

Lecture 24
(Diffraction I
Single-Slit Diffraction)

Physics 262-01 Spring 2019

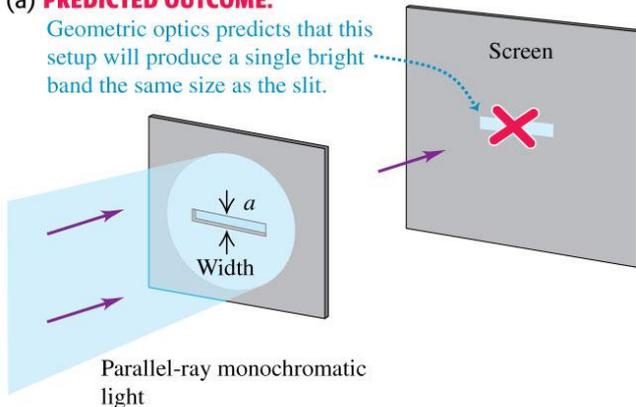
Douglas Fields

Single-Slit Diffraction

- As we have already hinted at, and seen, waves don't behave as we might have expected from our study of geometric optics.
- We can see interference fringes even when we only have a single slit!
- To understand this phenomena, we have to go back to Huygens' Principle and phasor diagrams.

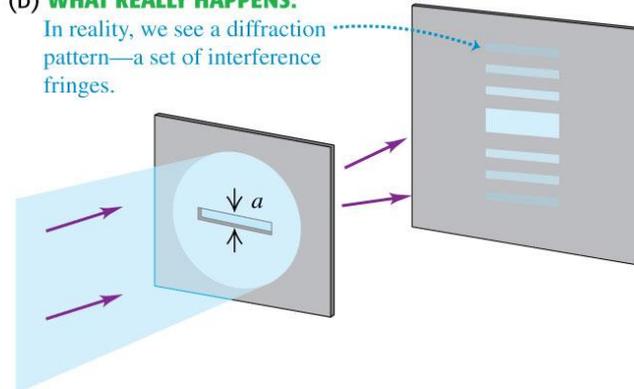
(a) **PREDICTED OUTCOME:**

Geometric optics predicts that this setup will produce a single bright band the same size as the slit.



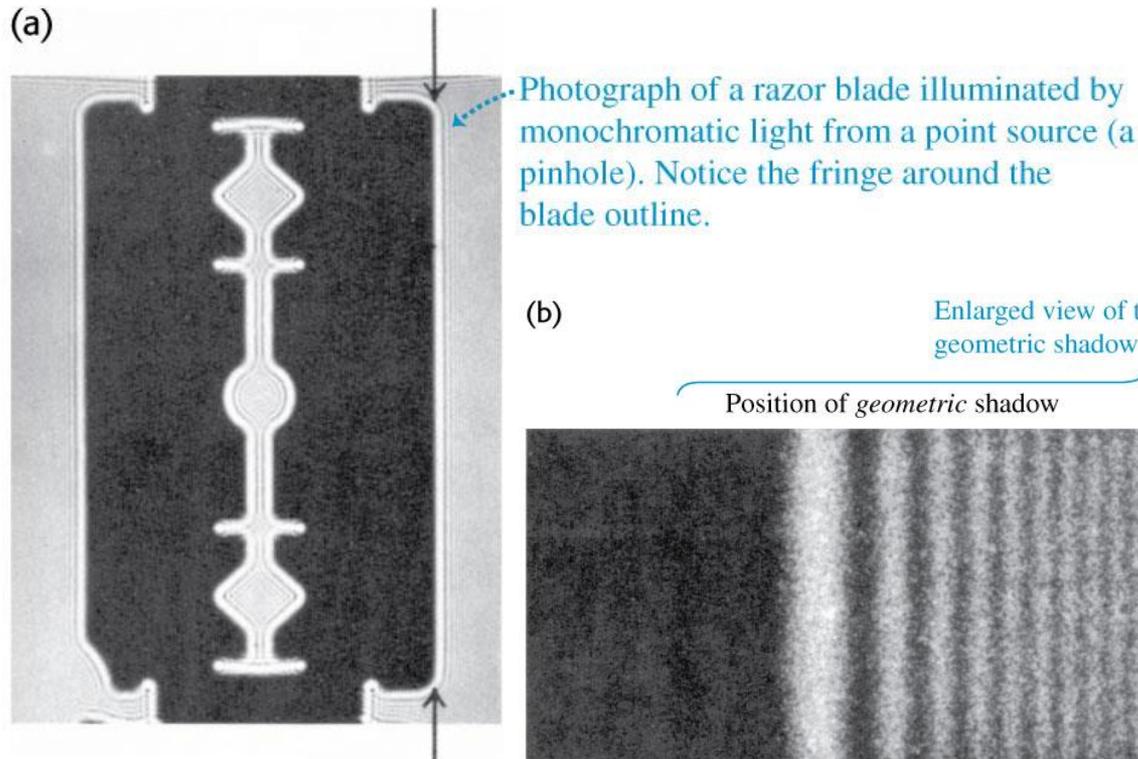
(b) **WHAT REALLY HAPPENS:**

In reality, we see a diffraction pattern—a set of interference fringes.

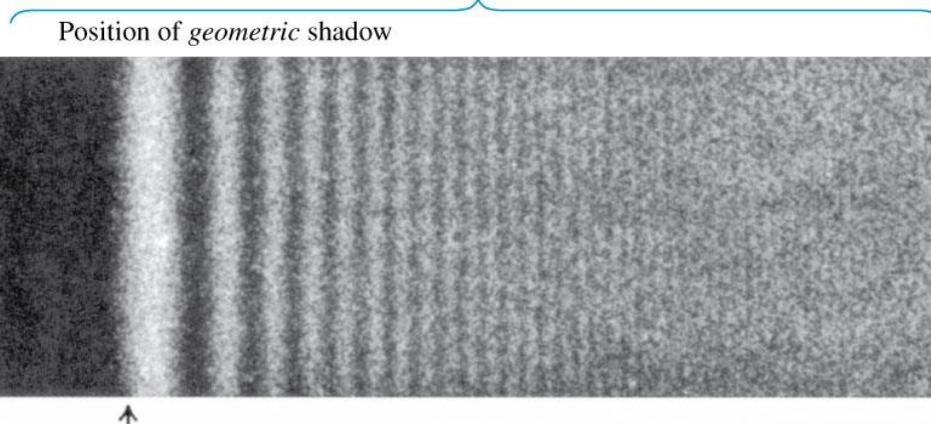


Doesn't have to be a slit...

- Any obstruction can cause an interference pattern.

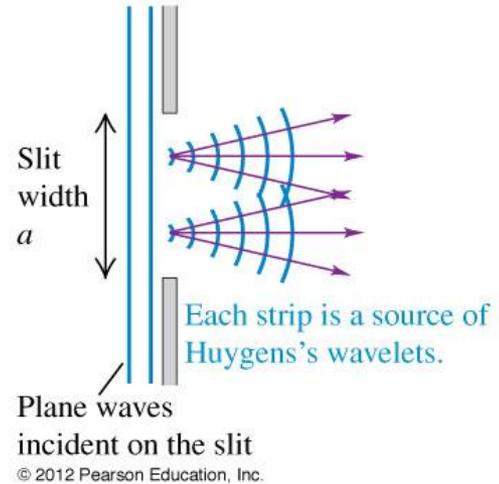


(b) Enlarged view of the area outside the geometric shadow of the blade's edge



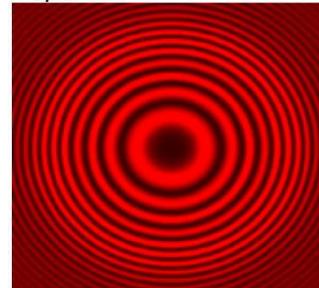
Single-Slit Diffraction

(a) A slit as a source of wavelets



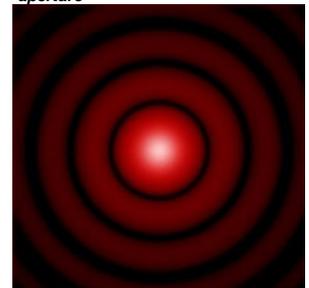
- Because real slits have finite width, then there is not just a single source of Huygens wavelets, but many (infinite?).
- As we saw with the two-slit problem, the geometry of the problem is much easier when we go to the limit that the distance to the screen (where the interference pattern is viewed) is much larger than the width of the slit. – Fraunhofer or far-field diffraction.
- When this isn't the case, you still get diffraction, it just looks different. – Fresnel or near-field diffraction.

Fresnel diffraction of circular aperture'



'Circular Aperture Fresnel Diffraction high res' by Gsiling - Own work. Licensed under CC BY-SA 3.0 via Commons - https://commons.wikimedia.org/wiki/File:Circular_Aperture_Fresnel_Diffraction_high_res.jpg#/media/File:Circular_Aperture_Fresnel_Diffraction_high_res.jpg

Fraunhofer diffraction of circular aperture

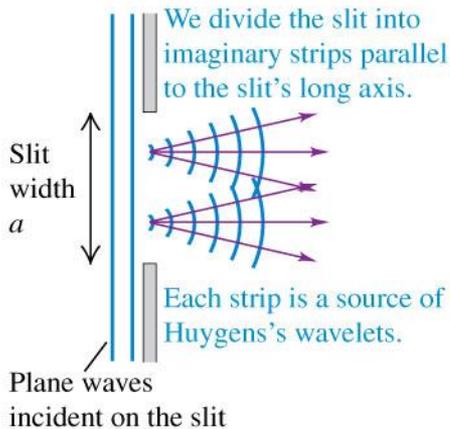


'Airy-pattern2' by Epzcow - Own work. Licensed under CC BY-SA 3.0 via Commons - <https://commons.wikimedia.org/wiki/File:Airy-pattern2.jpg#/media/File:Airy-pattern2.jpg>

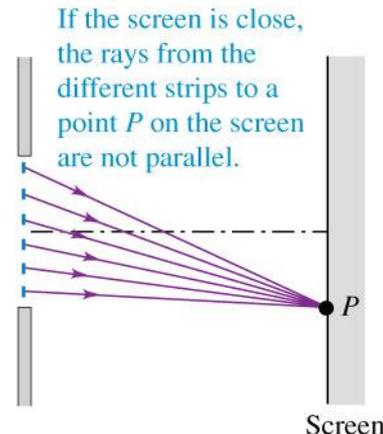
Fresnel and Fraunhofer Diffraction

- The only distinction is the distance between the slit and the screen.
- In Fraunhofer diffraction the distance is large enough to consider the rays to be parallel.
- We will limit our discussions to Fraunhofer diffraction (to make the geometry simpler).

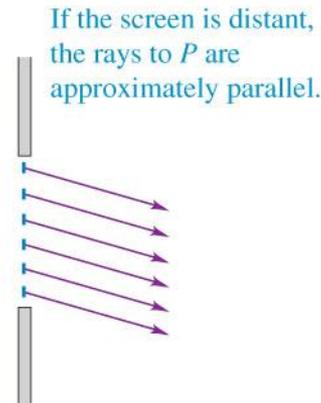
(a) A slit as a source of wavelets



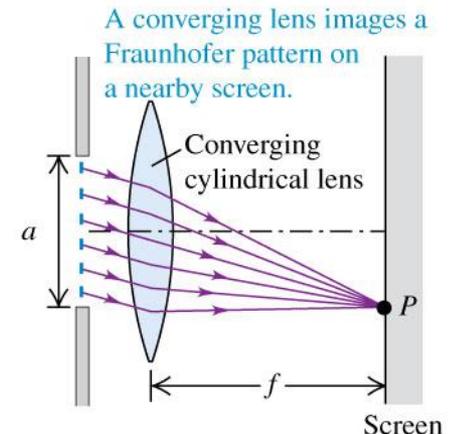
(b) Fresnel (near-field) diffraction



(c) Fraunhofer (far-field) diffraction

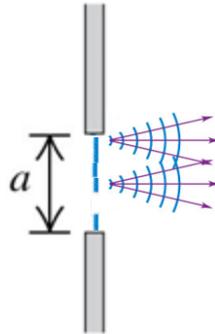


(d) Imaging Fraunhofer diffraction



Fraunhofer Diffraction

- Let's first start by looking at our slit of width a .
- We then look at two sources of Huygen's wavelets located at the top and the middle of the slit, separated by distance $a/2$.
- We will analyze these two wavelets,
- But our analysis will be the same for any two wavelets from "slits" separated by the same distance.



Fraunhofer Diffraction

- So, just as before, the path difference is just the slit spacing times the sin of the angle:

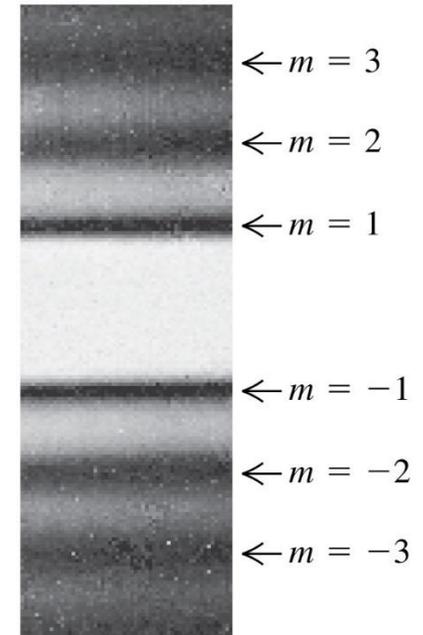
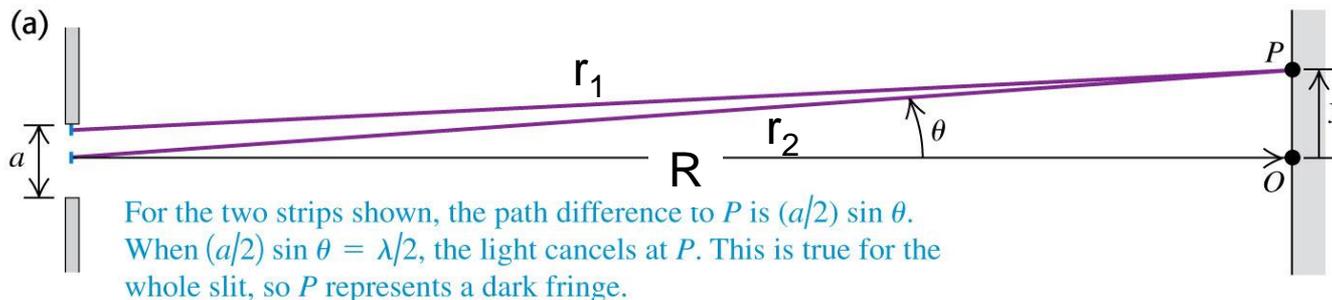
$$(r_1 - r_2) = \frac{a}{2} \sin \theta$$

- And, we use the same small angle approximations as before:

$$\tan \theta \approx \sin \theta = \frac{y}{R}$$

- To get for **destructive** interference:

$$\frac{a}{2} \frac{y}{R} = \frac{m\lambda}{2} \Rightarrow y = \frac{Rm\lambda}{a}, \quad m = \pm 1, \pm 2, \dots$$

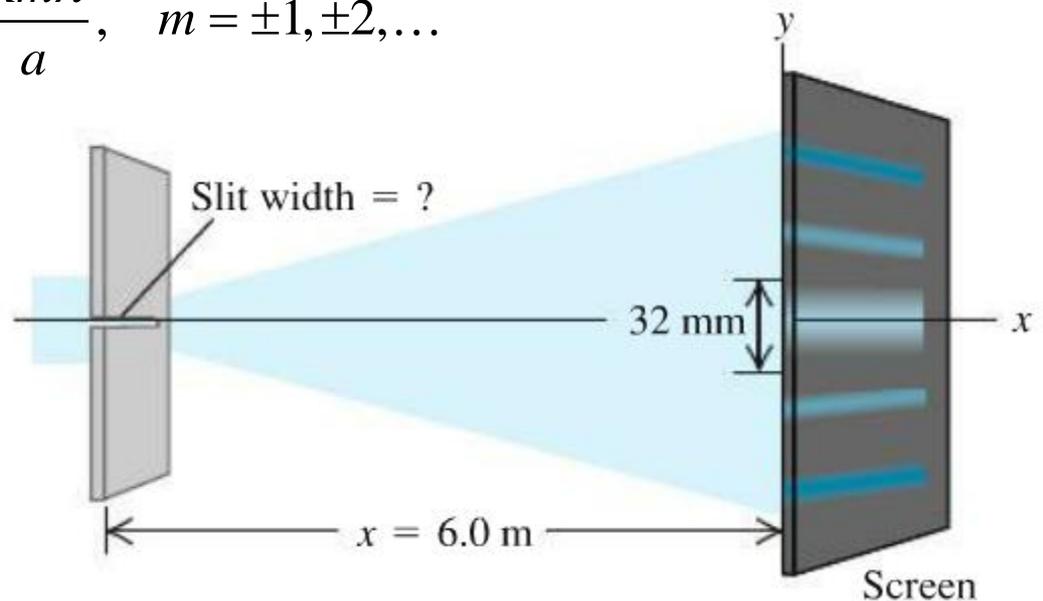


Example 36.1 Single-slit diffraction



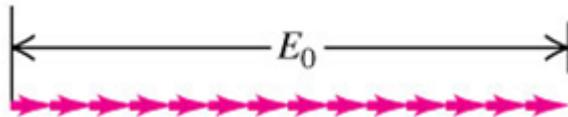
You pass 633-nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. The distance on the screen between the centers of the first minima on either side of the central bright fringe is 32 mm (Fig. 36.7). How wide is the slit?

$$y = \frac{Rm\lambda}{a}, \quad m = \pm 1, \pm 2, \dots$$

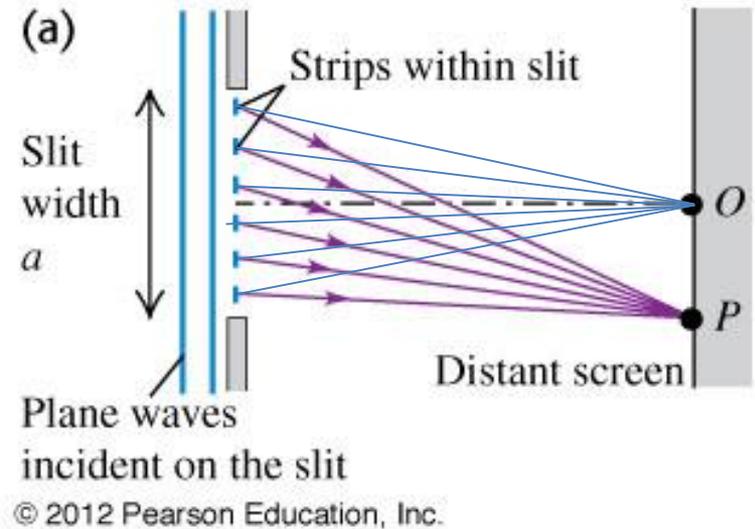


Intensity and Phasors

- At the perpendicular bisector on the screen, all the wavelets are in phase, **IF** the distance to the screen is large compared to the slit width.
- In this case, the phasor diagram at point O is simply:

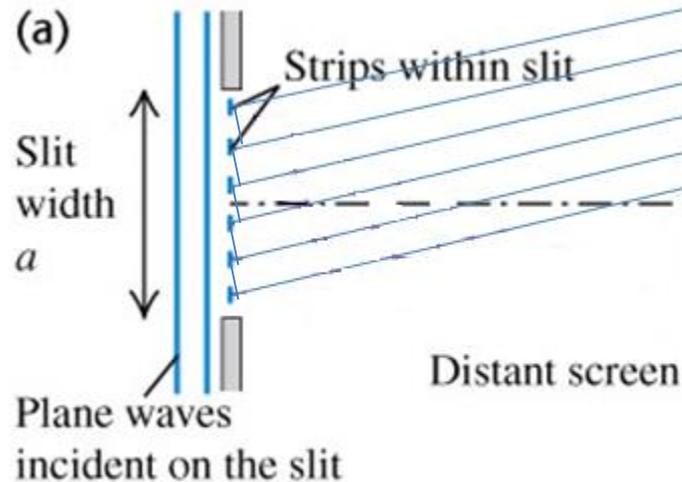


- We take the magnitude of the resultant wave at O to be E_0 .

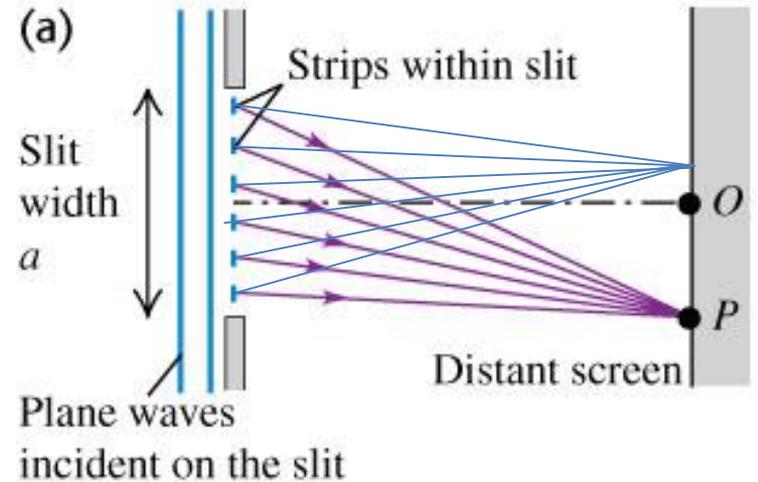


Slightly off center...

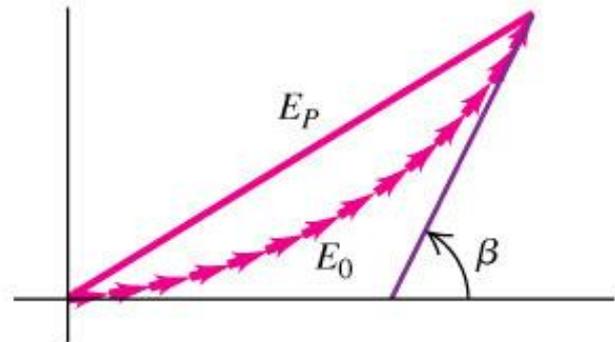
- If we move slightly off-center, each ray is slightly out of phase with the next.
- Remember that this is far-field, so that we can assume that each ray is parallel.



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(c) Phasor diagram at a point slightly off the center of the pattern; β = total phase difference between the first and last phasors.



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To infinity, and beyond!

- In the limit of infinite numbers of strips each infinitesimally long, it becomes an arc of a circle, with angular extent β – the difference between the phase of the first and last rays.

- Since the length of an arc is given by:

$$\ell = \beta r$$

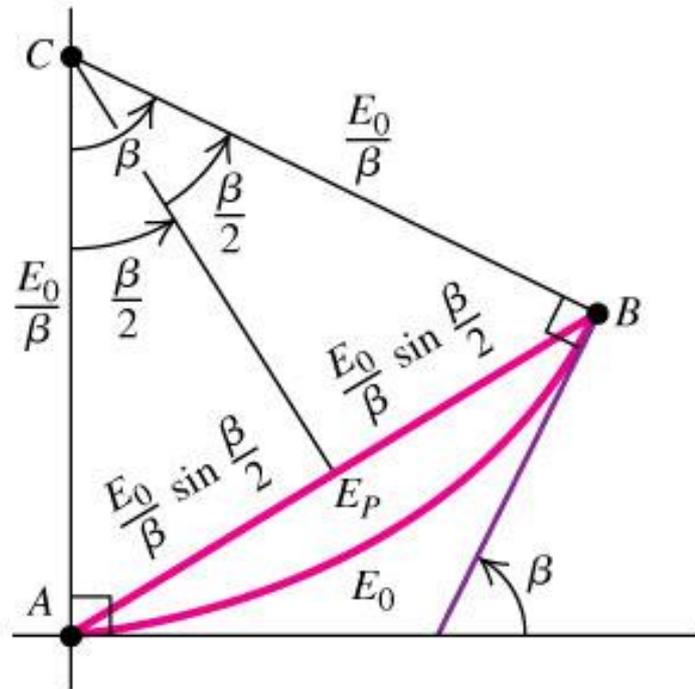
- And we know what the arc length is, E_0 , so:

$$E_0 = \beta r \Rightarrow r = \frac{E_0}{\beta}$$

- Then, with just a little simple trig, we can get the length of the resultant phasor:

$$E_P = 2 \frac{E_0}{\beta} \sin \frac{\beta}{2} = E_0 \frac{\sin(\beta/2)}{\beta/2}$$

(d) As in (c), but in the limit that the slit is subdivided into infinitely many strips



Resultant Amplitude

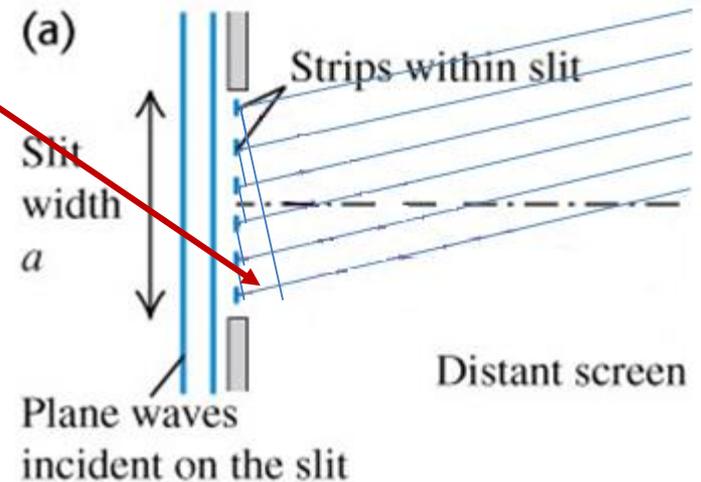
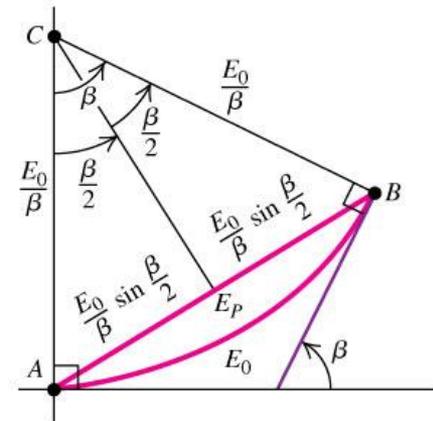
- Now, the total phase change between the first and last rays is just:

$$\beta = ka \sin \theta$$

- So,

$$\begin{aligned} E_P &= E_0 \frac{\sin(\beta/2)}{\beta/2} \\ &= E_0 \frac{\sin[(ka \sin \theta)/2]}{(ka \sin \theta)/2} \\ &= E_0 \frac{\sin[(\pi a \sin \theta)/\lambda]}{(\pi a \sin \theta)/\lambda} \end{aligned}$$

(d) As in (c), but in the limit that the slit is subdivided into infinitely many strips



Intensity

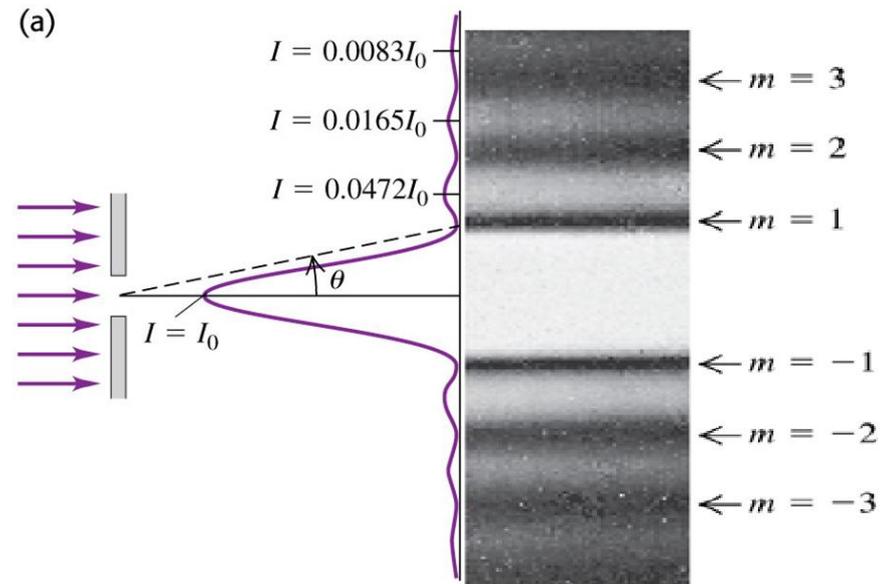
- Since the intensity is just proportional to the square of the electric field, then,

$$E_p = E_0 \frac{\sin\left[\frac{(\pi a \sin \theta)/\lambda}{(\pi a \sin \theta)/\lambda}\right]}{(\pi a \sin \theta)/\lambda} \Rightarrow$$

$$I = I_0 \left\{ \frac{\sin\left[\frac{(\pi a \sin \theta)/\lambda}{(\pi a \sin \theta)/\lambda}\right]}{(\pi a \sin \theta)/\lambda} \right\}^2$$

- Now, we have a complete picture of the diffraction pattern:

$$y = \frac{Rm\lambda}{a}, \quad m = \pm 1, \pm 2, \dots$$



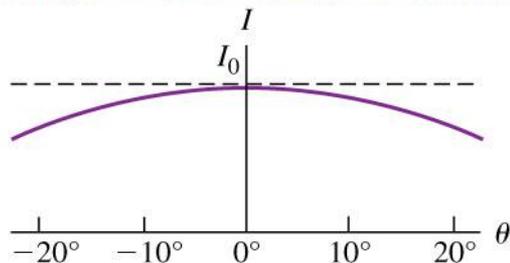
Width of Central Peak

- The positions of the first **minima** are given by: $\sin \theta = \frac{m\lambda}{a}$, $m = \pm 1$
- For small angles, $\sin \theta = \theta$, the width of the central peak is

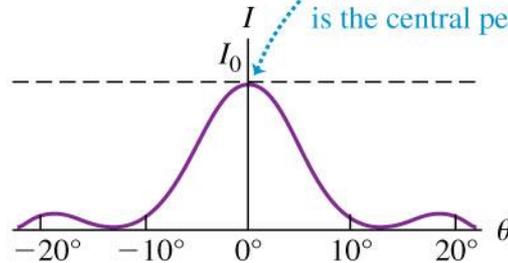
$$\theta_1 - \theta_{-1} = \frac{\lambda}{a} - \frac{-\lambda}{a} = \frac{2\lambda}{a}$$

- Also, as **a** approaches the wavelength, there are no minima, and the diffraction pattern is just one broad peak.

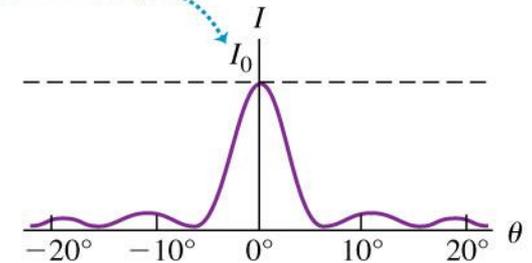
(a) $a = \lambda$



(b) $a = 5\lambda$



(c) $a = 8\lambda$



If the slit width is equal to or narrower than the wavelength, only one broad maximum forms.

The wider the slit (or the shorter the wavelength), the narrower and sharper is the central peak.

Light of wavelength λ passes through a single slit of width a . The diffraction pattern is observed on a screen that is very far from the slit.

Which of the following will give the greatest *increase* in the angular width of the central diffraction maximum?

- A. Double the slit width a and double the wavelength λ .
- B. Double the slit width a and halve the wavelength λ .
- C. Halve the slit width a and double the wavelength λ .
- D. Halve the slit width a and halve the wavelength λ .

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In a single-slit diffraction experiment with waves of wavelength λ , there will be *no* intensity minima (that is, no dark fringes) if the slit width is small enough.

What is the *maximum* slit width a for which this occurs?

A. $a = \lambda/2$

B. $a = \lambda$

C. $a = 2\lambda$

D. The answer depends on the distance from the slit to the screen on which the diffraction pattern is viewed.

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