

https://phet.colorado.edu/sims/radiating-charge/radiating-charge_en.html

Lecture 13

(Electromagnetic Waves)

Physics 262-01 Spring 2019

Douglas Fields

Reading Quiz

- Electromagnetic waves are caused by
 - A) Stationary charges.
 - B) DC currents.
 - C) Accelerating charges.
 - D) Energy.
 - E) Maxwell.

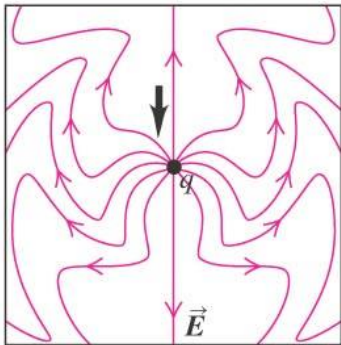
Qualities of the EM Wave

- The electric and magnetic fields are always perpendicular to the direction the wave is travelling – transverse waves.
- The electric field is always perpendicular to the magnetic field.
- The magnitudes of the electric and magnetic fields are governed by: $|\vec{E}| = c|\vec{B}|$
- The direction of the propagation of the wave is given by the right-hand-rule: $\vec{E} \times \vec{B}$
- The polarization direction (for linearly polarized waves) is defined by the direction of the electric field.

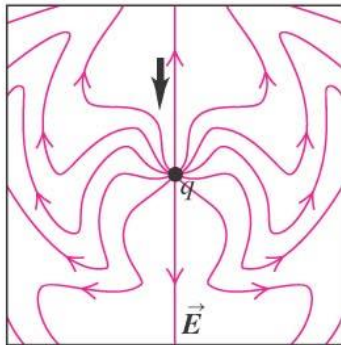
Sources of EM waves

- In analogy to string waves being caused by moving one end of a string up and down (or in 3D, a hummingbird's flapping wings causing a sound wave), EM waves are caused by a moving (actually accelerating) charge (or charge distribution).
- Just like for waves on a string, EM waves don't have to be strictly sinusoidal, they can be a pulse or whatever.
- But we will restrict our discussions mostly to sinusoidal waves...

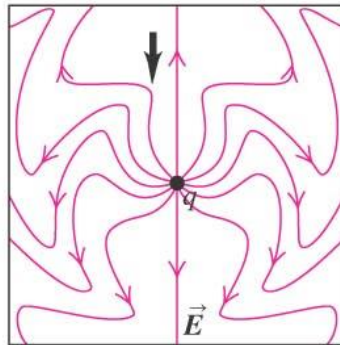
(a) $t = 0$



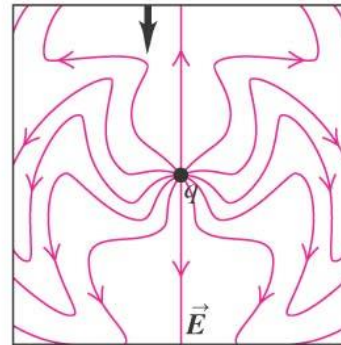
(b) $t = T/4$



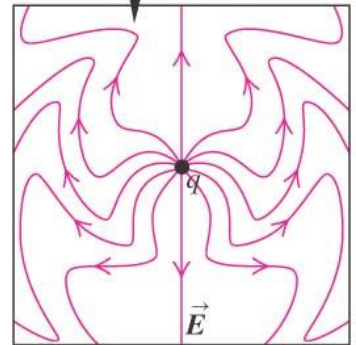
(c) $t = T/2$



(d) $t = 3T/4$

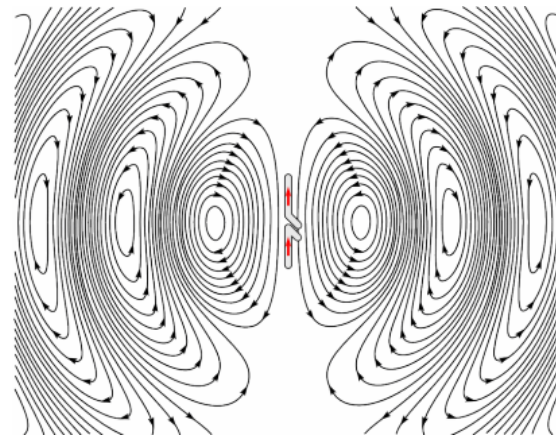
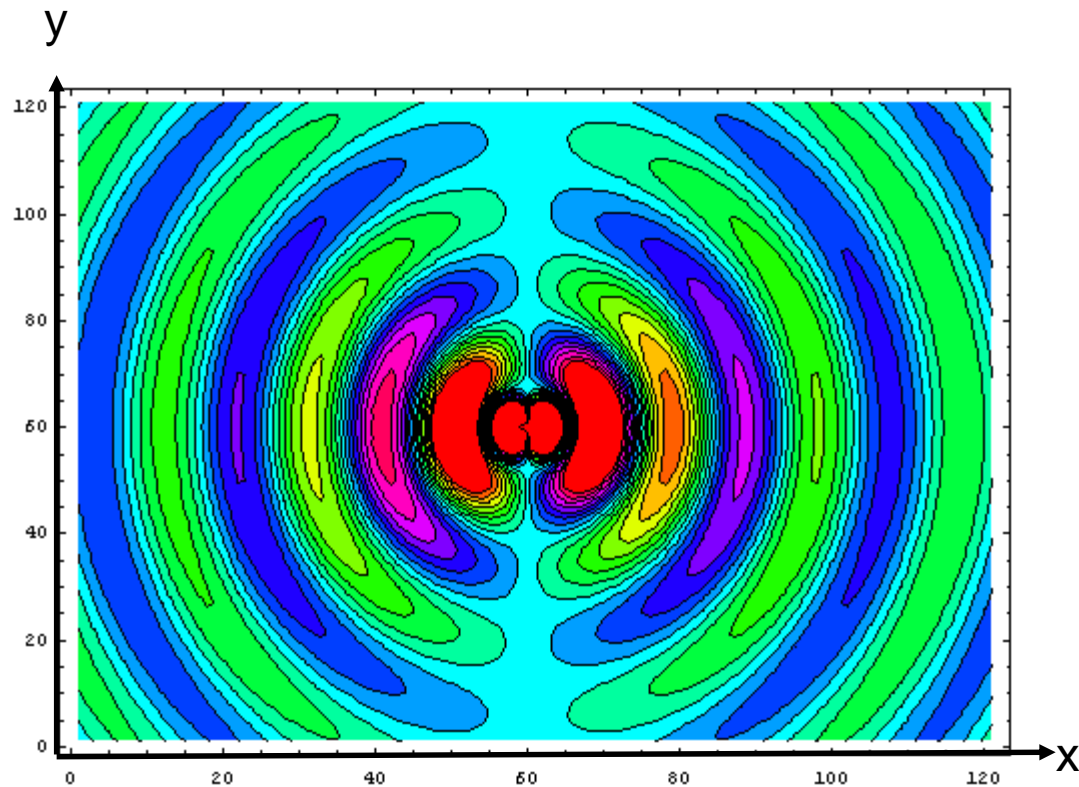


(e) $t = T$



Dipole Antenna

- Dipole oriented in +/- y-direction.
- Contours are B-field strength in z-direction (green/yellow and blue/red in opposite directions).
- Which direction is E-field?



At a certain point in space, the electric and magnetic fields of an electromagnetic wave at a certain instant are given by

$$\vec{E} = \hat{i} (6 \times 10^3 \text{ V/m})$$

$$\vec{B} = \hat{k} (2 \times 10^{-5} \text{ T})$$

At this point in space, the wave is propagating in the

- A. positive x -direction.
- B. negative x -direction.
- C. positive y -direction.
- D. negative y -direction.
- E. none of the above

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In a sinusoidal electromagnetic wave in a vacuum, the electric field has only an x -component. This component is given by $E_x = E_{\max} \cos (ky + \omega t)$.

The magnetic field of this wave

- A. has only an x -component.
- B. has only a y -component.
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- D. not enough information given to decide

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Electromagnetic Wave Equations (3D)

$$\nabla^2 B = \mu_0 \varepsilon_0 \frac{d^2 B}{dt^2} \qquad \nabla^2 E = \mu_0 \varepsilon_0 \frac{d^2 E}{dt^2}$$

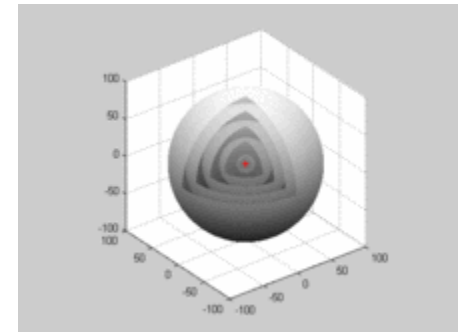
$$v = \frac{1}{\sqrt{(\mu_0 \varepsilon_0)}} = \frac{1}{\sqrt{(4\pi \times 10^{-7} \text{ H/m}) \times (8.854 \times 10^{-12} \text{ F/m})}} = 2.998 \times 10^8 \text{ m/s} = c$$

- This is a four-dimensional differential equation. It's solutions depend on the boundary conditions (in both time and space).
- For free-space (vacuum everywhere), and no angular dependence ($l=0$),

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

- the solutions look like:

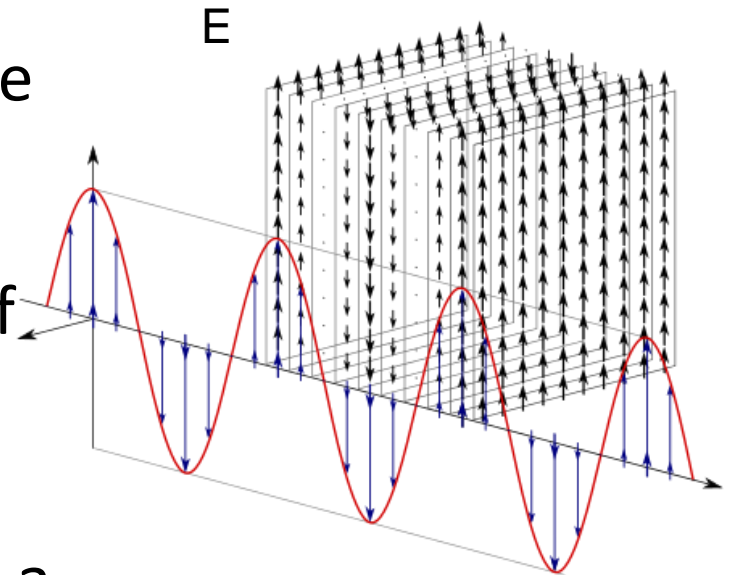
$$|\vec{B}(r,t)| = \frac{B_0}{r} \sin(kr - \omega t)$$



Can you confirm that this solution satisfies the wave equation?

Plane Waves

- Initially, we will limit our discussions to plane waves.
- Plane waves have the same value of the fields at all locations in space within a plane perpendicular to the direction of motion.
- How can that be?
- Think of the surface of a sphere, a large distance from the center compared to the distance on the surface you are sampling.



Electromagnetic Wave Equations (1D)

$$\nabla^2 B = \mu_0 \epsilon_0 \frac{d^2 B}{dt^2} \qquad \nabla^2 E = \mu_0 \epsilon_0 \frac{d^2 E}{dt^2}$$

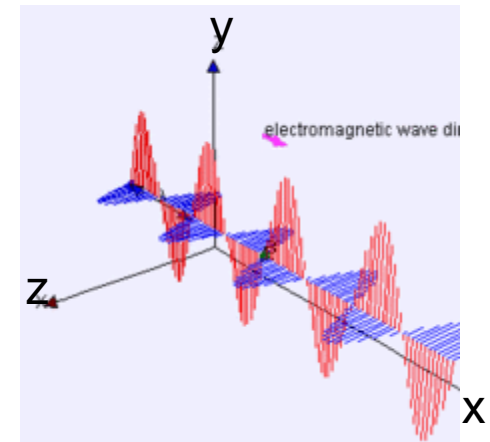
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- In the case of plane waves, we can restrict the solutions to the one-dimensional case.
- That is, ***moving*** in one dimension:

$$\vec{E}(x, t) = \hat{j} E_{\max} \sin(kx - \omega t + \phi_E)$$

$$\vec{B}(x, t) = \hat{k} B_{\max} \sin(kx - \omega t + \phi_B)$$

$$c = \frac{\omega}{k}, \quad k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \quad \phi_E = \phi_B$$



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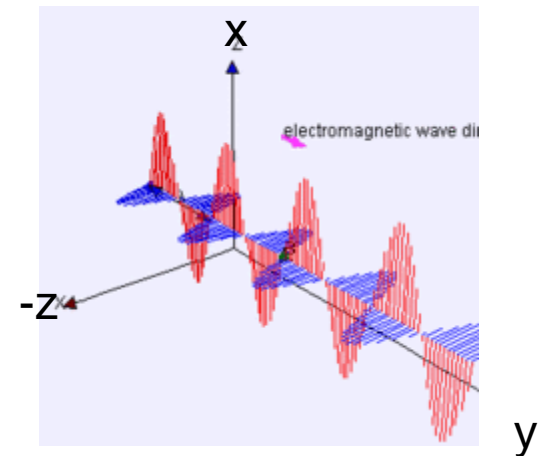
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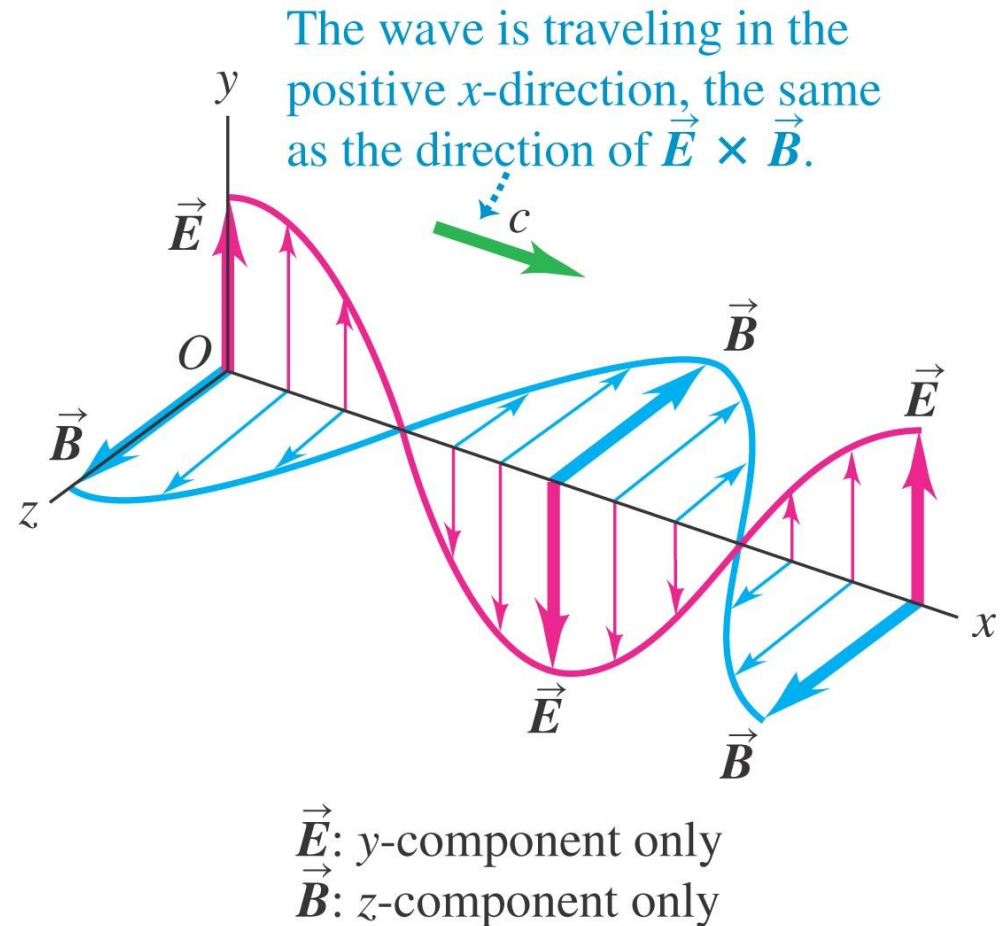
$$\vec{B}(x, t) = -\hat{k} B_{\max} \sin(ky - \omega t + \phi_B)$$

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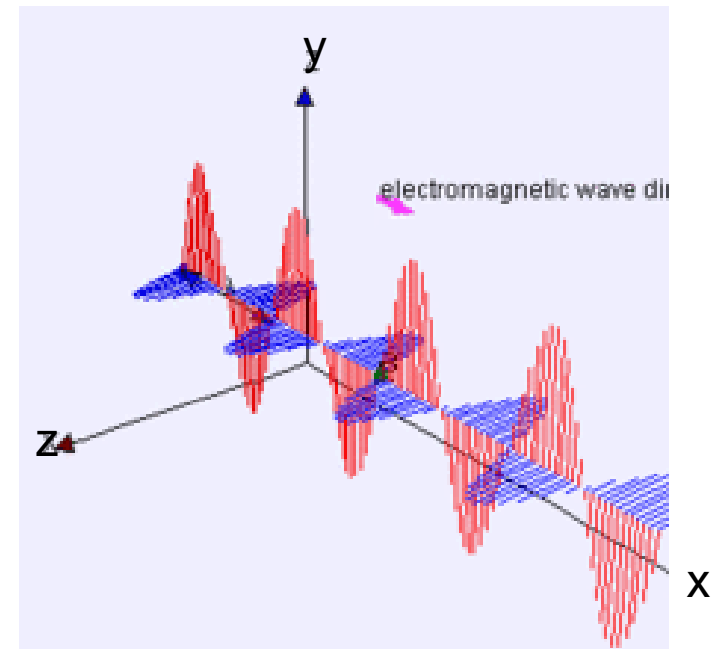
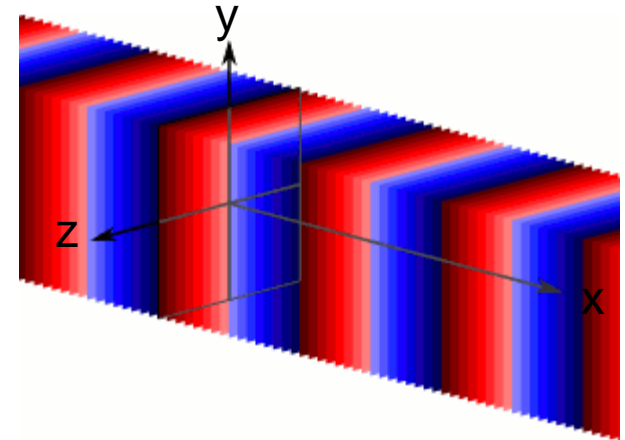
Confusion...

- Just like the electric field lines, the depiction of plane waves as in this figure can create conceptual confusion.



Plane Waves

- Don't forget that these planes move, so that at a specified point, the value of the field still oscillates.



Phase

- We can find the phase constant (same for E and B for plane waves) by identifying the position (x_1) of a maximum value of E along the x-axis at time $t=0$:

$$\vec{E}(x_1, 0) = \hat{j}E_{\max} \sin(kx_1 + \phi_E) = \hat{j}E_{\max}$$

or

$$\sin(kx_1 + \phi_E) = 1 \Rightarrow$$

$$kx_1 + \phi_E = \frac{\pi}{2}$$

Wavelength & Frequency

- Remember, in vacuum, the wave speed is fixed (speed of light = c), so that the wavelength and frequency both uniquely define the type of EM radiation.

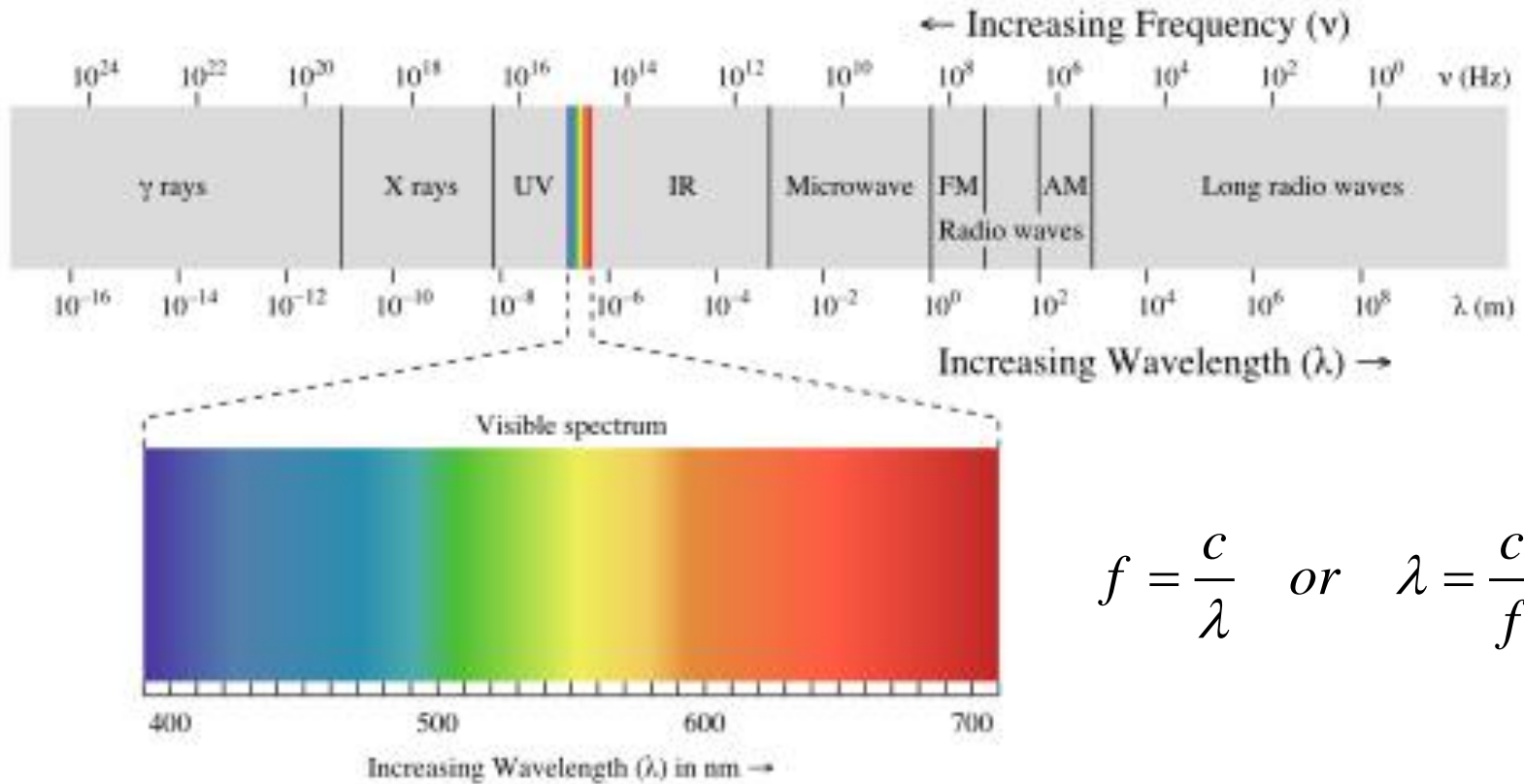
$$c = \frac{\omega}{k}, \quad k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f \Rightarrow$$

$$2\pi f = \frac{2\pi}{\lambda} c \Rightarrow$$

$$f = \frac{c}{\lambda} \quad \text{or} \quad \lambda = \frac{c}{f}$$

Electromagnetic Spectrum

- All are electromagnetic waves.



$$f = \frac{c}{\lambda} \quad \text{or} \quad \lambda = \frac{c}{f}$$

However...

- Remember from where the wave equations originate – Maxwell's equations.
- In matter (not vacuum, but no free charges), Maxwell's equations are modified:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

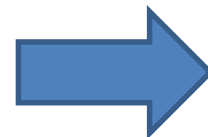
$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Relative permeability



$$\oint \vec{B} \cdot d\vec{l} = (\kappa_M \mu_0) (\kappa \epsilon_0) \frac{d\Phi_E}{dt}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Dielectric constant

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Speed of Light in Materials

- Then, because the only thing that has changed are these constants, we again get a wave equations from Maxwell's equations, but with a different wave speed:

$$\nabla^2 B = \mu\varepsilon \frac{d^2 B}{dt^2} \qquad \nabla^2 E = \mu\varepsilon \frac{d^2 E}{dt^2}$$

$$v = \frac{1}{\sqrt{(\mu\varepsilon)}} = \frac{1}{\sqrt{(\kappa\kappa_M)(\mu_0\varepsilon_0)}} = \frac{1}{\sqrt{(\kappa\kappa_M)}} c$$

Index of Refraction

- The index of refraction of a material is defined as:

$$v = \frac{1}{\sqrt{(\mu\varepsilon)}} = \frac{1}{\sqrt{(\kappa\kappa_M)(\mu_0\varepsilon_0)}} = \frac{1}{\sqrt{(\kappa\kappa_M)}} c \equiv \frac{c}{n} \Rightarrow$$

$$n = \sqrt{(\kappa\kappa_M)}$$

- So, the index of refraction (always > 1) has the effect of slowing the propagation of light in a material. This will have many consequences, as we will see later.