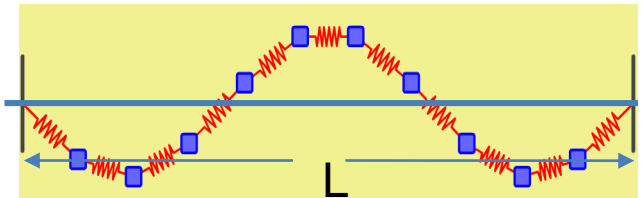
- By looking at a few more normal modes we can see the general form for k:



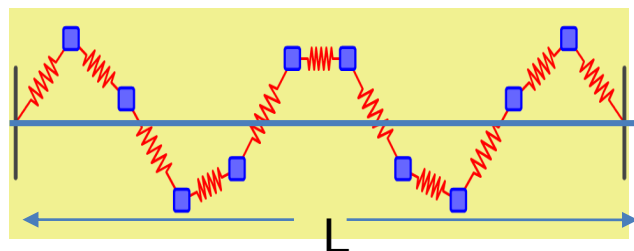
$$A_1(x_i) \propto \sin\left(\frac{\pi}{L} x_i\right)$$



$$A_2(x_i) \propto \sin\left(\frac{2\pi}{L} x_i\right)$$



$$A_3(x_i) \propto \sin\left(\frac{3\pi}{L} x_i\right)$$



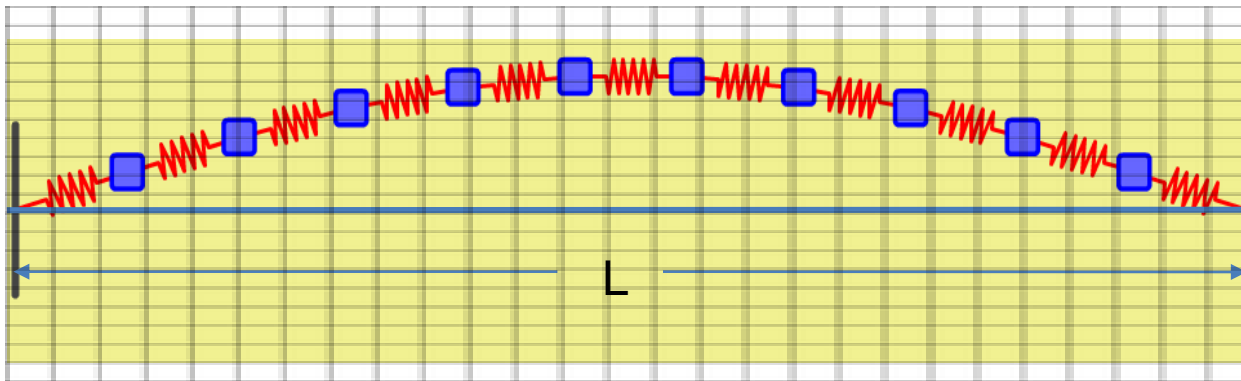
$$A_5(x_i) \propto \sin\left(\frac{5\pi}{L} x_i\right)$$

$$k_N = \frac{N\pi}{L}$$

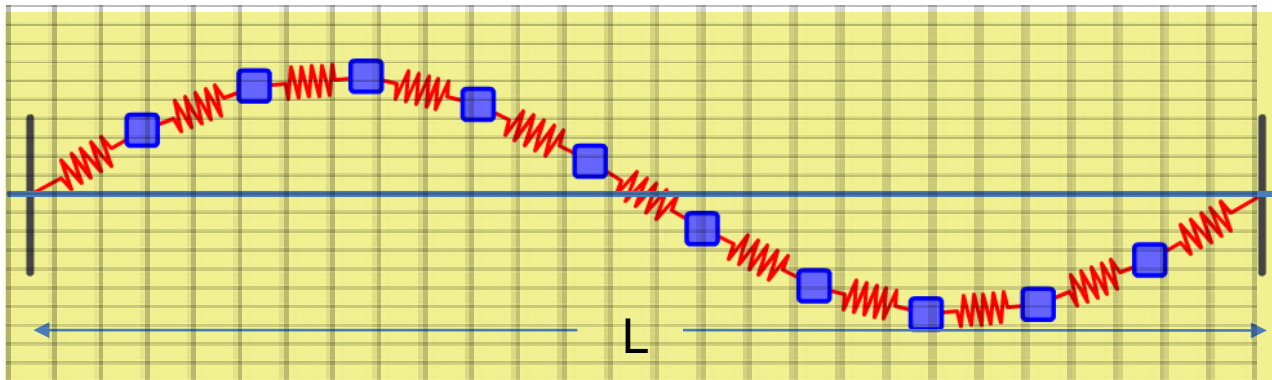
Why are they called **normal** modes?

- What is the answer to the following problem:

$$\sum_i A_1(x_i) A_2(x_i) = ?$$



$$A_1(x_i) \propto \sin\left(\frac{\pi}{L} x_i\right)$$

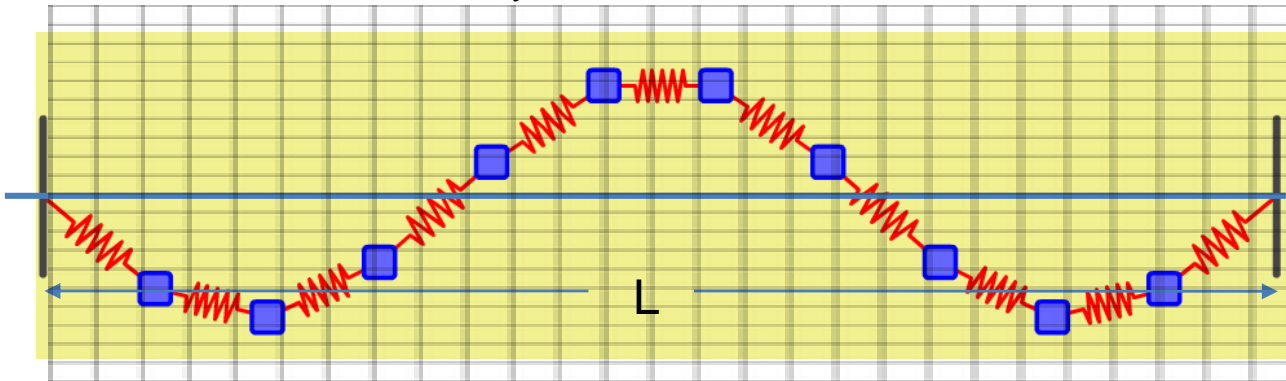


$$A_2(x_i) \propto \sin\left(\frac{2\pi}{L} x_i\right)$$

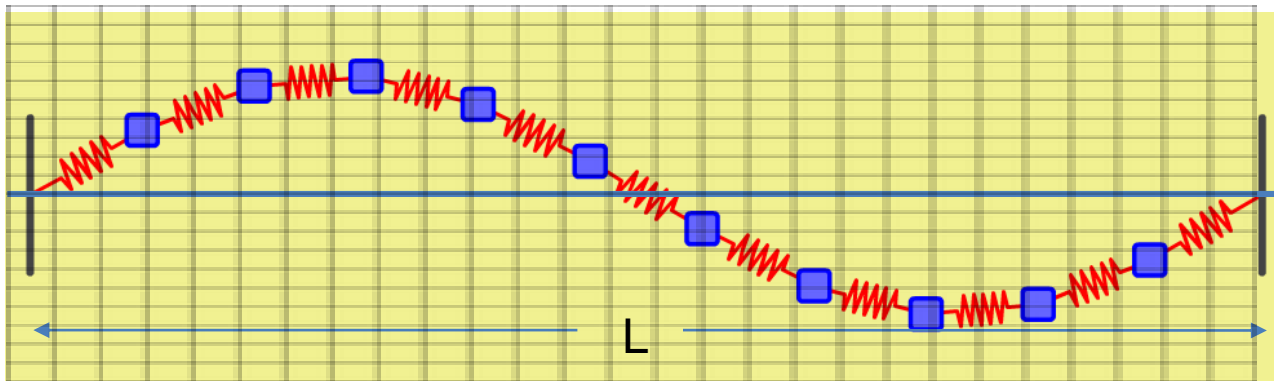
Why normal?

- What is the answer to the following problem:

$$\sum_i A_3(x_i) A_2(x_i) = ?$$



$$A_3(x_i) \propto \sin\left(\frac{3\pi}{L} x_i\right)$$

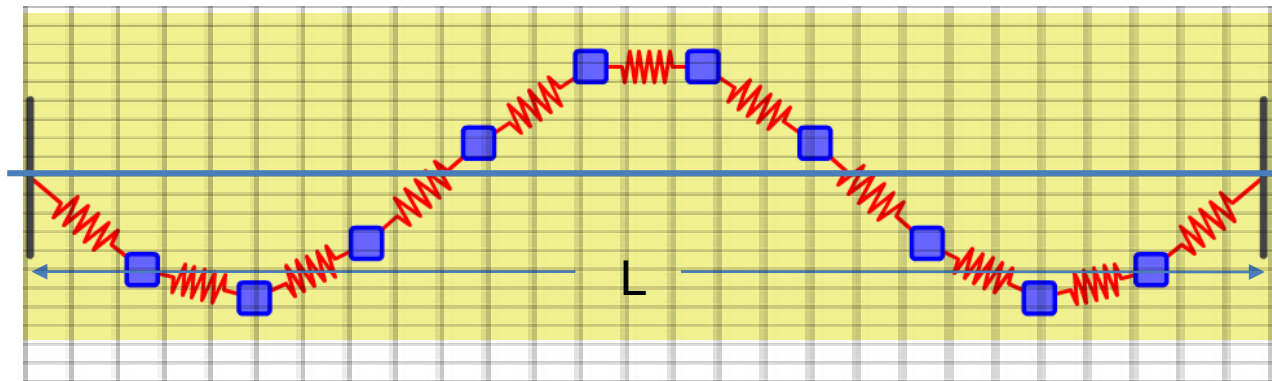


$$A_2(x_i) \propto \sin\left(\frac{2\pi}{L} x_i\right)$$

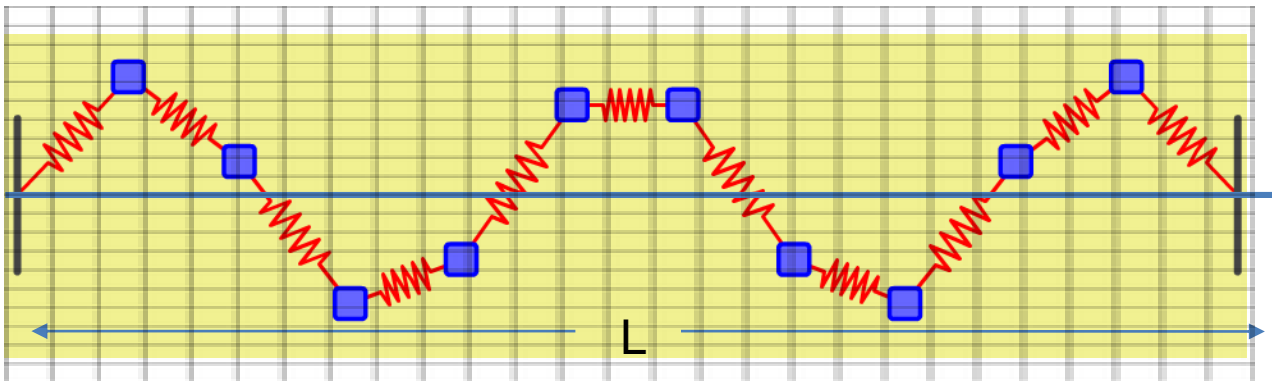
Why normal?

- What is the answer to the following problem:

$$\sum_i A_3(x_i) A_5(x_i) = ?$$



$$A_3(x_i) \propto \sin\left(\frac{3\pi}{L} x_i\right)$$



$$A_5(x_i) \propto \sin\left(\frac{5\pi}{L} x_i\right)$$

Why normal?

- They are called normal modes because:

$$\sum_i A_N(x_i) A_M(x_i) = \begin{cases} 0, & \text{if } N \neq M \\ \frac{L}{2}, & \text{if } N = M \end{cases}$$

- This is the condition for orthogonal functions.
- In fact, if we write these with a normalization constant of $2/L$, they are orthonormal functions!

Are the only motions the normal modes?

- NO!
- But all motions can be described in terms of the normal modes:

$$y(x_i, t) = \sum_N c_N y_N(x_i, t)$$

- Where the constants c_N tell you how much of each normal mode to use.

Generalization

- What happens if we continue to increase the number of masses, while keeping the total length and mass of the “string” of masses constant?
 - As we approach infinitesimal masses, the number of normal modes approaches infinity.
 - Waves! We will discuss this next time.

- <http://www.falstad.com/loadedstring/>