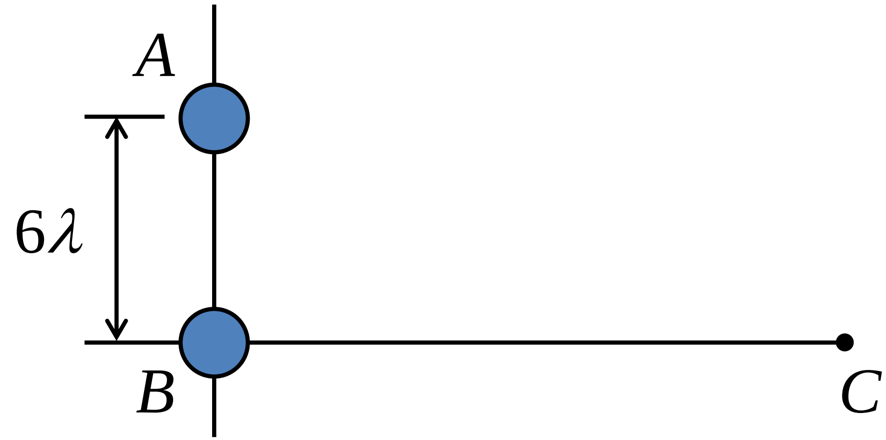
Lecture 22
(Interference II
Phasors and Interference Intensity)

Physics 262-01 Spring 2019

Douglas Fields

Two radio antennas radiating in phase are located at points *A* and *B*, which are 6 wavelengths apart. A radio receiver is moved along a line from point *B* to point *C*.



At what distances from point *B* will the receiver detect an intensity *maximum?*

a) 4.5λ

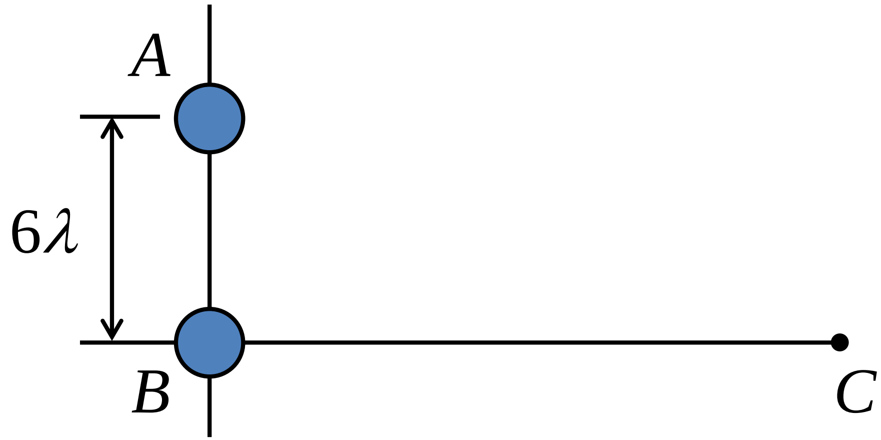
b) 8λ

c) 9λ

d) both a) and b)

e) all of a), b), and c)

Two radio antennas radiating in phase are located at points *A* and *B*, which are 6 wavelengths apart. A radio receiver is moved along a line from point *B* to point *C*.



At what distances from point *B* will the receiver detect an intensity *maximum*?

a) 4.5λ

b) 8λ

c) 9λ

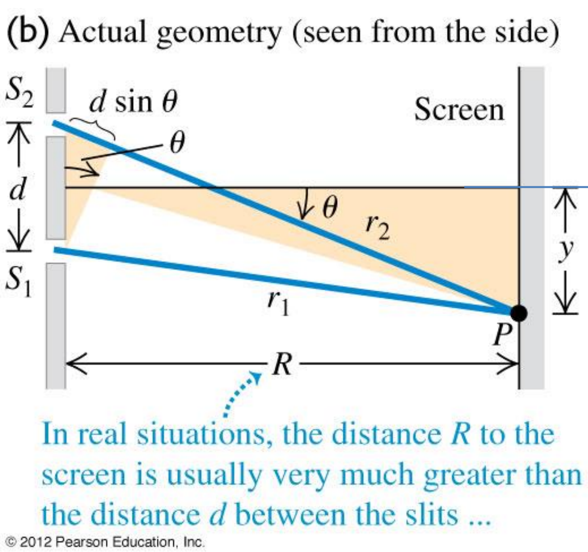
d) both a) and b)

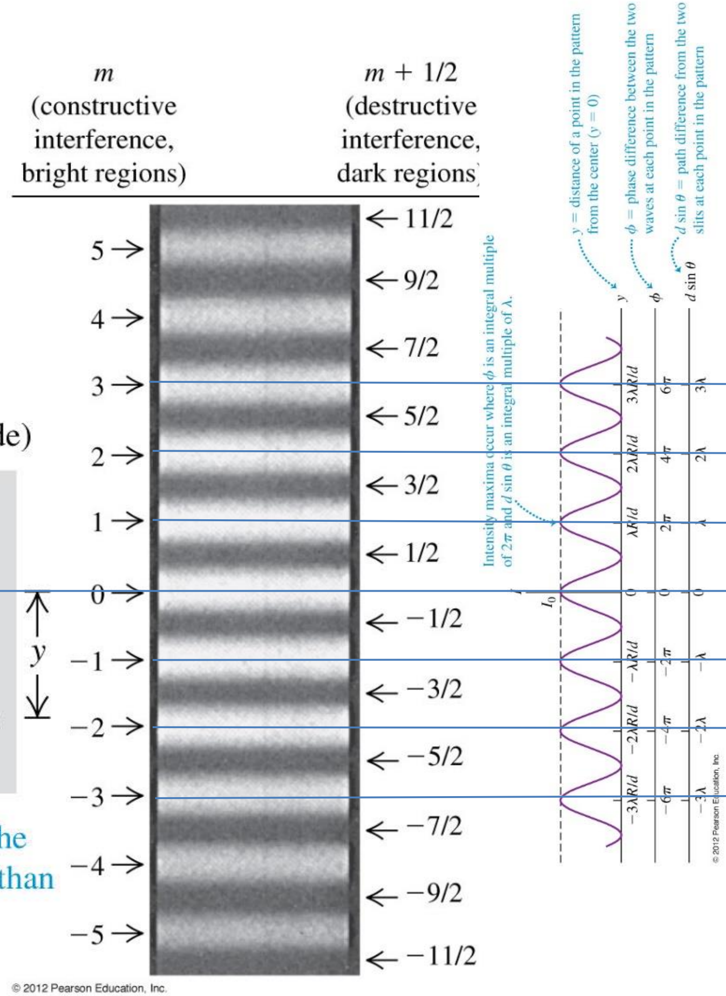
e) all of a), b), and c)

Review

$$\delta\phi = kr_1 - kr_2 + (\phi_1 - \phi_2)$$
$$= \frac{2\pi}{\lambda}(r_1 - r_2) + (\phi_1 - \phi_2)$$

$$(r_1 - r_2) = d\sin\theta$$





$d\sin\theta = 3\lambda$

$d\sin\theta = 2\lambda$

$d\sin\theta = 1\lambda$

$d\sin\theta = 0\lambda$

$$\sin\theta \approx \tan\theta = \frac{y}{R}$$

$$y_m = R\frac{m\lambda}{d}, \quad m = 0, \pm 1, \pm 2\ldots \quad \text{Constructive}$$

$$y_m = R\frac{\left(m + \frac{1}{2}\right)\lambda}{d}, \quad m = 0, \pm 1, \pm 2\ldots \quad \text{Destructive}$$

Young's Two-Slit Experiment

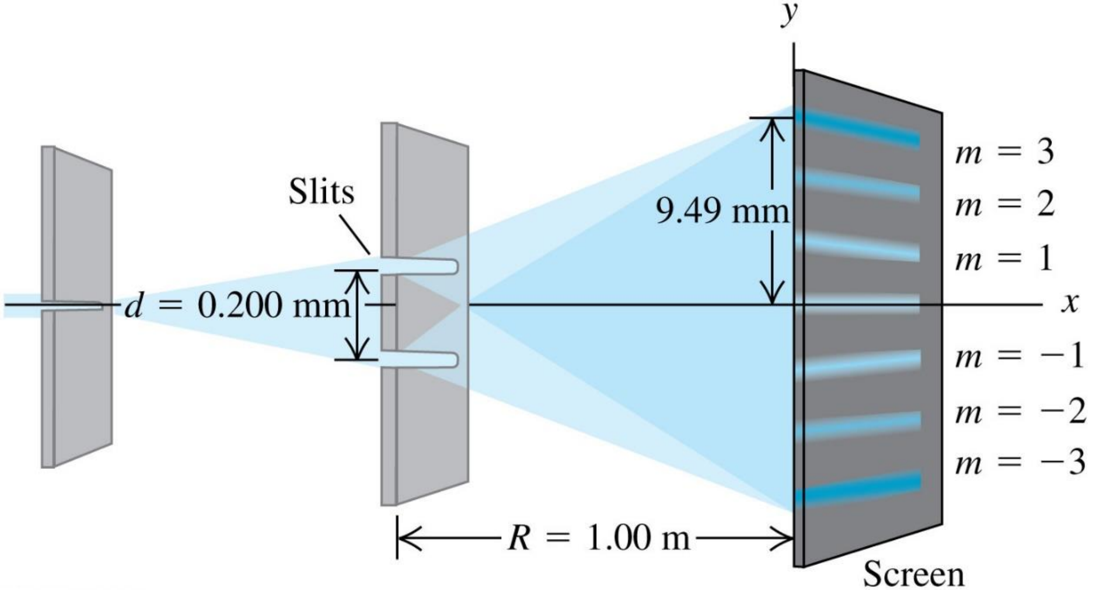
- What is the wavelength of the light in this example?

$$\frac{d}{R}y = m\lambda, \quad m = 0, \pm 1, \pm 2 \ldots \Rightarrow$$

$$y_m = R\frac{m\lambda}{d}, \quad m = 0, \pm 1, \pm 2 \ldots$$

$$\frac{d}{R}y = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2 \ldots$$

$$y_m = R\frac{\left(m + \frac{1}{2}\right)\lambda}{d}, \quad m = 0, \pm 1, \pm 2 \ldots$$



In Young’s experiment, coherent light passing through two slits (S_1 and S_2) produces a pattern of dark and bright areas on a distant screen. If the wavelength of the light is increased, how does the pattern change?

A. The bright areas move closer together.

B. The bright areas move farther apart.

C. The spacing between bright areas remains the same, but the color changes.

D. any of the above, depending on circumstances

E. none of the above

In Young's experiment, coherent light passing through two slits (S_1 and S_2) produces a pattern of dark and bright areas on a distant screen. If the wavelength of the light is increased, how does the pattern change?

A. The bright areas move closer together.

B. The bright areas move farther apart.

C. The spacing between bright areas remains the same, but the color changes.

D. any of the above, depending on circumstances

E. none of the above

In Young's experiment, coherent light passing through two slits (S_1 and S_2) produces a pattern of dark and bright areas on a distant screen.

What is the difference between the distance from S_1 to the $m = +3$ bright area and the distance from S_2 to the $m = +3$ bright area?

A. three wavelengths

B. three half-wavelengths

C. three quarter-wavelengths

D. not enough information given to decide

In Young's experiment, coherent light passing through two slits (S_1 and S_2) produces a pattern of dark and bright areas on a distant screen.

What is the difference between the distance from S_1 to the $m = +3$ bright area and the distance from S_2 to the $m = +3$ bright area?

A. three wavelengths

B. three half-wavelengths

C. three quarter-wavelengths

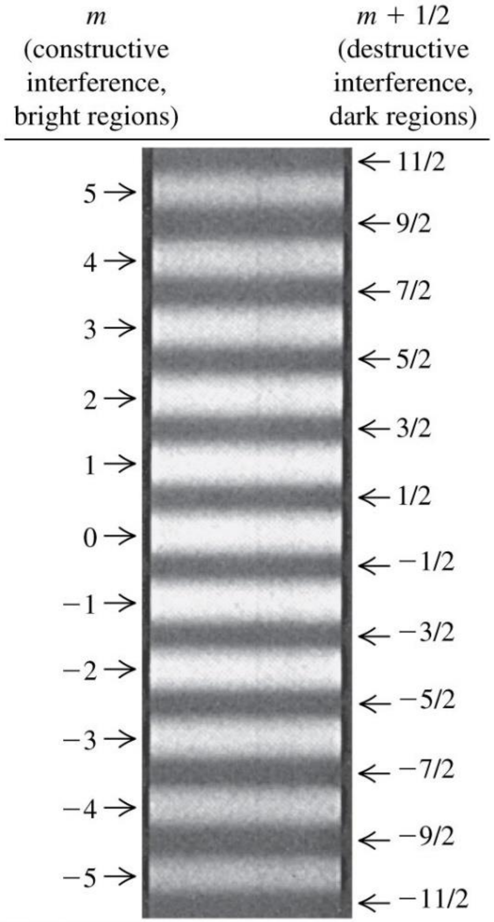
D. not enough information given to decide

Describing the Intensity Pattern

$$y_m = R\frac{m\lambda}{d}, \quad m = 0, \pm 1, \pm 2\ldots \quad \text{Constructive}$$

$$y_m = R\frac{\left(m+\frac{1}{2}\right)\lambda}{d}, \quad m = 0, \pm 1, \pm 2\ldots \quad \text{Destructive}$$

- OK, so now we know:
  - where the bright and dark spots are,
  - what happens if we change the distance between slits,
  - what happens when we change the wavelength of light
- But can we quantitatively describe the intensity pattern that we see?
- Yes, but we will need a tool that you saw earlier:
- Phasors.



What is the intensity at some point?

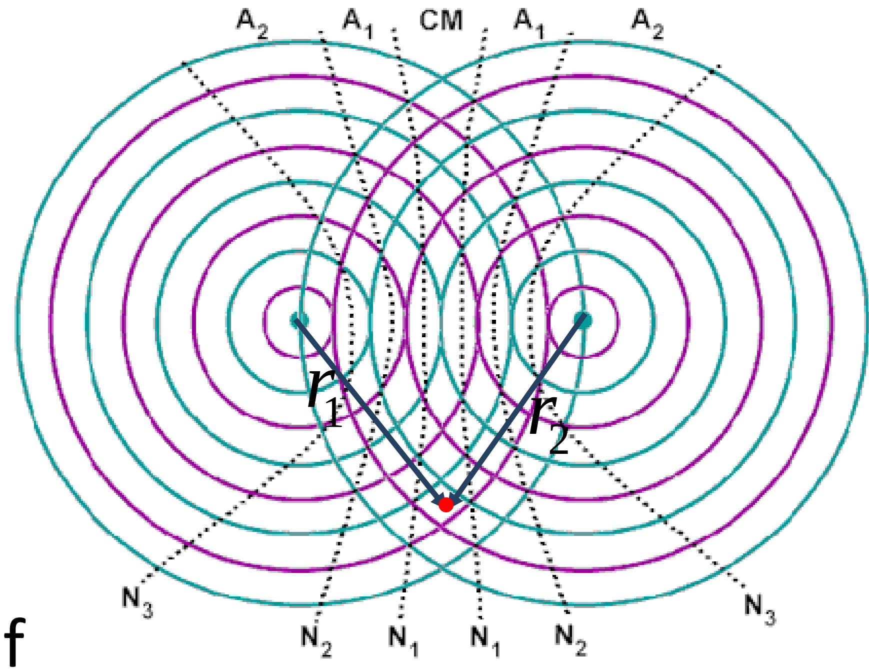
- The electric field (of an EM wave) is written generally as:

$$E_1(r,t) = E_1(r)\cos(kr - \omega t + \phi_1)$$

- But, if we want to look at the intensity at a point:

$$E_1(r_1,t) = E_1(r_1)\cos(kr_1 - \omega t + \phi_1)$$

- Then r_1 is a constant and kr_1 can be rolled into the phase of the wave at that point in space (it doesn't change with time).



$$E_1(r_1,t) = E_1(r_1)\cos\left[\omega t + (\phi_1 + kr_1)\right]$$

Superposition with Phasors

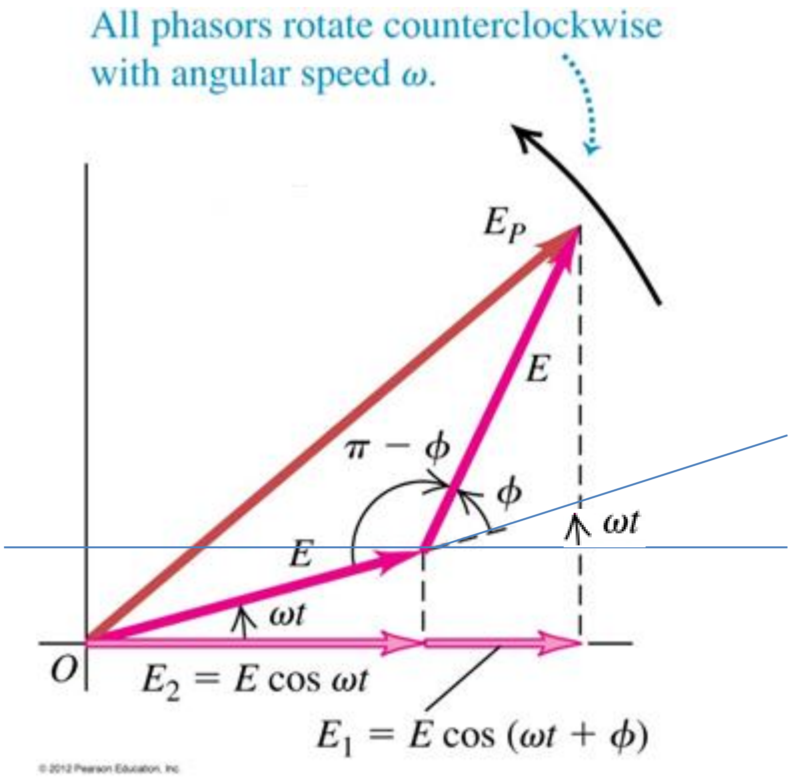
- Now, if we want to add two waves (superimpose) we have to add the phasors ***vectorially***, where the only important phase is the ***phase difference***:

$$E_1(r_1,t)=E_1(r_1)\cos\left[\omega t+(\phi_1+kr_1)\right]$$

$$E_2(r_2,t)=E_2(r_2)\cos\left[\omega t+(\phi_2+kr_2)\right]$$

$$\phi=(\phi_2+kr_2)-(\phi_1+kr_1)$$

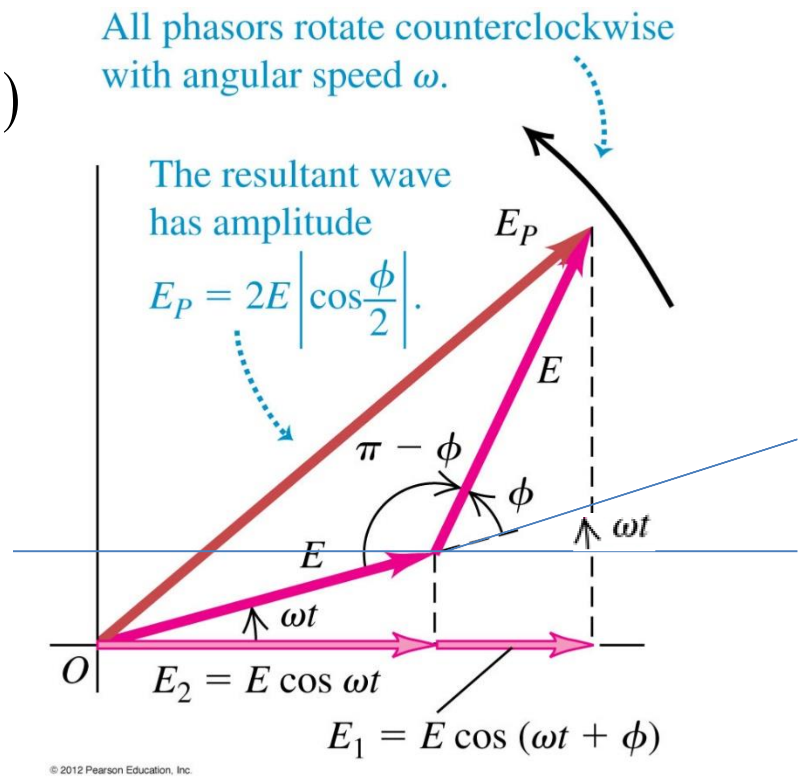
$$\phi=(\phi_2-\phi_1)+(kr_2-kr_1)$$



Superposition with Phasors

- Then, we can find the amplitude of the resultant wave as the vector sum of the two phasors:

$$E_P(t) = E_P\cos(\omega t) = E_1\cos(\omega t) + E_2\cos(\omega t + \phi)$$



- Now, if E_1 is not equal to E_2, regardless of the phase difference, there will be no (complete) destructive interference.
- With $E_1 = E_2 = E$, we can use trig to find E_P:

$$\begin{aligned} E_P^2 &= E^2 + E^2 - 2E^2\cos(\pi - \phi) \\ &= E^2 + E^2 + 2E^2\cos(\phi) \\ &= 2E^2(1 + \cos(\phi)) \\ &= 4E^2\cos^2\left(\frac{\phi}{2}\right) \Rightarrow \end{aligned}$$

$$\boxed{E_P = 2E\left|\cos\left(\frac{\phi}{2}\right)\right|}$$

$$\boxed{\phi = (\phi_2 - \phi_1) + (kr_2 - kr_1)}$$ Phase DIFFERENCE!

Amplitude and Intensity

- Now, $\boldsymbol{E_P}$ is the **amplitude** of the resultant phasor (it also rotates with angular frequency ω).

$$E_P\left(t\right) = E_P \cos\omega t = 2E\left|\cos\left(\frac{\phi}{2}\right)\right|\cos\omega t$$

- The intensity of light is just:

$$I = S_{Avg} = \frac{1}{2\mu_0 c}E_{Max}^2$$

$$E_{Max} = E_P = 2E\left|\cos\left(\frac{\phi}{2}\right)\right| \Rightarrow$$

$$I = \frac{1}{2\mu_0 c}\left[2E\cos\left(\frac{\phi}{2}\right)\right]^2$$

$$\boxed{I = \frac{2E^2}{\mu_0 c}\cos^2\left(\frac{\phi}{2}\right)}$$

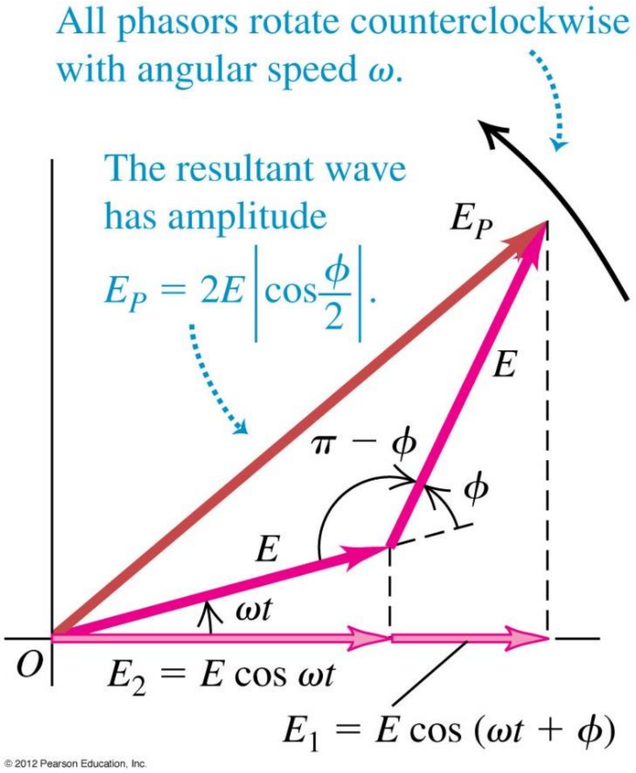
Sources in Phase

- If the two sources are in phase, $\phi_1 = \phi_2 = \phi_0$ and then the phase difference at the position of interest is just:

$$E_1(r_1,t) = E_1(r_1)\cos\left[\omega t + (\phi_0 + kr_1)\right]$$
$$E_2(r_2,t) = E_2(r_2)\cos\left[\omega t + (\phi_0 + kr_2)\right]$$

$$\phi = (\phi_0 + kr_2) - (\phi_0 + kr_1) = kr_2 - kr_1$$

$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1)$$

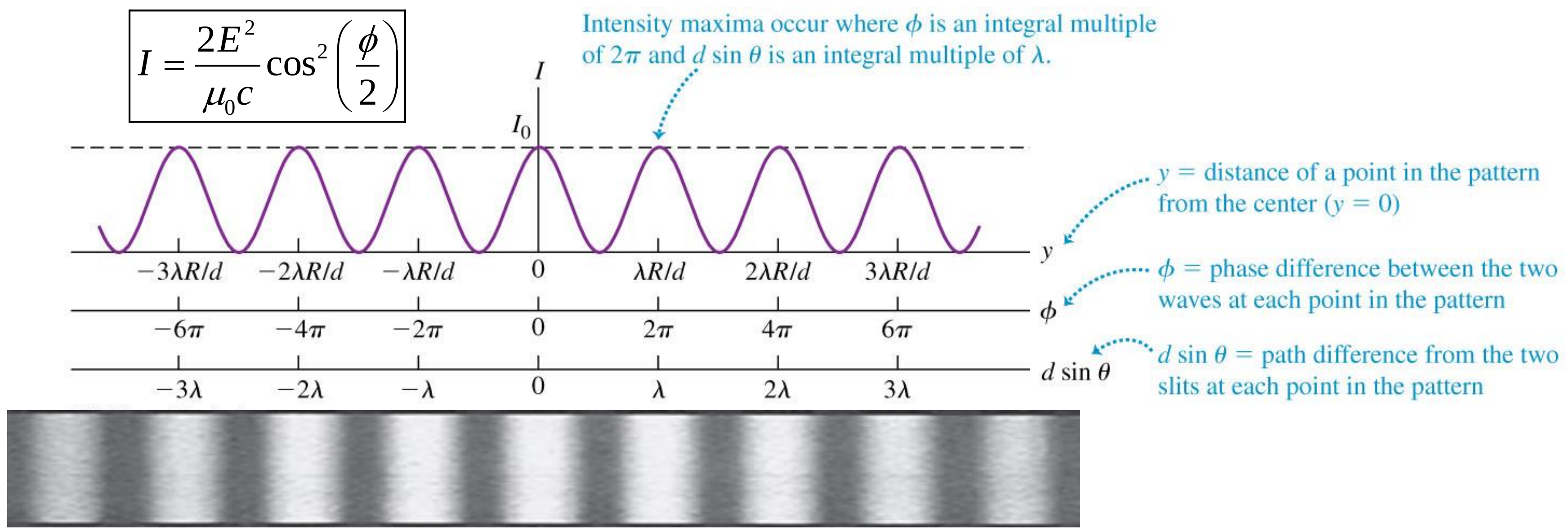


Sources in Phase

- Then, the conditions for constructive and destructive interference can be put in terms of the overall phase difference:
  - Constructive: $\phi = \cancel{(\phi_2 - \phi_1)} + (kr_2 - kr_1) = 2m\pi$
  - Destructive: $\phi = \cancel{(\phi_2 - \phi_1)} + (kr_2 - kr_1) = (2m+1)\pi$

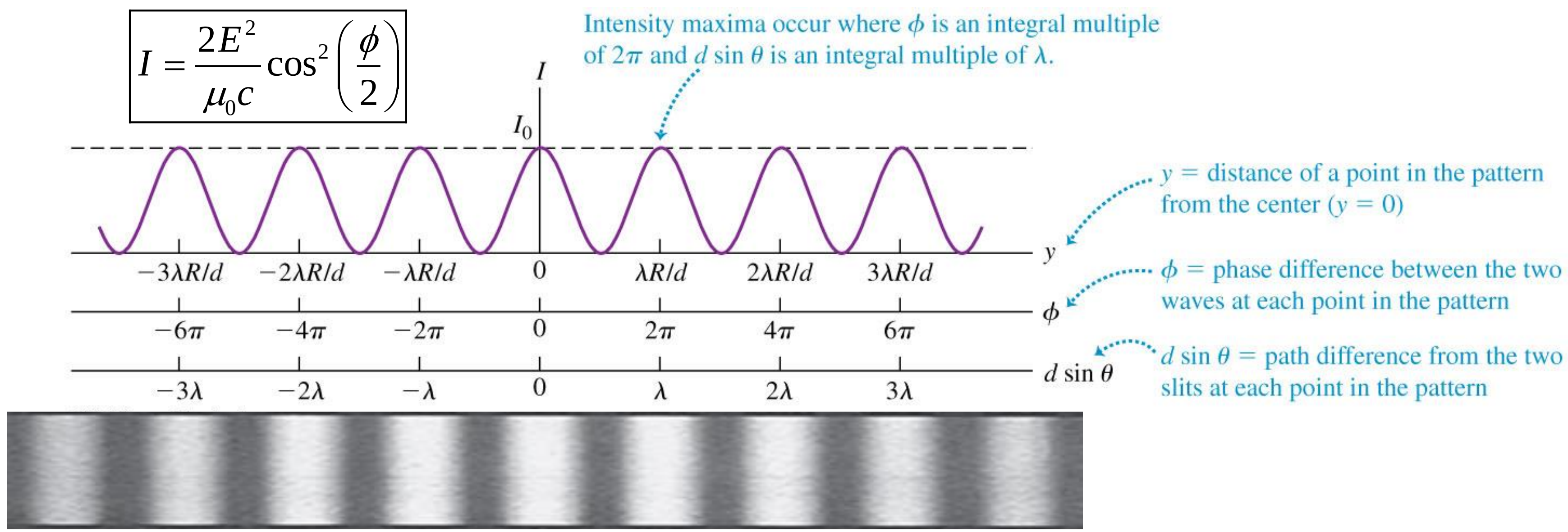
$m = 0, \pm 1, \pm 2, \ldots$

$$I = \frac{2E^2}{\mu_0 c}\cos^2\left(\frac{\phi}{2}\right)$$



Assumptions again…

- What do you see that is inconsistent between the picture and the graph?



Phasors, really?

- We will see in the coming lectures that you get interference patterns ***even when you only have one slit!***
- To understand this phenomena, and others, you have to understand the phasor diagrams obtained when you break up the source wave front into a continuous set of sources (a la Huygens).
- We won't begin this discussion today, but know that you need to understand how to set up a phasor diagram.